Appendix

Proposition 1. Social welfare is higher when the upstream technologies are in the public domain, relative to when they are granted patents, whenever SW^{public} is larger than SW^{ip} (where IP denotes "intellectual property") - that is:

$$\pi \cdot N(\pi \cdot \delta \cdot \Pi^m) \cdot [\delta \cdot SW^m + (1-\delta) \cdot SW^c] > \pi \cdot N(\pi \cdot \lambda \cdot \Pi^m) \cdot SW^m \tag{10}$$

There exists a value of λ, denoted λ^*, such that social welfare from the upstream technologies being in the public domain is the same as social welfare from the upstream technologies being granted patents. For values $\lambda < \lambda^*$, social welfare is strictly higher when the upstream technologies are in the public domain relative to when they are granted patents. For values $\lambda > \lambda^*$, social welfare is strictly higher when the upstream technologies are granted patents relative to when they are in the public domain. The value of λ^* is greater than δ, and is increasing in δ if and only if the elasticity ε of the supply of innovators with respect to the level of expected returns (that is, $\frac{x \cdot N'(x)}{N(x)}$) is such that

$$\varepsilon > \frac{\frac{SW^c}{SW^m} - 1}{1 + \frac{1-\delta}{\delta} \cdot \frac{SW^c}{SW^m}} \tag{11}$$

Proof of Proposition 1. The function $N(\cdot)$ is strictly increasing, which implies that social welfare when the upstream technologies have been granted patents is strictly increasing in λ.

At $\lambda = 0$, social welfare with patents is $\pi \cdot N(0) \cdot SW^m$. Comparing this level of social welfare with SW^{public}, we see that since $N(0)$ is strictly less than $N(\pi\delta\Pi^m)$ and SW^m is weakly less than $\delta \cdot SW^m + (1-\delta) \cdot SW^c$, at $\lambda = 0$ granting patents on the upstream technologies is strictly worse than the upstream technologies being in the public domain.

At $\lambda = 1$, social welfare with patents is $\pi \cdot N(\pi \cdot \Pi^m) \cdot SW^m$. Comparing this level of social welfare with SW^{public}, we see that in this case social welfare with patents is greater than SW^{public} if and only if the following condition holds:

$$\frac{N(\pi\Pi^m)}{N(\pi\delta\Pi^m)} > \delta + (1-\delta) \cdot \frac{SW^c}{SW^m} \tag{12}$$

If Equation (12) holds at $\lambda = 1$, then granting patents on the upstream technologies is preferable to the upstream technologies in the public domain.

At $\lambda = 1$, Equation (12) implies that granting patents on the upstream technologies is strictly worse than the upstream technologies being in the public domain as long as $\frac{N(\pi\Pi^m)}{N(\pi\delta\Pi^m)} < \delta + (1-\delta) \cdot \frac{SW^c}{SW^m}$. Since social welfare when the upstream technologies have been granted IP is strictly increasing in λ, this implies that for all values of lambda, granting patents on the upstream technologies is strictly worse than the upstream technologies being in the public domain as long as $\frac{N(\pi\Pi^m)}{N(\pi\delta\Pi^m)} < \delta + (1-\delta) \cdot \frac{SW^c}{SW^m}$.

Since social welfare when the upstream technologies have been granted patents is strictly lower than when the upstream technologies are in the public domain at $\lambda = 0$, strictly higher at $\lambda = 1$ when Equation (12) holds, and monotonically increasing in λ, the continuity of $N(\cdot)$ implies that when Equation (12) holds there exists a value of λ at which social welfare is the same when the upstream technologies have patents as when they are in the public domain.

The value of λ at which social welfare is equal across these two cases, denoted λ^*, is implicitly defined by the following equation:

$$\pi \cdot N(\pi \cdot \delta \cdot \Pi^m) \cdot [\delta \cdot SW^m + (1-\delta) \cdot SW^c] = \pi \cdot N(\pi \cdot \lambda^* \cdot \Pi^m) \cdot SW^m \tag{13}$$

Which we can re-write as:

$$N(\pi \cdot \lambda^* \cdot \Pi^m) = N(\pi \cdot \delta \cdot \Pi^m)\left[\delta + (1-\delta) \cdot \frac{SW^c}{SW^m}\right] \tag{14}$$

Note that the bracketed term, $\delta + (1-\delta) \cdot \frac{SW^c}{SW^m}$, is greater than one, implying $N(\pi \cdot \delta \cdot \Pi^m) < N(\pi \cdot \lambda^* \cdot \Pi^m)$, implying $\delta < \lambda^*$.

Differentiating both sides of Equation (14) with respect to δ gives the following:

$$N'(\pi \cdot \lambda^* \cdot \Pi^m) \cdot \frac{d\lambda^*}{d\delta} \cdot \pi \cdot \Pi^m = N'(\pi \cdot \delta \cdot \Pi^m)) \cdot \pi \cdot \Pi^m \cdot \left[\delta + (1-\delta) \cdot \frac{SW^c}{SW^m}\right] + N(\pi \cdot \delta \cdot \Pi^m) \cdot \left[1 - \frac{SW^c}{SW^m}\right] \tag{15}$$

Which can be re-written as:

$$\frac{d\lambda^*}{d\delta} = \frac{N'(\pi \cdot \delta \cdot \Pi^m) \cdot \pi \cdot \Pi^m \cdot \left[\delta + (1-\delta) \cdot \frac{SW^c}{SW^m}\right] + N(\pi \cdot \delta \cdot \Pi^m) \cdot \left[1 - \frac{SW^c}{SW^m}\right]}{N'(\pi \cdot \lambda^* \cdot \Pi^m) \cdot \pi \cdot \Pi^m} \tag{16}$$

The denominator on the right hand side of Equation (15) is always positive (since $N(\cdot)$ is strictly increasing). Thus, $\frac{d\lambda^*}{d\delta}$ is positive if and only if the numerator on the right hand side of Equation (15) is positive. That is, $\frac{d\lambda^*}{d\delta}$ is positive if and only if the following condition holds:

$$N'(\pi \cdot \delta \cdot \Pi^m)) \cdot \pi \cdot \Pi^m \cdot \left[\delta + (1-\delta) \cdot \frac{SW^c}{SW^m}\right] + N(\pi \cdot \delta \cdot \Pi^m) \cdot \left[1 - \frac{SW^c}{SW^m}\right] > 0 \tag{17}$$

Define ε to be the following, representing the elasticity of the supply of innovators with respect to the level of expected returns:

$$\varepsilon = \frac{N'(\pi \cdot \delta \cdot \Pi^m) \cdot \pi \cdot \delta \cdot \Pi^m}{N(\pi \cdot \delta \cdot \Pi^m)} \tag{18}$$

We can then re-write Equation (17) in terms of ε as follows:

$$\varepsilon + \varepsilon \cdot \left[\frac{1-\delta}{\delta} \cdot \frac{SW^c}{SW^m}\right] + [1 - \frac{SW^c}{SW^m}] > 0 \tag{19}$$

Equation (17) holds if and only if the following condition holds:

$$\varepsilon > \frac{\frac{SW^c}{SW^m} - 1}{1 + \frac{1-\delta}{\delta} \cdot \frac{SW^c}{SW^m}} \tag{20}$$

Q.E.D.