Appendix 3: Additional details of the random walk model

A. Data structure

| 7 11 2 4 14 2 4 14 14 14 14 14 | | |
|--------------------------------|----------|-------------------------|
| Study | No. Arms | Time points (in months) |
| Bjorgul (2013) | 3 | 24, 60, 84 |
| D'Antonio (2002) | 2 | 33 |
| Digas (2003) | 2 | 24, 36, 60, 120 |
| Howie (2005) | 2 | 60, 100 |
| Morison (2014) | 4 | 60 |
| Hamilton (2010) | 2 | 30 |
| Kadar (2011) | 4 | 12, 24, 60 |
| Ochs (2007) | 2 | 3,101 |
| Pabinger (2003) | 2 | 7, 24 |
| Bal (2005) | 2 | 24 |
| Bascarevic (2010) | 2 | 50 |
| Beaupre (2013) | 2 | 12, 60, 120 |
| Desmarchelier (2013) | 2 | 60, 98 |
| Jassim (2015) | 3 | 60 |
| Girard (2006) | 2 | 54, 93 |
| | | |

B. Modelling details

We used a binomial likelihood and a logistic link function to model the number of revisions in the j^{th} observed time period of the k^{th} arm or the i^{th} study:

$$r_{ikj} \sim Binomial(p_{ikj}, n_{ikj})$$

 p_{ikj} is the probability of revision; and n_{ikj} is the number of hips at risk at the start of observation period j.

The probability of revision p_{ikj} is defined by the exposure of the hips in each of the two time periods:

$$p_{ikj} = 1 - e^{-\theta_{ikj}}$$

As the time period for observation j may not match the time periods of the model (0-2 and 2-10 years), we split the contribution to the total hazard between the model time periods:

$$\theta_{ikj} = \lambda_{ik}^{0-2} E_{ikj}^{0-2} + \lambda_{ik}^{2-10} E_{ikj}^{2-10},$$

where E_{ikj}^{0-2} is exposure of observation j in 0-2 years; E_{ikj}^{2-10} is exposure of observation j in 2-10 years; and λ_{ik}^{0-2} and λ_{ik}^{2-10} are the hazards in these two time periods.

Log hazard ratios (treatment effects) in x=0-2 and x=2-10 years in study i and arm k were modelled as

$$\log(\lambda_{ik}^x) = \mu_{it} + d_{t_{ik}}^x.$$

The random walk model assumes treatment effects for implant t in period 2-10 were assumed normally distributed around effects in 0-2 years with between time period variance σ_d shared across implants:

$$d_t^{2-10} \sim N(d_t^{0-2}, \sigma_d)$$

More complex structural assumptions would not be identifiable as there are only two time periods. Vague prior distributions were assumed for treatment effects in period 0-2 years:

$$d_t^{0-2} \sim N(0, \sigma = \sqrt{1000})$$

We also ran several sensitivity analyses where we modelled the treatment effects as

$$\log(\lambda_{ik}^{x}) = \mu_{it} + d_{t_{ik}}^{0-2} + \varphi_{t_{k}}^{x}$$

where $arphi_{t_k}^{0-2}=0$, and $\mbox{we explored three different parameterisations for }arphi_{t_k}^{2-10}$, namely

 $\varphi_{t_k}^{2-10}=0$, $\varphi_{t_k}^{2-10}=f$ and $\varphi_{t_k}^{2-10}\sim N\!\left(f,\sigma_{\varphi}\right)$, with vague priors $f\sim N\!\left(0,\sqrt{1000}\right)$ and $\sigma_{\varphi}\sim U\!\left(0,5\right)$. The results showed the same trends but with greater precision, although the heterogeneity increased and DIC statistics supported our base case model.