

Supplementary Note 1: Polarization States of Waves in Anisotropic Media

The wave equation in a homogeneous non-magnetic anisotropic medium is¹

$$\nabla^2 \mathbf{E} + k_0^2 \|\varepsilon\| \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}), \quad (1)$$

where $\|\varepsilon\|$ is the relative dielectric tensor. In the coordinate system aligned to the optic axis of a uniaxial anisotropic crystal, $\|\varepsilon\|$ can be expressed as

$$\|\varepsilon\| = \begin{bmatrix} \varepsilon_{\perp} & 0 & 0 \\ 0 & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\parallel} \end{bmatrix}, \quad (2)$$

where ε_{\perp} is the relative dielectric constant perpendicular to the optic axis, and ε_{\parallel} is the relative dielectric constant parallel to the optic axis.

Supplementary Equation (1) actually contains three equations, one for each dimension. Explicitly, these equations can be expressed in the matrix form as

$$\begin{bmatrix} k_y^2 + k_z^2 - \varepsilon_{\perp} k_0^2 & -k_x k_y & -k_x k_z \\ -k_y k_x & k_x^2 + k_z^2 - \varepsilon_{\perp} k_0^2 & -k_y k_z \\ -k_z k_x & -k_z k_y & k_x^2 + k_y^2 - \varepsilon_{\parallel} k_0^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0 \quad (3)$$

where $\mathbf{k} = (k_x, k_y, k_z)$ is the wavevector and $\mathbf{E} = (E_x, E_y, E_z)$ is the electric field. For a homogeneous wave in the Y direction, we have $k_y = 0$, then Supplementary Equation (3) can be reduced to

$$\begin{bmatrix} k_z^2 - \varepsilon_{\perp} k_0^2 & 0 & -k_x k_z \\ 0 & k_x^2 + k_z^2 - \varepsilon_{\perp} k_0^2 & 0 \\ -k_z k_x & 0 & k_x^2 - \varepsilon_{\parallel} k_0^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0. \quad (4)$$

For Supplementary Equation (4) to have a non-trivial solution (i.e. non-zero electric field), the determinant of the matrix must be zero:

$$\begin{vmatrix} k_z^2 - \varepsilon_{\perp} k_0^2 & 0 & -k_x k_z \\ 0 & k_x^2 + k_z^2 - \varepsilon_{\perp} k_0^2 & 0 \\ -k_z k_x & 0 & k_x^2 - \varepsilon_{\parallel} k_0^2 \end{vmatrix} = 0. \quad (5)$$

Solving Supplementary Equation (5) we get two sets of solutions:

$$k_{z1} = \sqrt{\varepsilon_{\perp} k_0^2 - k_x^2}, \quad \mathbf{E}_{\text{TE}} = \begin{bmatrix} 0 \\ E_y \\ 0 \end{bmatrix} \quad (6)$$

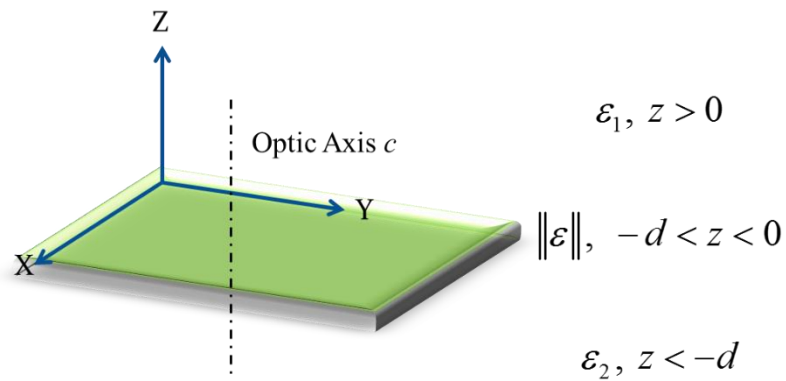
corresponding to the transverse electric (TE) polarized ordinary wave, and

$$k_{z2} = \sqrt{\frac{\varepsilon_{\perp}}{\varepsilon_{\parallel}} \sqrt{\varepsilon_{\parallel} k_0^2 - k_x^2}}, \quad \mathbf{E}_{\text{TM}} = \begin{bmatrix} E_x \\ 0 \\ E_z \end{bmatrix}. \quad (7)$$

corresponding to the transverse magnetic (TM) polarized extraordinary wave.

Supplementary Note 2: Eigenequations for Anisotropic Planar Waveguide

A three layer planar waveguide composed of an isotropic semi-infinite superstrate (ε_1), an anisotropic guiding layer ($\|\varepsilon\|$) with the thickness d , and an isotropic semi-infinite substrate (ε_2) is shown in Supplementary Fig. 1. The optic axis of the guiding layer is parallel to the Z axis.



Supplementary Figure 1 | Schematic of a three-layer planar waveguide. The superstrate and substrate are both semi-infinite and optically isotropic. The optic axis of the anisotropic guiding layer is perpendicular to the basal plane.

For a waveguide mode (ordinary or extraordinary) propagating in the X direction in the three-layer planar waveguide shown in Supplementary Fig. 1, the electric field can be expressed as

$$\begin{cases} \mathbf{E}_1(\mathbf{r}, t) = \mathbf{E}_1^0 e^{-\alpha_1 z} e^{i(\beta x - \omega t)} = (E_{1x}^0, E_{1y}^0, E_{1z}^0) e^{-\alpha_1 z} e^{i(\beta x - \omega t)} \\ \mathbf{E}_g(\mathbf{r}, t) = \mathbf{E}_g^0 \cos(\alpha_g z + \varphi) e^{i(\beta x - \omega t)} = (E_{gx}^0, E_{gy}^0, E_{gz}^0) \cos(\alpha_g z + \varphi) e^{i(\beta x - \omega t)} \\ \mathbf{E}_2(\mathbf{r}, t) = \mathbf{E}_2^0 e^{\alpha_2(z+d)} e^{i(\beta x - \omega t)} = (E_{2x}^0, E_{2y}^0, E_{2z}^0) e^{\alpha_2(z+d)} e^{i(\beta x - \omega t)} \end{cases} \quad (8)$$

where $\beta = k_x$ is the propagation constant of the waveguide mode (not to be confused with the angle in Figure 1 in the main text), $\alpha_g = k_z$.

For the electric fields in the isotropic superstrate and substrate, Supplementary Equation (1) can be reduced to

$$\nabla^2 \mathbf{E} + k_0^2 \varepsilon_{1,2} \mathbf{E} = 0. \quad (9)$$

Substituting \mathbf{E}_1 and \mathbf{E}_2 into Supplementary Equation (9), we get

$$\begin{cases} \alpha_1^2 = \beta^2 - k_0^2 \varepsilon_1 \\ \alpha_2^2 = \beta^2 - k_0^2 \varepsilon_2 \end{cases}. \quad (10)$$

Imposing Gauss's law $\nabla \cdot (\varepsilon_{1,2} \mathbf{E}) = 0$ on \mathbf{E}_1 and \mathbf{E}_2 , we get

$$\begin{cases} \mathbf{E}_1(\mathbf{r}, t) = (E_{1x}^0, E_{1y}^0, \frac{i\beta}{\alpha_1} E_{1x}^0) e^{-\alpha_1 z} e^{i(\beta x - \omega t)} \\ \mathbf{E}_2(\mathbf{r}, t) = (E_{2x}^0, E_{2y}^0, \frac{-i\beta}{\alpha_2} E_{2x}^0) e^{\alpha_2(z+d)} e^{i(\beta x - \omega t)} \end{cases}. \quad (11)$$

Imposing the Gauss's law $\nabla \cdot (\|\varepsilon\| \mathbf{E}) = 0$ on \mathbf{E}_g , we get

$$\mathbf{E}_g(\mathbf{r}, t) = \mathbf{E}_g^0 \cos(\alpha_g z + \varphi) e^{i(\beta x - \omega t)} = \left[\frac{\varepsilon_{\parallel} \alpha_g \sin(\alpha_g z + \varphi)}{\varepsilon_{\perp} i \beta \cos(\alpha_g z + \varphi)} E_{gz}^0, E_{gy}^0, E_{gz}^0 \right] \cos(\alpha_g z + \varphi) e^{i(\beta x - \omega t)}. \quad (12)$$

Using the Faraday's law $\nabla \times \mathbf{E} = i\omega \mu_0 \mathbf{H}$, we can get magnetic fields from Supplementary Equation (11) and (12)

$$\begin{cases} \mathbf{H}_1 = \frac{-i}{\omega\mu_0} (\alpha_1 E_{1y}^0, \frac{k_0^2 \varepsilon_1}{\alpha_1} E_{1x}^0, i\beta E_{1y}^0) e^{-\alpha_1 z} e^{i(\beta x - \omega t)} \\ \mathbf{H}_g = \frac{-i}{\omega\mu_0} [\alpha_g \sin(\alpha_g z + \varphi) E_{gy}^0, -i\beta \cos(\alpha_g z + \varphi) E_{gz}^0 - \alpha_g \sin(\alpha_g z + \varphi) E_{gx}^0, i\beta \cos(\alpha_g z + \varphi) E_{gy}^0] e^{i(\beta x - \omega t)} \\ \mathbf{H}_2 = \frac{-i}{\omega\mu_0} (-\alpha_2 E_{2y}^0, -\frac{k_0^2 \varepsilon_2}{\alpha_2} E_{2x}^0, i\beta E_{2y}^0) e^{\alpha_2(z+d)} e^{i(\beta x - \omega t)} \end{cases} \quad (13)$$

For a TE polarized ordinary waveguide mode, there are three field components (H_x, E_y, H_z).

Imposing the interface conditions on \mathbf{H}_x and \mathbf{E}_y at the two interfaces ($z=0, z=-d$), we get

$$\begin{cases} \alpha_1 E_{1y}^0 = \alpha_g \sin \varphi E_{gy}^0 \\ \alpha_g \sin(-\alpha_g d + \varphi) E_{gy}^0 = -\alpha_2 E_{2y}^0 \\ E_{1y}^0 = \cos \varphi E_{gy}^0 \\ \cos(-\alpha_g d + \varphi) E_{gy}^0 = E_{2y}^0 \end{cases}, \quad (14)$$

which can be reformulated into

$$\alpha_g d = \tan^{-1} \left(\frac{\alpha_1}{\alpha_g} \right) + \tan^{-1} \left(\frac{\alpha_2}{\alpha_g} \right) + m\pi \quad (\alpha_g = \sqrt{\varepsilon_{\perp} k_0^2 - \beta^2}). \quad (15)$$

Substituting α_1 , α_2 and α_g into Supplementary Equation (15) and let $\beta_o = \beta$, we can get the eigenvalue equation for TE polarized ordinary waveguide modes

$$\sqrt{\varepsilon_{\perp} k_0^2 - \beta_o^2} d = \tan^{-1} \left(\frac{\sqrt{\beta_o^2 - k_0^2 \varepsilon_1}}{\sqrt{\varepsilon_{\perp} k_0^2 - \beta_o^2}} \right) + \tan^{-1} \left(\frac{\sqrt{\beta_o^2 - k_0^2 \varepsilon_2}}{\sqrt{\varepsilon_{\perp} k_0^2 - \beta_o^2}} \right) + m\pi. \quad (16)$$

Similarly, for the TM polarized extraordinary waveguide mode (E_x, H_y, E_z), we have

$$\sqrt{\frac{\varepsilon_{\perp}}{\varepsilon_{\parallel}}} \sqrt{\varepsilon_{\parallel} k_0^2 - \beta_e^2} d = \tan^{-1} \left(\frac{\sqrt{\beta_e^2 - k_0^2 \varepsilon_1 \varepsilon_{\perp}}}{\sqrt{\frac{\varepsilon_{\perp}}{\varepsilon_{\parallel}}} \sqrt{\varepsilon_{\parallel} k_0^2 - \beta_e^2} \varepsilon_1} \right) + \tan^{-1} \left(\frac{\sqrt{\beta_e^2 - k_0^2 \varepsilon_2 \varepsilon_{\perp}}}{\sqrt{\frac{\varepsilon_{\perp}}{\varepsilon_{\parallel}}} \sqrt{\varepsilon_{\parallel} k_0^2 - \beta_e^2} \varepsilon_2} \right) + n\pi, \quad (17)$$

where $\beta_e = \beta$. In Supplementary Equation (16) and (17) m and n are non-negative integers.

Supplementary Note 3: Mode Profiles Normalization and Coupling Factors Calculation

According to Poynting's theorem, the power density in the X direction carried by the ordinary (TE) and extraordinary (TM) waveguide modes can be expressed as

$$P_o = \frac{1}{2} E_y(z) H_z(z) = \frac{\beta_o}{2\omega\mu_0} \int_{-\infty}^{+\infty} E_y^2(z) dz \quad (18)$$

and

$$P_e = -\frac{1}{2} E_z(z) H_y(z) = \frac{k_0^2}{2\omega\mu_0\beta_e} \left[\int_{-\infty}^{-d} \varepsilon_2 E_{2z}^2(z) dz + \int_{-d}^0 \varepsilon_{\parallel} E_{gz}^2(z) dz + \int_0^{+\infty} \varepsilon_1 E_{1z}^2(z) dz \right], \quad (19)$$

respectively. Thus, the ordinary and extraordinary mode profiles can be normalized as

$$\bar{E}_y(z) = \frac{E_y(z)}{\sqrt{P_o}} \quad (20)$$

and

$$\bar{E}_z(z) = \frac{E_z(z)}{\sqrt{P_e}}, \quad (21)$$

respectively. Of course, Supplementary Equation (20) and (21) can be further normalized by dividing the maximum of $\bar{E}_y(z)$ and $\bar{E}_z(z)$ — $\max\{\bar{E}_y(z), \bar{E}_z(z)\}$.

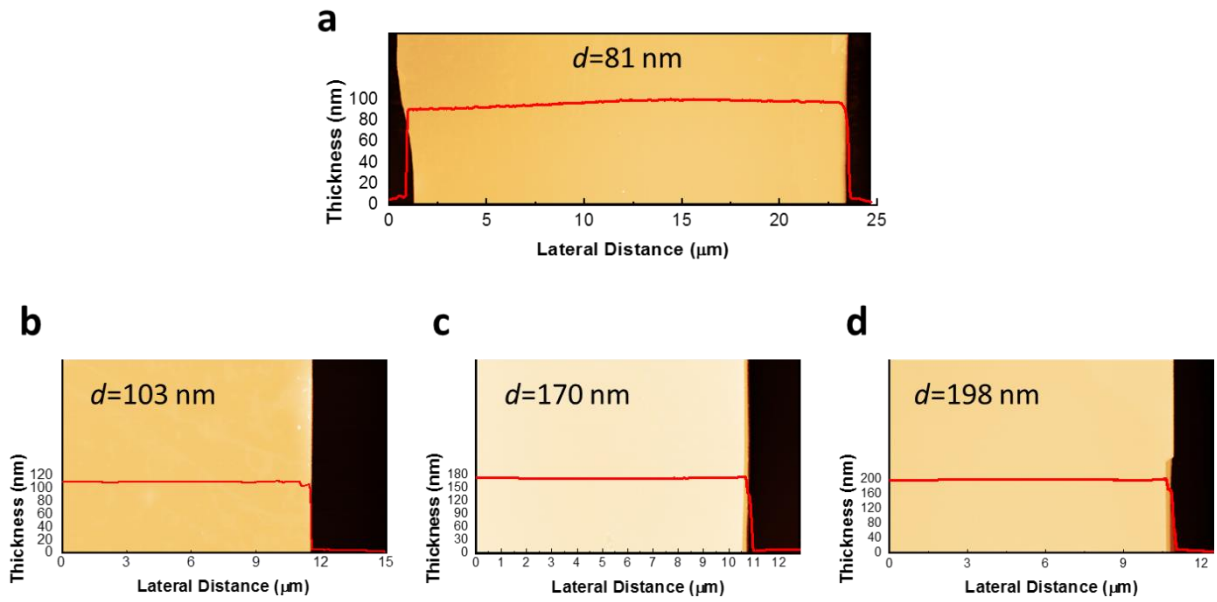
If we take the interval $0 \text{ nm} \leq z \leq 100 \text{ nm}$ to be the efficient coupling region of the waveguide modes and the s-SNOM tip-induced hot spot, the coupling factors can be defined as

$$CF_o = \frac{\int_0^{100} \bar{E}_y(z) dz}{\int_{-\infty}^{+\infty} \bar{E}_y(z) dz} \quad (22)$$

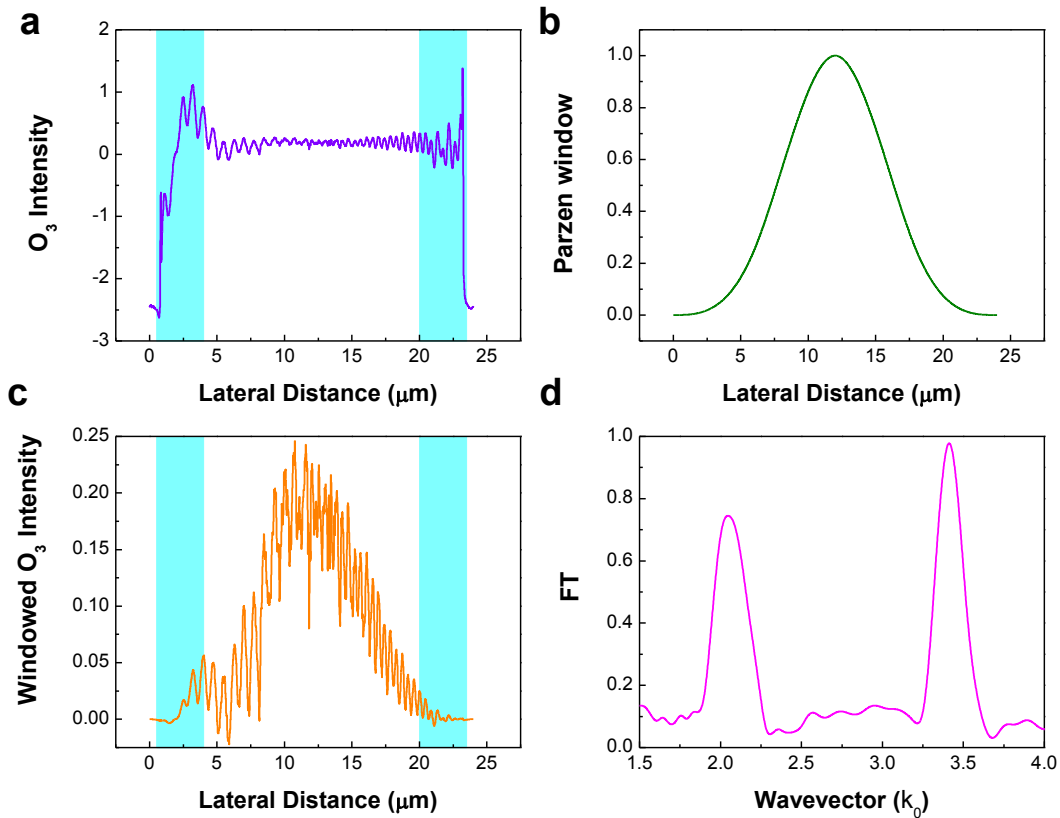
and

$$CF_e = \frac{\int_0^{100} \bar{E}_z(z) dz}{\int_{-\infty}^{+\infty} \bar{E}_z(z) dz} \quad (23)$$

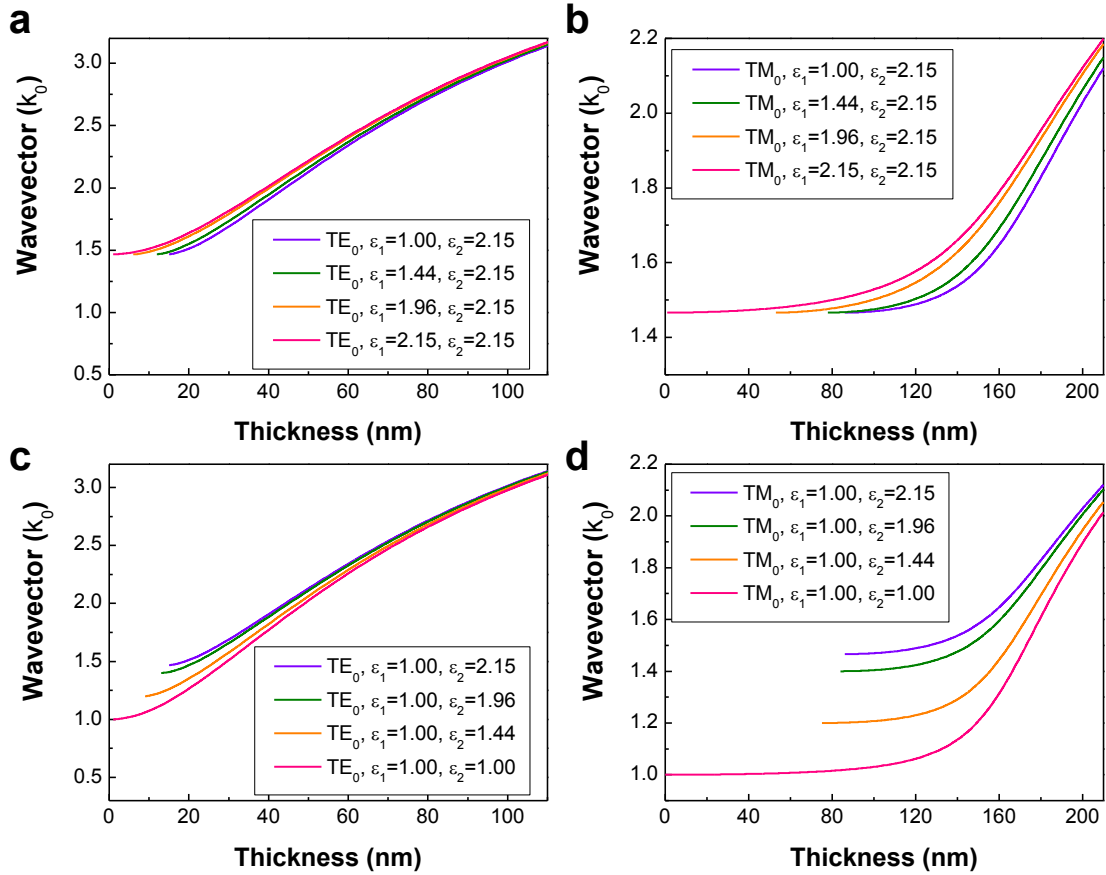
for the ordinary and extraordinary modes, respectively.



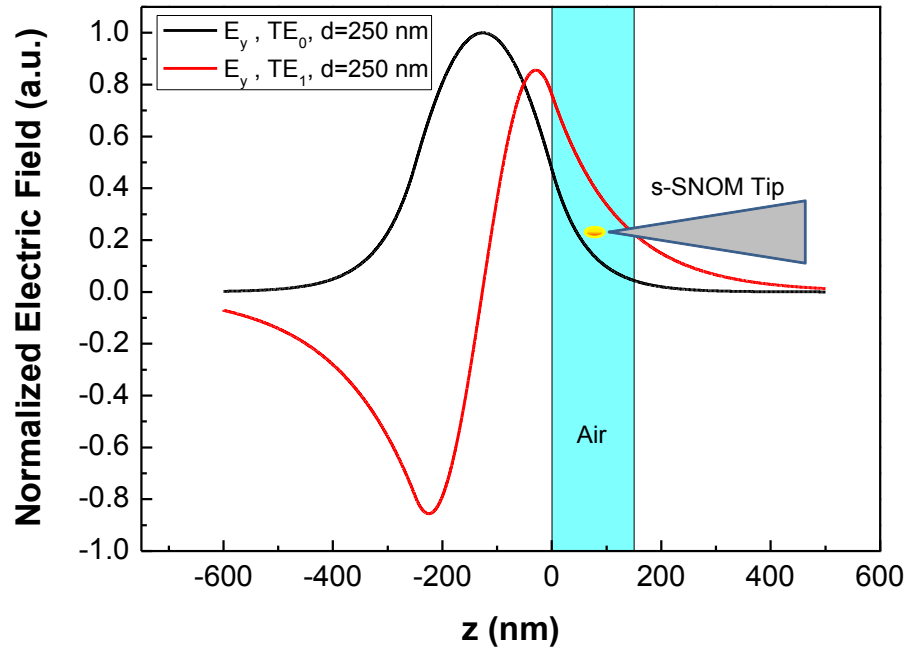
Supplementary Figure 2 | AFM images and height profiles for MoS₂ samples of different thicknesses. (a) $d=81$ nm; (b) $d=103$ nm; (c) $d=170$ nm; (d) $d=198$ nm.



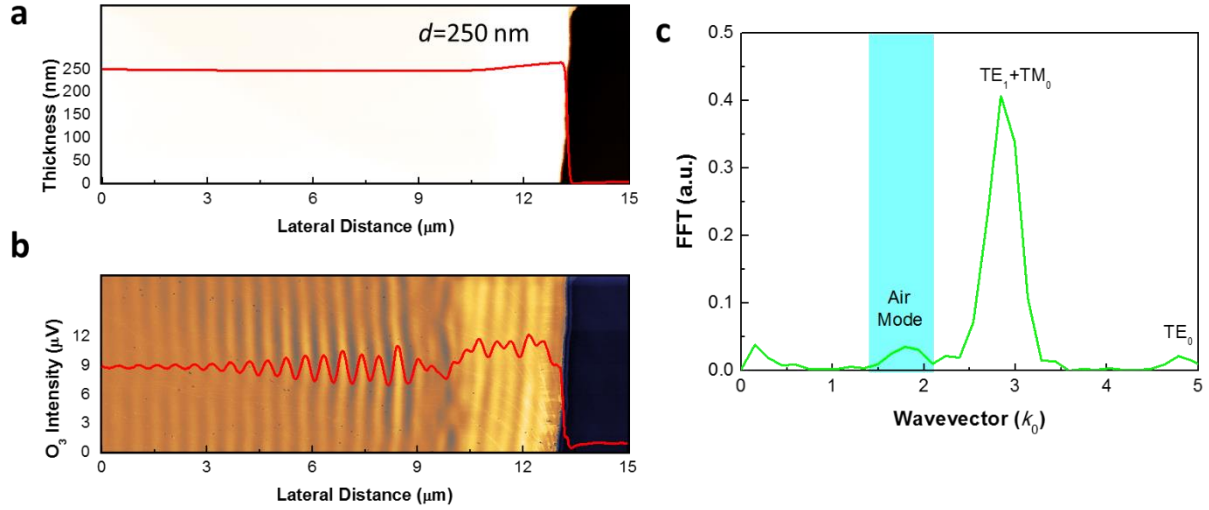
Supplementary Figure 3 | The schematic diagram of using a window function to suppress the edge effect in Fourier transform. (a) Subtract the average value of the whole profile from the original data to suppress the DC component in the momentum-space spectra, the shadowed areas indicate where the edge effect exists, widths of the shadows are 3 μm ; **(b)** The Parzen window function used to suppress the edge effect; **(c)** The windowed optical profile obtained by multiplying **a** with **b**, the shadowed areas indicate that the edge effect has been effectively suppressed; **(d)** The Fourier transform of **c**, reflecting mainly the spatial frequencies in the center areas of the near-field images.



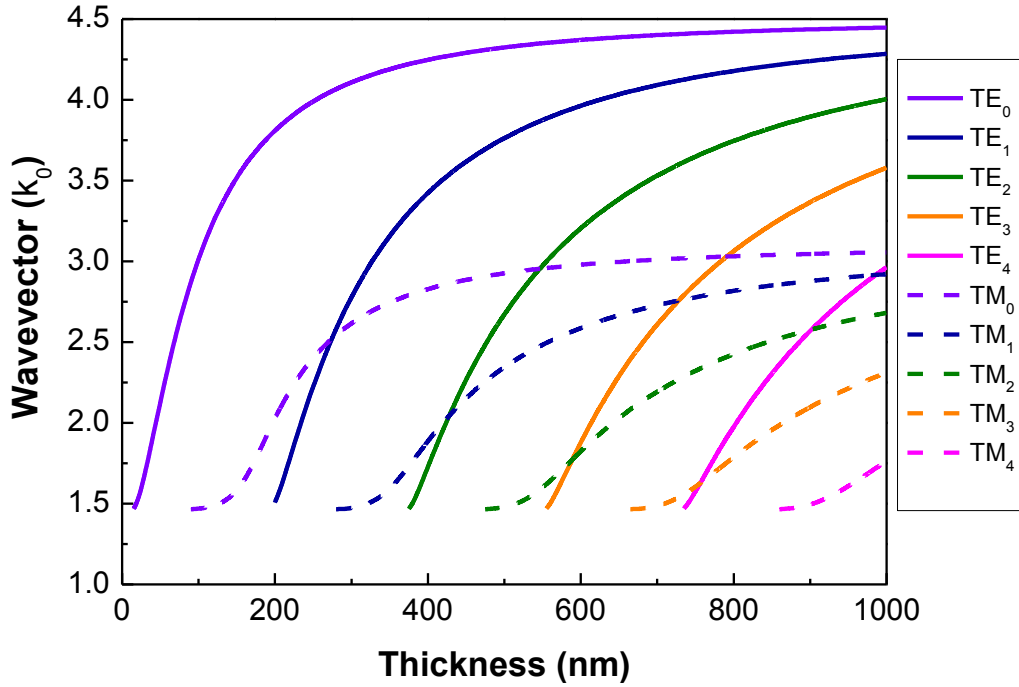
Supplementary Figure 4 | Thickness dispersions of MoS₂ waveguide with different degrees of symmetry. (a) and (b) Thickness dispersions of the TE₀ and TM₀ mode with increasing superstrate dielectric constant, respectively; (c) and (d) Thickness dispersions of the TE₀ and TM₀ mode with decreasing substrate dielectric constant, respectively. The cut-off thickness decreases with the increasing degree of symmetry.



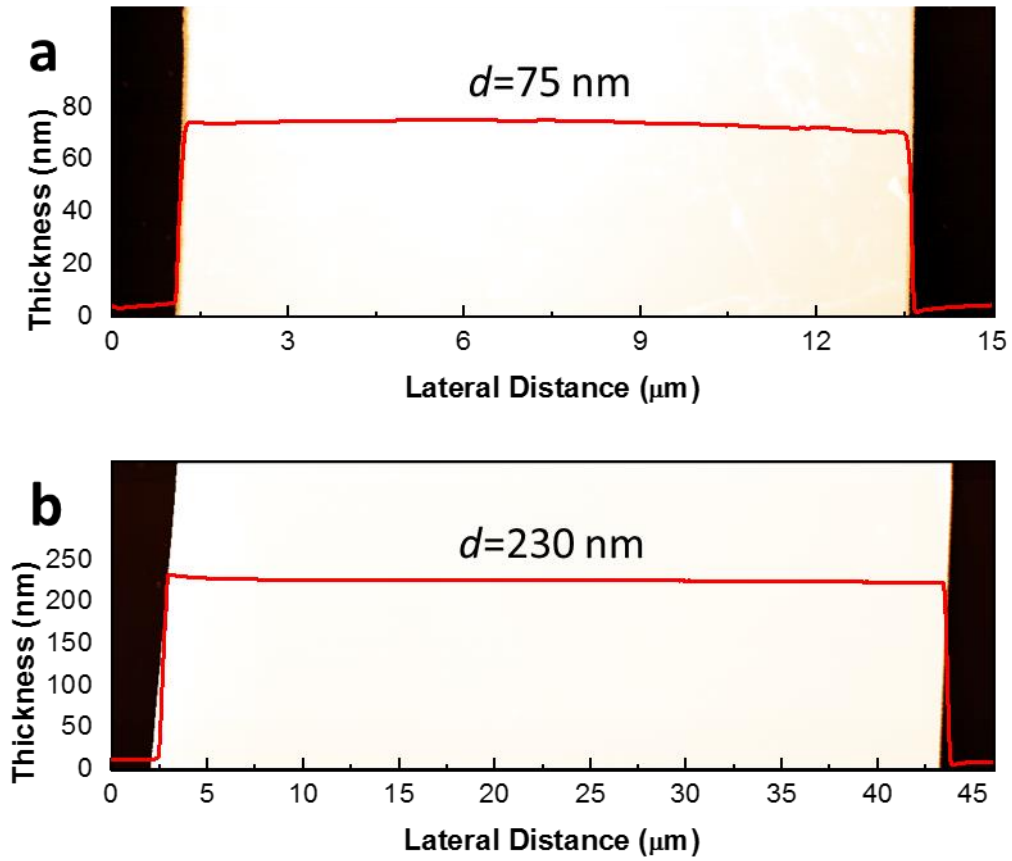
Supplementary Figure 5 | Electric field profiles of the fundamental and the first order TE waveguide modes. The high order modes possess smaller in-plane wavevectors compared with the fundamental modes, thus their evanescent fields extend much further out of the sample surface, leading to high excitation efficiencies of these high order waveguide modes and the enhanced interference visibilities in the s-SNOM images.



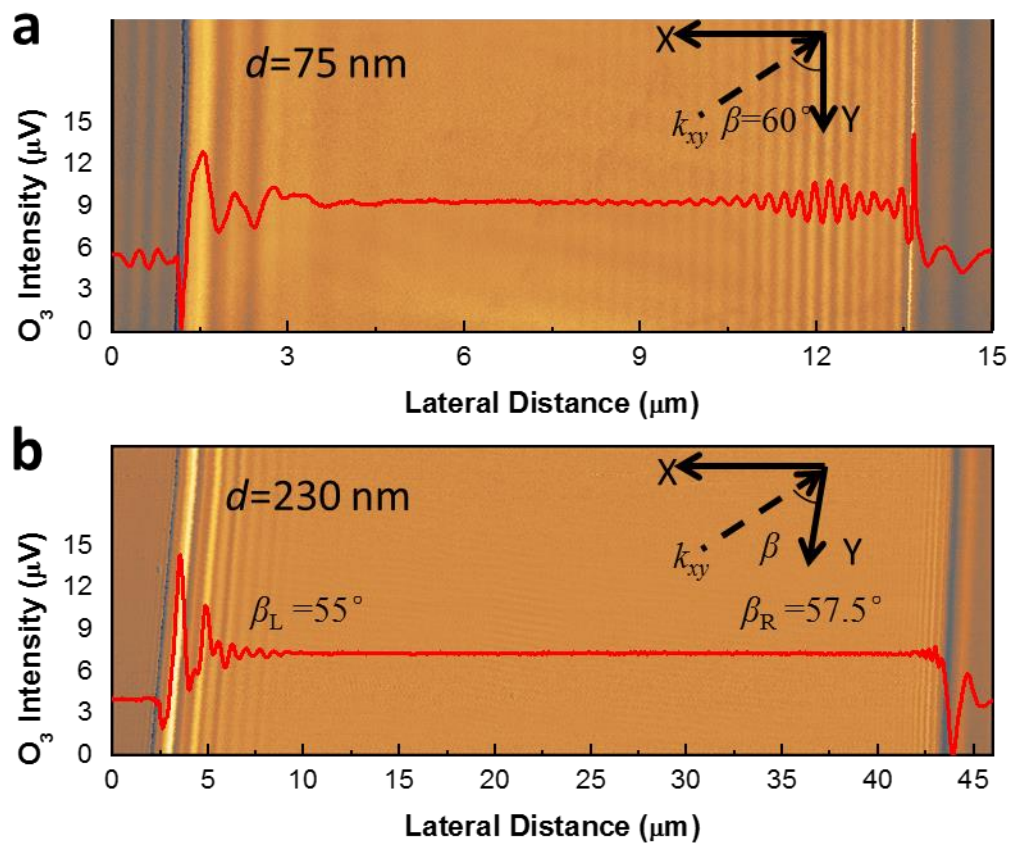
Supplementary Figure 6 | Experimental verification of enhanced interference visibility of TE_1 mode. We used a 250-nm-thick MoS_2 sample supporting the first order TE mode to verify our explanation of the varying interference visibilities in Supplementary Fig 5 and main text. **(a)** AFM image and height profile for a 250-nm-thick MoS_2 sample; **(b)** Near-field image and optical profile of the 250-nm-thick MoS_2 sample; **(c)** Spatial frequency domain spectrum of the optical profile in **b**. Note that the frequency difference between TE_1 and TM_0 is expected to be $0.177k_0$, too small to be resolved in **c**.



Supplementary Figure 7 | Thickness dispersions of air-MoS₂-SiO₂ waveguide. The MoS₂ thickness range from 0 to 1000 nm. There are five TE modes (m=0~4) and five TM modes (n=0~4) for a waveguide of a 1000-nm-thick MoS₂ guide layer.



Supplementary Figure 8 | AFM images and height profiles for h-BN samples of different thicknesses. (a) $d=75$ nm; (b) $d=230$ nm.



Supplementary Figure 9 | Large-area near-field images and real-space fringe profiles of h-BN samples. (a) $d=75 \text{ nm}$; (b) $d=230 \text{ nm}$.

Supplementary References

- 1 Born, M. *et al. Principles of Optics*; Cambridge University Press: Cambridge, U.K., 1999.