Supplementary Information: Parallel magnetic field suppresses dissipation in superconducting nanostrips

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(Dated: July 16, 2017)

I. ALIGNMENT OF MAGNETIC FIELD ORIENTATION IN TRIPLE-AXIS VECTOR MAGNET

The sample was placed in a superconducting triple-axis vector magnet with maximum magnetic fields of 1 Tesla, 1 Tesla and 6 Tesla in the three principle directions X_0, Y_0 and Z_0, respectively, which allow us to apply magnetic field in any desired orientation. The sample was mounted close to the X_0-Y_0 plane with a margin-of-error (usually below 1°) from the X_0-Y_0 plane, as shown in Fig.S1a. The field alignment is conducted through a coordinate transformation using Euler rotations. In the transformed coordinate system for a given magnetic field $H' = [H'_x, H'_y, H'_z]$, its corresponding coordinate $H = [H_x, H_y, H_z]$ in the original coordinate system is directly related to the three principle axes of vector magnet and can be obtained as $H = R^{-1}(\alpha, \beta, \gamma)H'$, α, β, γ are the rotation angles around Z axis, X' axis and Z'' axis, respectively. All the field directions mentioned in the main article is in the transformed coordinate system H'. Figures S1b, S1c and S1d show the three steps of the alignment procedure, as described below.

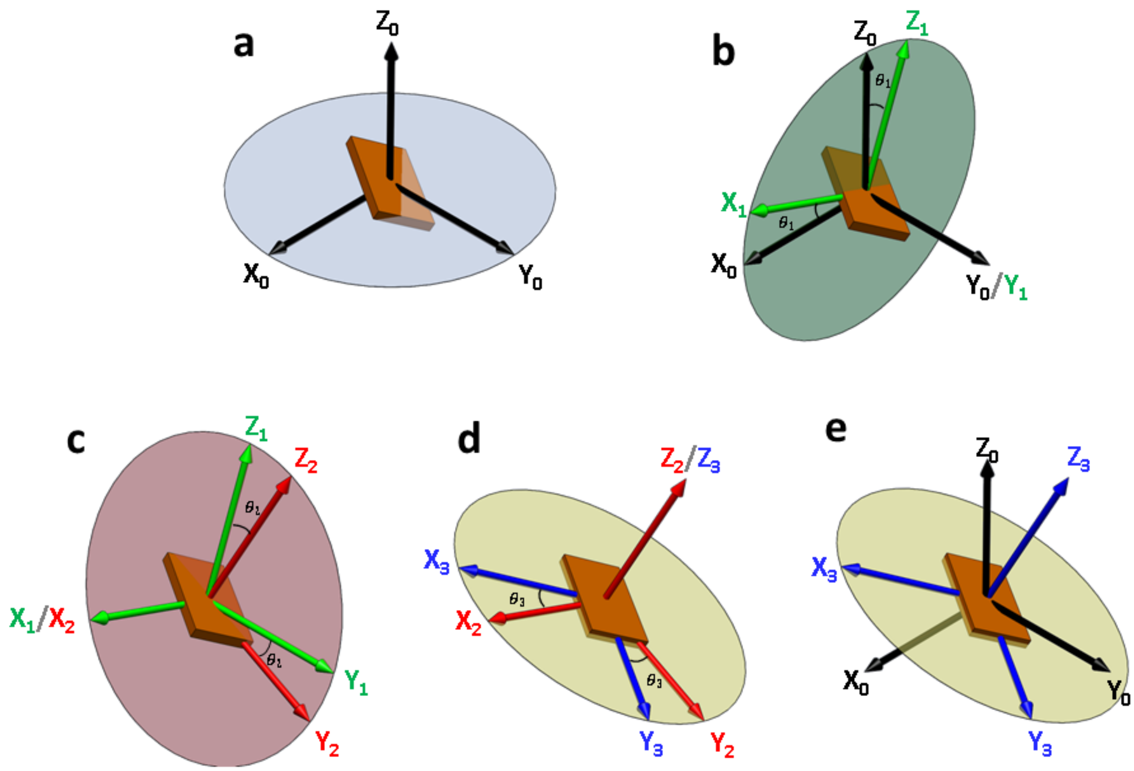

FIG. S1. **Magnetic field orientation alignments a**) Sample in vector magnet with tilted angle with primary $X - Y$ plane in original coordinate system $X_0 - Y_0 - Z_0$. **b**), align X axis in sample plane. **c**), align Y axis in sample plane. **d**), align Y axis with the current direction along the long length of sample. **e**), the final coordinate system $X_3 - Y_3 - Z_3$.

1. *Align the X axis in the sample plane (Fig. S1b)*: The deviation angle θ_1 between the sample film plane and the X axis can be determined by measuring the angle dependent magneto-resistance in the $X_0 - Z_0$ plane. The angle of the minimum resistance is the in-plane direction of the film. To align the X axis in the sample plane, we apply an Euler rotation with Euler angle $\alpha = 90°, \beta = -\theta_1, \gamma = 0°$. Figure S1b shows the transformation from the original coordinate system $X_0 - Y_0 - Z_0$ to a new coordinate system $X_1 - Y_1 - Z_1$ following step 1. Figure S2a shows the angle dependent magneto-resistance before and after aligning the X axis to the sample plane.

2. *Align the Y axis to the sample plane (Fig. S1c)*: In coordinate system X_1-Y_1-Z_1, conduct angle dependent magneto-resistance measurements in the Y_1-Z_1 plane and determine the angle deviation θ_2 between the Y axis and film plane. The Euler rotation with Euler angles $\alpha = 0°$, $\beta = \theta_2$, $\gamma = 0°$ will transform the coordinate system X_1-Y_1-Z_1 to X_2-Y_2-Z_2 with Y_2 in the film plane, as shown in Fig.S1c. Figure S2b shows the angle dependent magneto-resistance before and after aligning the Y axis to the sample plane.

3. *Align the Y axis to current direction along the long length of the sample (Fig. S1d)*: In the coordinate system X_2-Y_2-Z_2, an angle dependent magneto-resistance measurement to determine the deviation angle θ_3 between Y axis and the current direction (long length direction) wa conducted by finding orientation for the minimum resistance. Since the dissipation for in-plane magnetic fields is small, slightly tilt the field to an out-of-plane direction, such as 1° from the in-plane direction, can increase the resistance value. As shown in Fig S1d, the Euler rotation with Euler angles $\alpha = \beta = 0°, \gamma = \theta_3$ transforms coordinate system $X_2 - Y_2 - Z_2$ to coordinate system $X_3 - Y_3 - Z_3$ with Y_3 in the current direction. Figure S2c shows the angle dependent magneto-resistance before and after aligning the Y axis to the current flow direction.

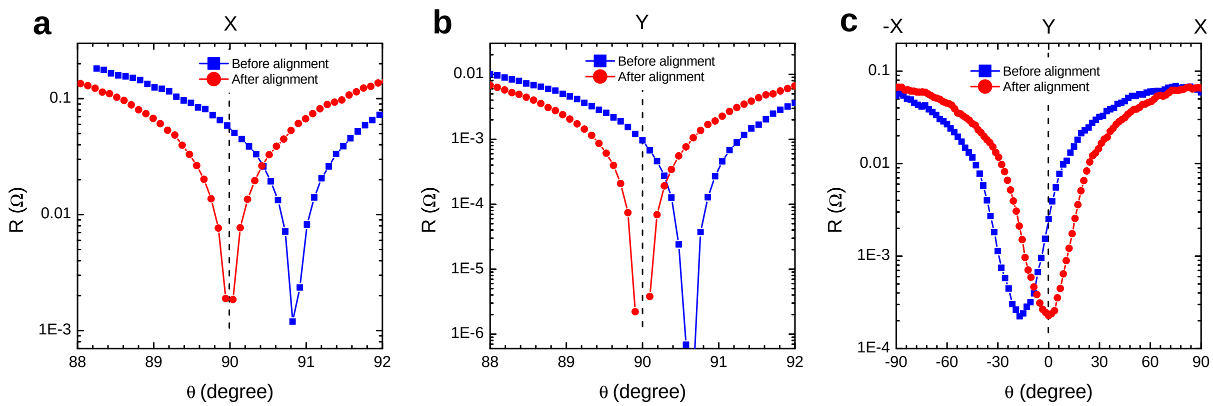

FIG. S2. **Angle dependent magneto-resistance.** **a**) rotate magnetic field in $X - Z$ plane with 90° indicating X direction. **b**), rotate magnetic field in $Y - Z$ plane with 90° indicating Y direction. **c**), rotate magnetic field in $X - Y$ plane with 0° indicating Y direction.

The above three steps transform the original coordinate system X_0-Y_0-Z_0 to a final coordinate system X_3-Y_3-Z_3, as shown in Fig. S1e, in which Z_3 is the out-of-plane direction of the sample film, X_3 is the in-plane direction perpendicular to current flow, and Y_3 is the in-plane direction parallel to current flow.

II. RESULTS OF 50 NM THICK SAMPLE

Experimental results for sample with thickness of 50 nm are shown in Fig. S3.

III. TEMPERATURE DEPENDENCE OF THE RESISTANCE FOR DIFFERENT CURRENTS

Measurements for the temperature dependence of the resistance are presented in Fig. S4. Panel (A) shows the resistance curves at three different magnetic fields at an applied current of 0.2 mA, while panel (B) shows the same curves for an applied current of 1.0 mA (as Fig. 2A in the paper).

IV. CRITICAL CURRENT, SIMULATION

Besides the magneto-resistance, we also calculated the critical current in the magneto-current curve, indicating the reentrance behavior. The critical current can be calculated using two different methods: by a simple constant voltage cutoff with arbitrary threshold $V < 10^{-3}$ or by a small percentage of the vortex creep flow aB_xJ_x, where $a \ll 1$ (usually 0.01). Both criteria yielded essentially the same curve for $a < 0.05$. Fig. S5 shows the dependence of the critical current as function of the applied field for different defect concentrations, in analogy with Fig. 4C of the main text.

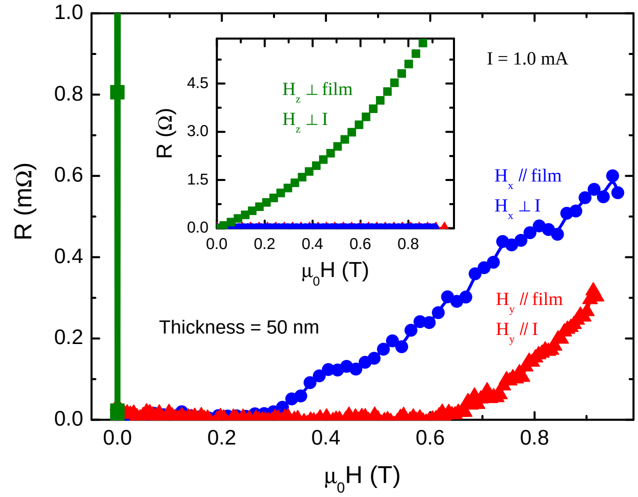

FIG. S3. Magneto-resistance measured at $T = 5.4$ K for a sample of thickness 50 nm in magnetic fields in three orthogonal directions. There are no intermediate resistive or reentrant superconducting states.

V. SUPPLEMENTARY MOVIES, SIMULATION

1. Clean sample [Movie1_yfield_ramp_clean.mp4, 24MB]: The field H_y is increased in a clean sample generation more-and-more vortex rows before breaking down due to fluctuations
2. A lot of impurities [Movie2_yfield_ramp_many_imp.mp4, 32MB]: The field H_y is increased in a sample with $N_{\mathrm{imp}} = 100$ impurities. No re-entrance is observed. The sample become resistive quickly.
3. Field ramp [Movie3_yfield_ramp.mp4, 5MB]: The field H_y is increased in a sample showing reentrance behavior.
4. Intermediate state [Movie4_yfield_intermediate_state.mp4, 20MB]: The field is fixed at $H_y = 0.12$ in the intermediate state, showing the periodic oscillatory motion causing the resistance at intermediate fields before becoming superconducting again in the reentrance region.

Note, the H_y component of the field is shown either as x-component or B_z in the movies.

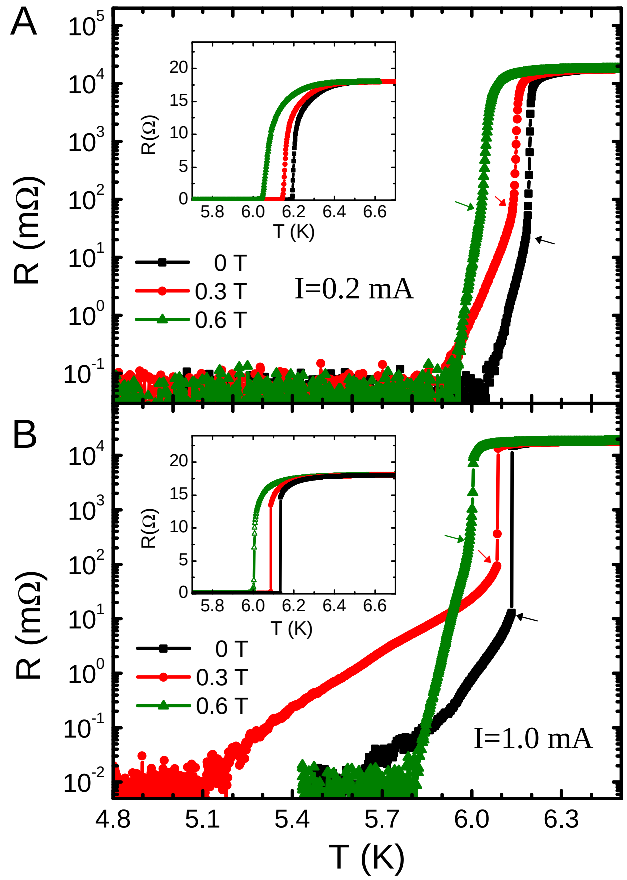

FIG. S4. Temperature dependence of the resistance under parallel magnetic fields of 0 T (black-square), 0.3 T (red-dot) and 0.6 T (green-triangle) obtained with an applied current of 0.2 mA (A) and 1.0 mA (B). The main panels in (A) and (B) are semi-log plots and the two insets are linear plots. The resistance states below the kinks pointed out by arrows are typical features originated from vortex motion.

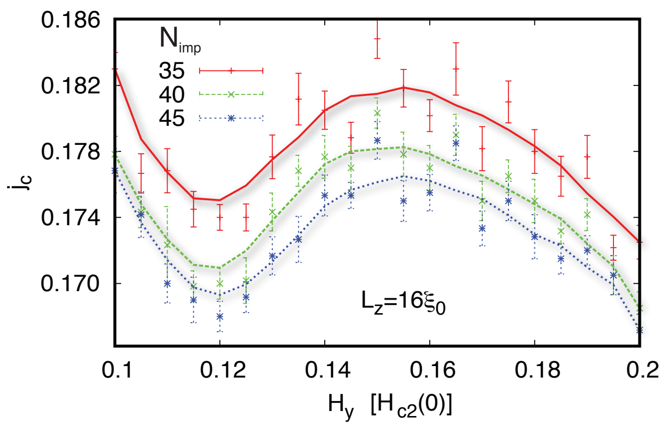

FIG. S5. The magnetic field dependence of the critical current, recovering the instability seen in the magneto-resistance curves (the graph labels correspond to different defect concentrations). Lines are trend-lines obtained by a weighted adjacent average.