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Supplementary Materials for "Blood pressure and the risk of Chronic Kidney Disease progression using multistate marginal structural models in the CRIC Study"

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1. Estimating marginal probabilities $P(Y_{i,J+1}^{\bar{a}} = k)$

Below we show how to obtain the $P(Y_{i,J+1}^{\bar{a}} = k)$ marginal probabilities from the MS-MSM estimates. We first note that the marginal probabilities of potential outcome states are determined by

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$$
\begin{aligned}
P\left(Y_{j+1}^{\bar{a}}=k\right) &= \sum_{\omega\in\Omega} P\left(Y_{j+1}^{\bar{a}}=k|\bar{Y}_j^{\bar{a}}=y_j^{\bar{a}}\right)P\left(\bar{Y}_j^{\bar{a}}=y_j^{\bar{a}}\right)\\
&= \sum_{k'_j=1}^{K} P\left(Y_{j+1}^{\bar{a}}=k|Y_J^{\bar{a}}=k'_j\right)P\left(Y_J^{\bar{a}}=k'_j\right)\\
&= \sum_{k'_j=1}^{K} P\left(Y_{j+1}^{\bar{a}}=k|Y_J^{\bar{a}}=k'_j\right)\left\{\sum_{k'_{j-1}=1}^{K} P\left(Y_j^{\bar{a}}=k'_j|Y_{j-1}^{\bar{a}}=k'_{j-1}\right)P\left(Y_{j-1}^{\bar{a}}=k'_{j-1}\right)\right\}\\
&= \sum_{k'_j=1}^{K} P\left(Y_{j+1}^{\bar{a}}=k|Y_J^{\bar{a}}=k'_j\right)\left\{\sum_{k'_{j-1}=1}^{K} P\left(Y_j^{\bar{a}}=k'_j|Y_{j-1}^{\bar{a}}=k'_{j-1}\right)\left\{\sum_{k'_{j-2}=1}^{K} P\left(Y_{j-1}^{\bar{a}}=k'_{j-1}|Y_{j-2}^{\bar{a}}=k'_{j-2}\right)\left\{....\right.\right.\right.\\
&\quad \left.\left.\left.\left\{\sum_{k'_1=1}^{K} P\left(Y_1^{\bar{a}}=k'_1|Y_0^{\bar{a}}=k_0\right)P\left(Y_0^{\bar{a}}=k_0\right)\right\}\right\}\right\}\right\} \qquad (1)
\end{aligned}
$$

where Ω describes the set of possible values of the history of responses $\bar{Y}^{\bar{a}}$, and the first equality follows from the first-order Markov assumption. The second and third equalities follow from repeated application of the law of total probability. It remains to show that the product of transition intensity matrices $\Lambda_{J+1}^{\bar{a}} = \prod_{j=0}^{J} \lambda_{j+1}^{\bar{a}}$, yields ((1).

Let $\gamma_0 = (\gamma_0^1, \gamma_0^2, ..., \gamma_0^K)^{\mathrm{T}} = (P(Y_0 = 1), P(Y_0 = 2), ..., P(Y_0 = K))^{\mathrm{T}}$ define the marginal distribution of Y_0 and $\lambda_1^{\bar{a}}$ be the $K \times K$ transition intensity matrix that characterizes the state transitition from time $j = 0$ to time $j+1 = 1$ under the joint exposure $\bar{a}$, and $\lambda_2^{\bar{a}}$ similarly characterizes the transition intensity matrix from time $j = 1$ to time $j+1 = 2$. We have

$$\Lambda^{\bar{a}}_{J+1} = \gamma_0^{\mathrm{T}} \prod_{j=0}^{J} \lambda^{\bar{a}}_{j+1} \tag{2}$$

$$= \begin{bmatrix} \gamma_0^1 & \gamma_0^2 & \dots & \gamma_0^K \end{bmatrix} \begin{bmatrix} \lambda_1^{\bar{a},1,1} & \lambda_1^{\bar{a},1,2} & \dots & \lambda_1^{\bar{a},1,K} \\ \lambda_1^{\bar{a},2,1} & \lambda_1^{\bar{a},2,2} & \dots & \lambda_1^{\bar{a},2,K} \\ \vdots & \ddots & \ddots & \vdots \\ \lambda_1^{\bar{a},K,1} & \lambda_1^{\bar{a},K,2} & \dots & \lambda_1^{\bar{a},K,K} \end{bmatrix} \begin{bmatrix} \lambda_2^{\bar{a},1,1} & \lambda_2^{\bar{a},1,2} & \dots & \lambda_2^{\bar{a},1,K} \\ \lambda_2^{\bar{a},2,1} & \lambda_2^{\bar{a},2,2} & \dots & \lambda_2^{\bar{a},2,K} \\ \vdots & \ddots & \ddots & \vdots \\ \lambda_2^{\bar{a},K,1} & \lambda_2^{\bar{a},K,2} & \dots & \lambda_2^{\bar{a},K,K} \end{bmatrix} \times \dots \tag{3}$$

$$\times \begin{bmatrix} \lambda_2^{\bar{a},1,1} & \lambda_2^{\bar{a},1,2} & \dots & \lambda_2^{\bar{a},1,K} \\ \lambda_2^{\bar{a},2,1} & \lambda_2^{\bar{a},2,2} & \dots & \lambda_2^{\bar{a},2,K} \\ \vdots & \ddots & \ddots & \vdots \\ \lambda_2^{\bar{a},K,1} & \lambda_2^{\bar{a},K,2} & \dots & \lambda_2^{\bar{a},K,K} \end{bmatrix} \tag{4}$$

$$= \begin{bmatrix} \sum_{k'_0=1}^{K} \gamma_0^{k'_0} \lambda_1^{\bar{a},k'_0,1} & \sum_{k'_0=1}^{K} \gamma_0^{k'_0} \lambda_2^{\bar{a},k'_0,2} & \dots & \sum_{k'_0=1}^{K} \gamma_0^{k'_0} \lambda_1^{\bar{a},k'_0,K} \end{bmatrix} \times \begin{bmatrix} \lambda_2^{\bar{a},1,1} & \lambda_2^{\bar{a},1,2} & \dots & \lambda_2^{\bar{a},1,K} \\ \lambda_2^{\bar{a},2,1} & \lambda_2^{\bar{a},2,2} & \dots & \lambda_2^{\bar{a},2,K} \\ \vdots & \ddots & \ddots & \vdots \\ \lambda_2^{\bar{a},K,1} & \lambda_2^{\bar{a},K,2} & \dots & \lambda_2^{\bar{a},K,K} \end{bmatrix} \times \dots \tag{5}$$

$$\times \begin{bmatrix} \lambda_J^{\bar{a},1,1} & \lambda_J^{\bar{a},1,2} & \dots & \lambda_J^{\bar{a},1,K} \\ \lambda_J^{\bar{a},2,1} & \lambda_J^{\bar{a},2,2} & \dots & \lambda_J^{\bar{a},2,K} \\ \vdots & \ddots & \ddots & \vdots \\ \lambda_J^{\bar{a},K,1} & \lambda_J^{\bar{a},K,2} & \dots & \lambda_J^{\bar{a},K,K} \end{bmatrix} \tag{6}$$

$$= \begin{bmatrix} \sum_{k'_1=1}^{K} \lambda^{\bar{a},k'_1,1} \left\{ \sum_{k'_0=1}^{K} \gamma_0^{k'_0} \lambda_1^{\bar{a},k'_0,k'_1} \right\} & \sum_{k'_1=1}^{K} \lambda^{\bar{a},k'_1,2} \left\{ \sum_{k'_0=1}^{K} \gamma_0^{k'_0} \lambda_1^{\bar{a},k'_0,k'_1} \right\} & \dots & \sum_{k'_1=1}^{K} \lambda^{\bar{a},k'_1,K} \left\{ \sum_{k'_0=1}^{K} \gamma_0^{k'_0} \lambda_1^{\bar{a},k'_0,k'_1} \right\} \end{bmatrix} \times \dots \tag{7}$$

$$\times \begin{bmatrix} \lambda_J^{\bar{a},1,1} & \lambda_J^{\bar{a},1,2} & \dots & \lambda_J^{\bar{a},1,K} \\ \lambda_J^{\bar{a},2,1} & \lambda_J^{\bar{a},2,2} & \dots & \lambda_J^{\bar{a},2,K} \\ \vdots & \ddots & \ddots & \vdots \\ \lambda_J^{\bar{a},K,1} & \lambda_J^{\bar{a},K,2} & \dots & \lambda_J^{\bar{a},K,K} \end{bmatrix} \tag{8}$$

$$= \begin{bmatrix} \sum_{k'_J=1}^{K} \lambda^{\bar{a},k'_J,1} \left\{ \sum_{k'_{J-1}=1}^{K} \lambda^{\bar{a},k'_{J-1},k'_J} \dots \left\{ \sum_{k'_1=1}^{K} \lambda^{\bar{a},k'_1,k'_2} \left\{ \sum_{k'_0=1}^{K} \gamma_0^{k'_0} \lambda_1^{\bar{a},k'_0,k'_1} \right\} \right\} \right\} \\ \sum_{k'_J=1}^{K} \lambda^{\bar{a},k'_J,2} \left\{ \sum_{k'_{J-1}=1}^{K} \lambda^{\bar{a},k'_{J-1},k'_J} \dots \left\{ \sum_{k'_1=1}^{K} \lambda^{\bar{a},k'_1,k'_2} \left\{ \sum_{k'_0=1}^{K} \gamma_0^{k'_0} \lambda_1^{\bar{a},k'_0,k'_1} \right\} \right\} \right\} \\ \vdots \\ \sum_{k'_J=1}^{K} \lambda^{\bar{a},k'_J,K} \left\{ \sum_{k'_{J-1}=1}^{K} \lambda^{\bar{a},k'_{J-1},k'_J} \dots \left\{ \sum_{k'_1=1}^{K} \lambda^{\bar{a},k'_1,k'_2} \left\{ \sum_{k'_0=1}^{K} \gamma_0^{k'_0} \lambda_1^{\bar{a},k'_0,k'_1} \right\} \right\} \right\} \end{bmatrix}^{\mathrm{T}} \tag{9}$$

where the k^{th} element of the $1 \times K$ vector $\Lambda^{\bar{a}}_{J+1}$ indicates $P(Y^{\bar{a}}_{i,J+1} = k)$.

2. Distirbution of inverse probability weights pre- and post-truncation in application of MS-MSMs to the CRIC study