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Supplementary Materials for "Blood pressure and the risk of Chronic Kidney Disease progression using multistate marginal structural models in the CRIC Study"

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1. Estimating marginal probabilities $P(Y_{i,I+1}^{\bar{a}} = k)$

Below we show how to obtain the $P(Y_{i,J+1}^{\bar{a}} = k)$ marginal probabilities from the MS-MSM estimates. We first note that the marginal probabilities of potential outcome states are determined by

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$$\begin{split} P\left(Y_{j+1}^{\bar{a}} = k\right) &= \sum_{\omega \in \Omega} P\left(Y_{j+1}^{\bar{a}} = k | \bar{Y}_{j}^{\bar{a}} = y_{j}^{\bar{a}}\right) P\left(\bar{Y}_{j}^{\bar{a}} = y_{j}^{\bar{a}}\right) \\ &= \sum_{k'_{j}=1}^{K} P\left(Y_{j+1}^{\bar{a}} = k | Y_{j}^{\bar{a}} = k'_{j}\right) P\left(Y_{j}^{\bar{a}} = k'_{j}\right) \\ &= \sum_{k'_{j}=1}^{K} P\left(Y_{j+1}^{\bar{a}} = k | Y_{j}^{\bar{a}} = k'_{j}\right) \left\{ \sum_{k'_{j-1}=1}^{K} P\left(Y_{j}^{\bar{a}} = k'_{j}| Y_{j-1}^{\bar{a}} = k'_{j-1}\right) P\left(Y_{j-1}^{\bar{a}} = k'_{j-1}\right) \right\} \\ &= \sum_{k'_{j}=1}^{K} P\left(Y_{j+1}^{\bar{a}} = k | Y_{j}^{\bar{a}} = k'_{j}\right) \left\{ \sum_{k'_{j-1}=1}^{K} P\left(Y_{j}^{\bar{a}} = k'_{j}| Y_{j-1}^{\bar{a}} = k'_{j-1}\right) \left\{ \sum_{k'_{j-2}=1}^{K} P\left(Y_{j-1}^{\bar{a}} = k'_{j-1}| Y_{j-2}^{\bar{a}} = k'_{j-2}\right) \right\} \\ &\left\{ \sum_{k'_{1}=1}^{K} P\left(Y_{1}^{\bar{a}} = k'_{1}| Y_{0}^{\bar{a}} = k_{0}\right) P\left(Y_{0}^{\bar{a}} = k_{0}\right) \right\} \right\} \right\} \end{split}$$

where Ω describes the set of possible values of the history of responses $\bar{Y}^{\bar{a}}$, and the first equality follows from the first-order Markov assumption. The second and third equalities follow from repeated application of the law of total probability. It remains to show that the product of transition intensity matrices $\Lambda_{I+1}^{\bar{a}} = \prod_{i=0}^{J} \lambda_{i+1}^{\bar{a}}$ yields ((1).

Let $\gamma_0 = (\gamma_0^1, \gamma_0^2, ..., \gamma_0^K)^T = (P(Y_0 = 1), P(Y_0 = 2), ..., P(Y_0 = K))^T$ define the marginal distribution of Y_0 and $\lambda_1^{\bar{a}}$ be the $K \times K$ transition intensity matrix that characterizes the state transition from time j = 0 to time j + 1 = 1under the joint exposure \bar{a} , and $\lambda_2^{\bar{a}}$ similarly characterizes the transition intensity matrix from time j=1 to time j + 1 = 2. We have

$$\Lambda_{J+1}^{\bar{a}} = \gamma_0^{\mathrm{T}} \prod_{j=0}^{J} \lambda_{j+1}^{\bar{a}} \tag{2}$$

$$= \begin{bmatrix} \gamma_0^1 & \gamma_0^2 & \dots & \gamma_0^K \end{bmatrix} \begin{bmatrix} \lambda_1^{\bar{a},1,1} & \lambda_1^{\bar{a},1,2} & \dots & \lambda_1^{\bar{a},1,K} \\ \lambda_1^{\bar{a},2,1} & \lambda_1^{\bar{a},2,2} & \dots & \lambda_1^{\bar{a},2,K} \\ \vdots & \ddots & \ddots & \vdots \\ \lambda_1^{\bar{a},K,1} & \lambda_1^{\bar{a},K,2} & \dots & \lambda_1^{\bar{a},K,K} \end{bmatrix} \begin{bmatrix} \lambda_2^{\bar{a},1,1} & \lambda_2^{\bar{a},1,2} & \dots & \lambda_2^{\bar{a},1,K} \\ \lambda_2^{\bar{a},2,1} & \lambda_2^{\bar{a},2,2} & \dots & \lambda_2^{\bar{a},2,K} \\ \vdots & \ddots & \ddots & \vdots \\ \lambda_2^{\bar{a},K,1} & \lambda_2^{\bar{a},K,2} & \dots & \lambda_2^{\bar{a},K,K} \end{bmatrix} \times \dots$$

$$(3)$$

$$\times \begin{bmatrix}
\lambda_{2}^{\bar{a},1,1} & \lambda_{2}^{\bar{a},1,2} & \dots & \lambda_{2}^{\bar{a},1,K} \\
\lambda_{2}^{\bar{a},2,1} & \lambda_{2}^{\bar{a},2,2} & \dots & \lambda_{2}^{\bar{a},2,K} \\
\vdots & \ddots & \ddots & \vdots \\
\lambda_{2}^{\bar{a},K,1} & \lambda_{2}^{\bar{a},K,2} & \dots & \lambda_{2}^{\bar{a},K,K}
\end{bmatrix}$$
(4)

$$= \left[\sum_{k'_{0}=1}^{K} \gamma_{0}^{k'_{0}} \lambda_{1}^{\bar{a},k'_{0},1} \sum_{k'_{0}=1}^{K} \gamma_{0}^{k'_{0}} \lambda_{2}^{\bar{a},k'_{0},2} \dots \sum_{k'_{0}=1}^{K} \gamma_{0}^{k'_{0}} \lambda_{1}^{\bar{a},k'_{0},K} \right] \times \left[\begin{array}{ccccc} \lambda_{2}^{a,1,1} & \lambda_{2}^{a,1,2} & \dots & \lambda_{2}^{a,1,K} \\ \lambda_{2}^{\bar{a},2,1} & \lambda_{2}^{\bar{a},2,2} & \dots & \lambda_{2}^{\bar{a},2,K} \\ \vdots & \ddots & \ddots & \vdots \\ \lambda_{2}^{\bar{a},K,1} & \lambda_{2}^{\bar{a},K,2} & \dots & \lambda_{2}^{\bar{a},K,K} \end{array} \right] \times \dots$$
 (5)

$$\times \begin{bmatrix} \lambda_{J}^{\bar{a},1,1} & \lambda_{J}^{\bar{a},1,2} & \dots & \lambda_{J}^{\bar{a},1,K} \\ \lambda_{J}^{\bar{a},2,1} & \lambda_{J}^{\bar{a},2,2} & \dots & \lambda_{J}^{\bar{a},2,K} \\ \vdots & \ddots & \ddots & \vdots \\ \lambda_{J}^{\bar{a},K,1} & \lambda_{J}^{\bar{a},K,2} & \dots & \lambda_{J}^{\bar{a},K,K} \end{bmatrix}$$
(6)

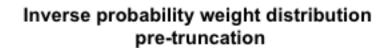
$$= \left[\sum_{k_{1}'=1}^{K} \lambda^{\bar{a},k_{1}',1} \left\{ \sum_{k_{0}'=1}^{K} \gamma_{0}^{k_{0}'} \lambda_{1}^{\bar{a},k_{0}',k_{1}'} \right\} - \sum_{k_{1}'=1}^{K} \lambda^{\bar{a},k_{1}',2} \left\{ \sum_{k_{0}'=1}^{K} \gamma_{0}^{k_{0}'} \lambda_{1}^{\bar{a},k_{0}',k_{1}'} \right\} - \dots - \sum_{k_{1}'=1}^{K} \lambda^{\bar{a},k_{1}',K} \left\{ \sum_{k_{0}'=1}^{K} \gamma_{0}^{k_{0}'} \lambda_{1}^{\bar{a},k_{0}',k_{1}'} \right\} \right] \times \dots (7)$$

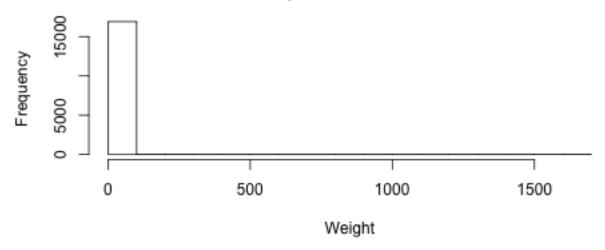
$$\times \begin{bmatrix}
\lambda_{I}^{\bar{a},1,1} & \lambda_{I}^{\bar{a},1,2} & \dots & \lambda_{I}^{\bar{a},1,K} \\
\lambda_{I}^{\bar{a},2,1} & \lambda_{I}^{\bar{a},2,2} & \dots & \lambda_{I}^{\bar{a},2,K} \\
\vdots & \ddots & \ddots & \vdots \\
\lambda_{I}^{\bar{a},K,1} & \lambda_{I}^{\bar{a},K,2} & \dots & \lambda_{I}^{\bar{a},K,K}
\end{bmatrix}$$
(8)

$$= \begin{bmatrix}
\sum_{k'_{j}=1}^{K} \lambda^{\bar{a},k'_{j},1} \left\{ \sum_{k'_{j-1}=1}^{K} \lambda^{\bar{a},k'_{j-1},k'_{j}} \dots \left\{ \sum_{k'_{1}=1}^{K} \lambda^{\bar{a},k'_{1},k'_{2}} \left\{ \sum_{k'_{0}=1}^{K} \gamma_{0}^{k'_{0}} \lambda_{1}^{\bar{a},k'_{0},k'_{1}} \right\} \right\} \right\} \\
= \begin{bmatrix}
\sum_{k'_{j}=1}^{K} \lambda^{\bar{a},k'_{j},2} \left\{ \sum_{k'_{j-1}=1}^{K} \lambda^{\bar{a},k'_{j-1},k'_{j}} \dots \left\{ \sum_{k'_{1}=1}^{K} \lambda^{\bar{a},k'_{1},k'_{2}} \left\{ \sum_{k'_{0}=1}^{K} \gamma_{0}^{k'_{0}} \lambda_{1}^{\bar{a},k'_{0},k'_{1}} \right\} \right\} \right\} \\
\vdots \\
\sum_{k'_{j}=1}^{K} \lambda^{\bar{a},k'_{j},K} \left\{ \sum_{k'_{j-1}=1}^{K} \lambda^{\bar{a},k'_{j-1},k'_{j}} \dots \left\{ \sum_{k'_{1}=1}^{K} \lambda^{\bar{a},k'_{1},k'_{2}} \left\{ \sum_{k'_{0}=1}^{K} \gamma_{0}^{k'_{0}} \lambda_{1}^{\bar{a},k'_{0},k'_{1}} \right\} \right\} \right\}$$
(9)

where the k^{th} element of the $1 \times K$ vector $\Lambda_{I+1}^{\bar{a}}$ indicates $P(Y_{i,I+1}^{\bar{a}} = k)$.

2. Distirbution of inverse probability weights pre- and post-truncation in application of MS-MSMs to the CRIC study





Inverse probability weight distribution post-truncation

