S2 Appendix: More general digestion networks

To see why the tragedy continues to hold for more general digestion network, we consider solutions of system the following system:

$$\begin{aligned}
\frac{ds}{dt}(t) &= D(t)(S^0(t) - s) - k_1 es + k_{-1}c & (1)\\
\frac{dp}{dt}(t) &= k_2 c - (x_1 + x_2)\, f(p) - D(t)p & (2)\\
\frac{de}{dt}(t) &= (1-q)x_1 f(p) - k_1 es + k_{-1}c + k_2 c - D(t)e & (3)\\
\frac{dc}{dt}(t) &= k_1 es - k_{-1}c - k_2 c - D(t)c & (4)\\
\frac{dx_1}{dt}(t) &= x_1\,(qf(p) - D(t)) & (5)\\
\frac{dx_2}{dt}(t) &= x_2\,(f(p) - D(t))\,, & (6)
\end{aligned}$$

for which it is easily verified that the variable:

$$m = s + p + e + 2c + x_1 + x_2,$$

still satisfies equation

$$\frac{dm}{dt}(t) = D(t)(S^0(t) - m), \tag{7}$$

implying that the family of compact sets Ω_ϵ, defined earlier, is forward invariant for system (1) − (6), for all $\epsilon \geq 0$, when **H2** holds. Consequently, the proof of Theorem 1 in S4 remains valid for the above chemostat model (1) − (6). Indeed, the first proof only crucially depends on the dynamics of x_1 and x_2 to show that $x_1(t)$ converges to zero, after which the convergence of e, p and x_2 is obtained by elementary comparison arguments. For the digestion network presented here, the dynamics of x_1 and x_2 remain unchanged, hence we can still conclude that $x_1(t)$ converges to zero. After that, it follows from a comparison argument that $e + c$ converges to zero, and then similarly that p and x_2 converge to zero as well. One could also easily adapt the steps of the second proof to obtain the same conclusion.