Davenport, K., B. Mosher, B. Brost, D. Henderson, N. Denkers, A. Nalls, E. McNulty, C. Mathiason, and E. Hoover. Distinguishing the shedding and detection of CWD prions in deer saliva with occupancy modeling.

Appendix A. Multi-scale false positive occupancy model specification and associated full-conditional distributions.

1 Model Statement

Let y_{ijk} be a binary observation representing a detection/non-detection, where $i = 1, \ldots, N$ indexes individual deer, $j = 1, \ldots, J_i$ indexes saliva samples nested within deer, and $k = 1, \ldots K_{ij}$ indexes replicates per saliva sample. Also let v_c be the number of detections out of M_c trials obtained in the "negative control" dataset for detection method c (i.e., concentration methods PTA and IOME).

$$
\begin{aligned}
y_{ijk} &\sim \begin{cases} (1-\phi_c)\,\text{Bern}\,(p_{ijk}) + \phi_c 1_{\{y_{ijk}=1\}}, & a_{ij}=1 \\ \text{Bern}\,(\phi_c)\,, & a_{ij}=0 \end{cases} \\
a_{ij} &\sim \begin{cases} \text{Bern}\,(\theta_{ij})\,, & z_i = 1 \\ 0, & z_i = 0 \end{cases} \\
z_i &\sim \text{Bern}\,(\psi_i) \\
\psi_i &= \text{logit}^{-1}\,(\mathbf{x}_i'\boldsymbol{\beta}) \\
\theta_{ij} &= \text{logit}^{-1}\left(\mathbf{u}_{ij}'\boldsymbol{\gamma}\right) \\
p_{ijk} &= \text{logit}^{-1}\left(\mathbf{w}_{ijk}'\boldsymbol{\alpha}\right) \\
\boldsymbol{\beta} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\beta, \sigma_\beta^2\mathbf{I}\right) \\
\boldsymbol{\gamma} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\gamma, \sigma_\gamma^2\mathbf{I}\right) \\
\boldsymbol{\alpha} &\sim \mathcal{N}\left(\boldsymbol{\mu}_\alpha, \sigma_\alpha^2\mathbf{I}\right) \\
v_c &\sim \text{Binom}\,(M_c, \phi_c) \\
\phi_c &\sim \text{Beta}\,(a, b)
\end{aligned}
$$

The parameters are as follows:

ϕ_c	Probability of a false positive error for detection method c.
p_{ijk}	Probability of detection for replicate k obtained from saliva sample j within deer i.
a_{ij}	Latent use state.
θ_{ij}	Probability of use for saliva sample j within deer i.
z_i	Latent occupancy state. Note that we set $z_i = 1$ given the experimental design of the chronic wasting disease applications.
ψ_i	Probability of occupancy for deer i (not estimated given the known occupancy state).
$\boldsymbol{\beta}$	Vector of coefficients that describe the affect of covariates $\mathbf{x}_i$ on probability of use (not estimated given the known occupancy state).
$\boldsymbol{\gamma}$	Vector of coefficients that describe the affect of covariates $\mathbf{u}_{ij}$ on probability of use.
$\boldsymbol{\alpha}$	Vector of coefficients that describe the affect of covariates $\mathbf{w}_{ijk}$ on probability of detection.
$\boldsymbol{\mu}_\beta$	Prior mean for occupancy coefficients.
$\boldsymbol{\mu}_\gamma$	Prior mean for use coefficients.
$\boldsymbol{\mu}_\alpha$	Prior mean for detection coefficients.

σ_β	Prior standard deviation for occupancy coefficients.
σ_γ	Prior standard deviation for use coefficients.
σ_α	Prior standard deviation for detection coefficients.
a, b	Prior shape parameters for false positive error rate.

2 Full-conditional Distributions

2.1 *Occupancy state* (z_i)

$$
\begin{aligned}
[z_i \mid \cdot] &\propto \prod_{j=1}^{J_i} [a_{ij} \mid \theta_{ij}, z_i] [z_i] \\
&\propto \prod_{j=1}^{J_i} \left(\theta_{ij}^{a_{ij}} (1-\theta_{ij})^{1-a_{ij}}\right)^{z_i} \left(1_{\{a_{ij}=0\}}^{1-z_i}\right) \psi_i^{z_i} (1-\psi_i)^{1-z_i} \\
&\propto \prod_{j=1}^{J_i} \left(\psi_i \theta_{ij}^{a_{ij}} (1-\theta_{ij})^{1-a_{ij}}\right)^{z_i} \left((1-\psi_i) 1_{\{a_{ij}=0\}}\right)^{1-z_i} \\
&= \text{Bern}\left(\tilde{\psi}_i\right),
\end{aligned}
$$

where,

$$
\tilde{\psi}_i = \frac{\psi_i \prod_{j=1}^{J_i} \theta_{ij}^{a_{ij}} (1-\theta_{ij})^{1-a_{ij}}}{\psi_i \prod_{j=1}^{J_i} \theta_{ij}^{a_{ij}} (1-\theta_{ij})^{1-a_{ij}} + (1-\psi_i) \prod_{j=1}^{J_i} 1_{\{a_{ij}=0\}}}.
$$

2.2 *Use state* (a_{ij})

Note that the mixture specification for a_{ij} in the model statement above is equivalent to $a_{ij} \sim \text{Bern}(z_i \theta_{ij})$, an alternate specification that simplifies the update for a_{ij}.

$$
\begin{aligned}
[a_{ij} \mid \cdot] &\propto \prod_{k=1}^{K_{ij}} [y_{ijk} \mid p_{ijk}, a_{ij}, \phi_c] [a_{ij}] \\
&\propto \prod_{k=1}^{K_{ij}} \left((1-\phi_c) p_{ijk}^{y_{ijk}} (1-p_{ijk})^{1-y_{ijk}} + \phi_c 1_{\{y_{ijk}=1\}}\right)^{a_{ij}} \left(\phi_c^{y_{ijk}} (1-\phi_c)^{1-y_{ijk}}\right)^{1-a_{ij}} (z_i\theta_{ij})^{a_{ij}} (1-z_i\theta_{ij})^{1-a_{ij}} \\
&\propto \prod_{k=1}^{K_{ij}} \left[(z_i\theta_{ij}) \left((1-\phi_c) p_{ijk}^{y_{ijk}} (1-p_{ijk})^{1-y_{ijk}} + \phi_c 1_{\{y_{ijk}=1\}}\right)\right]^{a_{ij}} \left((1-z_i\theta_{ij}) \phi_c^{y_{ijk}} (1-\phi_c)^{1-y_{ijk}}\right)^{1-a_{ij}} \\
&= \text{Bern}\left(\tilde{\theta}_{ij}\right),
\end{aligned}
$$

where,

$$
\tilde{\theta}_{ij} = \frac{z_i\theta_{ij} \prod_{k=1}^{K_{ij}} \left[(1-\phi_c) p_{ijk}^{y_{ijk}} (1-p_{ijk})^{1-y_{ijk}} + \phi_c 1_{\{y_{ijk}=1\}}\right]}{z_i\theta_{ij} \prod_{k=1}^{K_{ij}} \left[(1-\phi_c) p_{ijk}^{y_{ijk}} (1-p_{ijk})^{1-y_{ijk}} + \phi_c 1_{\{y_{ijk}=1\}}\right] + (1-z_i\theta_{ij}) \prod_{k=1}^{K_{ij}} \phi_c^{y_{ijk}} (1-\phi_c)^{1-y_{ijk}}}.
$$

2.3 *Occupancy coefficients* ($\boldsymbol{\beta}$)

$$
\begin{aligned}
[\boldsymbol{\beta} \mid \cdot] &\propto \prod_{i=1}^{N} [z_i \mid \psi_i] [\boldsymbol{\beta}] \\
&\propto \prod_{i=1}^{N} \text{Bern}(z_i \mid \psi_i) \mathcal{N}\left(\boldsymbol{\beta} \mid \boldsymbol{\mu}_\beta, \sigma_\beta^2 \mathbf{I}\right).
\end{aligned}
$$

The update for $\boldsymbol{\beta}$ proceeds using Metropolis-Hastings.

2.4 *Use coefficients ($\boldsymbol{\gamma}$)*

$$
\begin{aligned}
[\boldsymbol{\gamma} \mid \cdot] &\propto \prod_{i=1}^{N}\prod_{j=1}^{J_i}[a_{ij} \mid \theta_{ij}, z_i]\,[\boldsymbol{\gamma}] \\
&\propto \prod_{i=1}^{N}\prod_{j=1}^{J_i}\text{Bern}\left(a_{ij} \mid \theta_{ij}\right)^{z_i}\mathcal{N}\left(\boldsymbol{\gamma} \mid \boldsymbol{\mu}_{\gamma}, \sigma_{\gamma}^{2}\mathbf{I}\right).
\end{aligned}
$$

The update for $\boldsymbol{\gamma}$ proceeds using Metropolis-Hastings. Note that the product over i only includes instances of i such that $z_i = 1$.

2.5 *Detection coefficients ($\boldsymbol{\alpha}$)*

$$
\begin{aligned}
[\boldsymbol{\alpha} \mid \cdot] &\propto \prod_{i=1}^{N}\prod_{j=1}^{J_i}\prod_{k=1}^{K_{ij}}[y_{ijk} \mid p_{ijk}, a_{ij}, \phi_c]\,[\boldsymbol{\alpha}] \\
&\propto \prod_{i=1}^{N}\prod_{j=1}^{J_i}\prod_{k=1}^{K_{ij}}\left[(1-\phi_c)\,p_{ijk}^{y_{ijk}}\left(1-p_{ijk}\right)^{1-y_{ijk}} + \phi_c 1_{\{y_{ijk}=1\}}\right]^{a_{ij}}\mathcal{N}\left(\boldsymbol{\alpha} \mid \boldsymbol{\mu}_{\alpha}, \sigma_{\alpha}^{2}\mathbf{I}\right).
\end{aligned}
$$

The update for $\boldsymbol{\alpha}$ proceeds using Metropolis-Hastings. Note that the product over i and j only includes instances of i and j such that $a_{ij} = 1$.

2.6 *Probability of a false positive detection (ϕ_c)*

$$
\begin{aligned}
[\phi_c \mid \cdot] &\propto [v_c \mid M_c, \phi_c]\,[\phi_c] \\
&\propto \text{Binom}\left(v_c \mid M_c, \phi_c\right)\text{Beta}\left(a, b\right) \\
&\propto \phi_c^{v}\left(1-\phi_c\right)^{M_c - v_c}\phi_c^{a-1}(1-\phi)^{b-1} \\
&\propto \phi_c^{v_c + a - 1}\left(1-\phi_c\right)^{M_c - v_c + b - 1} \\
&= \text{Beta}\left(v_c + a, M_c - v_c + b\right)
\end{aligned}
$$