Supporting Information

File S1. Detailed derivation of results

In this Supporting Information we will give a detailed derivation of the results which in the main text have been abbreviated to keep formulas concise. First, Equation 23 showing that $\Delta_N^{(n)}$ is bounded for all $n \in \mathbb{N}$ is derived as

$$
\begin{aligned}
\lim_{N\to\infty} \Delta_N^{(n)} &= \lim_{N\to\infty} (N_n - N_{n+1}) \\
&= \lim_{N\to\infty} \left(N\left(1-\rho\frac{\psi^2}{N^\gamma}\right)^n - N\left(1-\rho\frac{\psi^2}{N^\gamma}\right)^{n+1} \right) \\
&= \lim_{N\to\infty} N\left(1-\rho\frac{\psi^2}{N^\gamma}\right)^n \left(1-\left(1-\rho\frac{\psi^2}{N^\gamma}\right)\right) \\
&= \lim_{N\to\infty} N\left(1-\rho\frac{\psi^2}{N^\gamma}\right)^n \left(\rho\frac{\psi^2}{N^\gamma}\right) \\
&\leq \lim_{N\to\infty} N\rho\frac{\psi^2}{N^\gamma} \\
&= 0,
\end{aligned} \tag{S1}
$$

given $\gamma > 1$. Note that in the second to last line we have used that $\Delta_N^{(n)}$ is always the largest for $n = 0$ (i.e., the population grows the largest from the previous to the current generation). Second, the step function $F_N(s)$ (eq. 31) for the ancestral process is derived as

$$
\begin{aligned}
F_N(s) &= \sum_{n=1}^{s} c_N^{(n)} \\
&= \sum_{n=1}^{s} \frac{\psi^2}{N_n^\gamma} \\
&= \frac{\psi^2}{N^\gamma} \sum_{n=1}^{s} \left(\frac{1}{\left(1-\rho\frac{\psi^2}{N^\gamma}\right)^\gamma} \right)^n \\
&= \frac{\psi^2}{N^\gamma} \left(1-\rho\frac{\psi^2}{N^\gamma}\right)^{-\gamma} \frac{\left(\left(1-\rho\frac{\psi^2}{N^\gamma}\right)^{-\gamma s} - 1\right)}{\left(1-\rho\frac{\psi^2}{N^\gamma}\right)^{-\gamma} - 1}.
\end{aligned} \tag{S2}
$$

Then, by solving for s the time-change function $\mathcal{G}_N^{-1}(t)$ (eq. 32) for the ancestral process can be derived as

$$
\begin{aligned}
\mathcal{G}_N^{-1}(t) &= \inf\{s > 0 : F_N(\lfloor s \rfloor) > t\} - 1 \\
&\sim \inf\left\{ s > 0 : [s] > \frac{\log\left[1 + \frac{tN^\gamma}{\psi^2}\left(1 - (1 - \frac{\rho\gamma\psi^2}{N^\gamma})\right]\right.}{-\gamma\log\left[\left(1-\frac{\rho\psi^2}{N^\gamma}\right)\right]} \right\} - 1 \\
&\sim \left\lceil \frac{\log\left[1+\rho\gamma t\right]}{-\gamma\log\left[\left(1-\frac{\rho\psi^2}{N^\gamma}\right)\right]} \right\rceil \\
&\sim \left\lceil \frac{\log\left[1+\rho\gamma t\right]}{\rho\gamma} \frac{N^\gamma}{\psi^2} \right\rceil,
\end{aligned} \tag{S3}
$$

where we have used that $\log\left[1 - \frac{\rho\psi^2}{N^\gamma}\right] \sim -\frac{\rho\psi^2}{N^\gamma}$ for sufficiently large N. Finally, the derivation showing that the ancestral process of the Moran model converges to a (time-inhomogeneous) psi-coalescent (eq. 38) is given by

$$
\begin{aligned}
\lim_{N\to\infty} \sum_{n=1}^{\mathcal{G}_N^{-1}(t)} \Phi_1^{(N)}(n;b) &= \lim_{N\to\infty} \sum_{n=1}^{\mathcal{G}_N^{-1}(t)} G_{b,b} \\
&= \lim_{N\to\infty} \sum_{n=1}^{\mathcal{G}_N^{-1}(t)} \binom{b}{b} \sum_{u=2}^{N} P_U(u) \frac{(u)_b (N_n - u)_{b-b}}{(N_n)_b} \\
&= \lim_{N\to\infty} \sum_{n=1}^{\mathcal{G}_N^{-1}(t)} \frac{(1 - N_n^{\gamma})(2)_b + N_n^{-\gamma}(N_n\psi)_b}{(N_n)_b} \\
&= \lim_{N\to\infty} \sum_{n=1}^{\mathcal{G}_N^{-1}(t)} \frac{N_n^{-\gamma}(N_n\psi)^b)}{N_n^b} \\
&= \lim_{N\to\infty} \sum_{n=1}^{\mathcal{G}_N^{-1}(t)} \frac{\psi^b}{N_n^{\gamma}} \\
&= \lim_{N\to\infty} \frac{\psi^b}{N^{\gamma}} \sum_{n=1}^{\mathcal{G}_N^{-1}(t)} \left(\left(1 - \frac{\rho\psi^2}{N^{\gamma}}\right)^{-\gamma} \right)^n \\
&= \lim_{N\to\infty} \frac{\psi^b}{N^{\gamma}} \frac{\left(1 - \frac{\rho\psi^2}{N^{\gamma}}\right)^{-\gamma\mathcal{G}_N^{-1}(t)} - 1}{1 - \left(1 - \frac{\rho\psi^2}{N^{\gamma}}\right)^{\gamma}} \\
&= \lim_{N\to\infty} \frac{\psi^b}{N^{\gamma}} \frac{\left(1 - \frac{\rho\psi^2}{N^{\gamma}}\right)^{-\gamma\mathcal{G}_N^{-1}(t)} - 1}{\frac{\rho\gamma\psi^2}{N^{\gamma}}} \\
&= \lim_{N\to\infty} \psi^{b-2} \frac{\left(1 - \frac{\rho\psi^2}{N^{\gamma}}\right)^{-\gamma\mathcal{G}_N^{-1}(t)} - 1}{\rho\gamma} \\
&= \lim_{N\to\infty} \psi^{b-2} \frac{\exp\left(\frac{\rho\gamma\psi^2}{N^{\gamma}} \mathcal{G}_N^{-1}(t)\right) - 1}{\rho\gamma} \\
&= \lim_{N\to\infty} \psi^{b-2} \frac{\exp\left(\frac{\rho\gamma\psi^2}{N^{\gamma}} \frac{N^{\gamma}}{\rho\gamma\psi^2} \log(1 + \rho\gamma t)\right) - 1}{\rho\gamma} \\
&= \psi^{b-2} \frac{\exp(\log(1 + \rho\gamma t)) - 1}{\rho\gamma} \\
&= \psi^{b-2} t.
\end{aligned} \tag{S4}
$$