Supplement 1

Case 1: Centered Two-Sided Truncation Provided both x_a and x_b are symmetrically allocated around the mean of P and F_1, it follows that i and k (Eqs. 4 and 5) are reduced to

$$\begin{aligned} i &= 0; \\ k &= \frac{\beta\phi(\beta) - \alpha\phi(\alpha)}{\Phi(\beta) - \Phi(\alpha)}. \end{aligned} \tag{S1.1}$$

Correspondingly Eq. 10 is simplified to

$$\frac{pr}{P(Y)} = \Phi\left(\frac{\beta}{\sqrt{1-0.5Bkh^4}}\right) - \Phi\left(\frac{\alpha}{\sqrt{1-0.5Bkh^4}}\right). \tag{S1.2}$$

Due to the symmetry of the cdf, the above equation yields

$$\frac{pr}{P(Y)} = \left(1 - \Phi\left(\frac{\alpha}{\sqrt{1-0.5Bkh^4}}\right)\right) - \Phi\left(\frac{\alpha}{\sqrt{1-0.5Bkh^4}}\right), \tag{S1.3}$$

which can be further simplified as

$$\Phi^{-1}\left(0.5 - \frac{pr}{2P(Y)}\right) = \frac{\alpha}{\sqrt{1-0.5Bkh^4}}, \tag{S1.4}$$

where $\Phi^{-1}(x)$ is the quantile function (inverse of the cdf) and h^2 is then calculated as

$$h^2 = \frac{\sqrt{2}}{|Q|}\left(\frac{Q^2-\alpha^2}{Bk}\right)^{\frac{1}{2}}, \tag{S1.5}$$

where $Q = \Phi^{-1}\left(0.5 - \frac{pr}{2P(Y)}\right)$.

Case 2: Right-Sided Truncation When $x_b \to +\infty$ (Fig. 1), i and k (Eqs. 4 and 5) reduce to

$$\begin{aligned} i &= \frac{\phi(\alpha)}{1-\Phi(\alpha)}; \\ k &= i(i-\alpha). \end{aligned} \tag{S1.6}$$

Under these circumstances, i and k correspond to the standard tabulated values for the right-truncated selection (e.g. (Falconer and Mackay 1996)) and the rearrangement of Eq. 10 leads to

$$\frac{pr}{P(Y)} = 1 - \Phi\left(\frac{\alpha - Bih^2}{\sqrt{1-0.5Bkh^4}}\right), \tag{S1.7}$$

and

$$\Phi^{-1}\left(1 - \frac{pr}{P(Y)}\right) = \frac{\alpha - Bih^2}{\sqrt{1-0.5Bkh^4}}. \tag{S1.8}$$

Further arrangement leads to

$$h^2 = \frac{2\alpha Bi \pm \left[4\alpha^2B^2i^2 - 2\left(2B^2i^2 + BQ'^2k\right)\left(\alpha^2 - Q'^2\right)\right]^{\frac{1}{2}}}{\left(2B^2i^2 + BQ'^2k\right)}, \tag{S1.9}$$

where $Q' = \Phi^{-1}\left(1 - \frac{pr}{P(Y)}\right)$. There are two solutions of the above equation and verification using Eq. S1.7 provides the unique one.

Variance of the h^2 ***estimate*** Here we use the general h^2 estimate (Eq. 10) and further, we assume that p, i, and k are constants as they are all functions of the known input parameters x_a, x_b, μ, and σ_p^2. Thus, to evaluate the variance of the h^2 estimate, we assume that h^2 is the exclusive function of r. Utilizing the maximum likelihood and linearization methods, we derived the following standard errors.

The standard error associated of the general h^2 estimate (Eq. 10) is

$$\text{s.e.}\{h^2\} = \frac{\left(1 - 0.5h^4kB\right)^{\frac{3}{2}}}{\left|\phi\left(b\right)\left(-iB + 0.5h^2\beta kB\right) - \phi\left(a\right)\left(-iB + 0.5h^2\alpha kB\right)\right|} \frac{p}{P(Y)} \sqrt{r(1-r)} \frac{1}{\sqrt{n}}, \tag{S1.10}$$

where n is the number of progeny in p, $h^2 \neq 0$ under symmetric two-sided truncation, and a and b are

$$\begin{aligned} a &= \frac{\alpha - Bih^2}{\sqrt{1 - 0.5Bkh^4}}, \\ b &= \frac{\beta - Bih^2}{\sqrt{1 - 0.5Bkh^4}}. \end{aligned} \tag{S1.11}$$

The standard error of the h^2 estimate for the scenario of right-sided truncation (Eq. 12) is

$$\text{s.e.}\{h^2\} = \frac{\left(1 - 0.5h^4kB\right)^{\frac{3}{2}}}{\left|-\phi\left(a\right)\left(-iB + 0.5h^2\alpha kB\right)\right|} \frac{p}{P(Y)} \sqrt{r(1-r)} \frac{1}{\sqrt{n}}. \tag{S1.12}$$