A Online appendix for *Individual Survival Curves Comparing Subjective and Observed Mortality Risks*

A.1 Derivation of the expected survival probability

We can derive the expected survival probability from age a to age t as follows:

$$S_a(t|\boldsymbol{x_i}) = \int \frac{S(t|\boldsymbol{x_i}, \nu)}{S(a|\boldsymbol{x_i}, \nu)} f(v|T > a) dv$$

which, given that

$$f(v|T > a) = S(a|\boldsymbol{x_i}, \nu) f(v)/S(a|\boldsymbol{x_i})$$

yields

$$S_a(t|\boldsymbol{x_i}) = \frac{1}{S(a|\boldsymbol{x_i})} \int S(t|\boldsymbol{x_i}, \nu) f(v) dv = \frac{S(t|\boldsymbol{x_i})}{S(a|\boldsymbol{x_i})}.$$

We can use known results for the integration of the hazard over the gamma distribution to obtain a closed-form solution (see for instance Cameron and Trivedi, 2005, pp. 615-616),

$$S_a(t|\boldsymbol{x_i}) = \left(\frac{\delta + \Lambda(a|\boldsymbol{x_i})}{\delta + \Lambda(t|\boldsymbol{x_i})} \right)^{\delta}. \tag{21}$$

A.2 Distribution of the frailty terms for respondents aged a

Given that we need the distribution of survival probabilities to age t given survival to a in order to model rounding, denote the conditional distribution of subjective survival rates $F_s(s^S|T > a, \boldsymbol{x_i})$. To obtain an expression for this distribution, we can use $F_{\nu s}(v|T > a, \boldsymbol{x_i})$, the distribution of ν^S conditional on surviving to time a:

$$F_s(s^S|T > a, \boldsymbol{x_i}) = 1 - F_{\nu s} \left(-\frac{\ln s^S}{\Lambda_a^S(t|\boldsymbol{x_i})} \middle| T > a, \boldsymbol{x_i} \right).$$

This is due to the fact that for two random variables (Y, X), if $Y = g(X)$, then $F(y) = F_X(g^{-1}(y))$.

Remember that the Gamma distribution $\mathcal{G}(c, d)$ is given by

$$F(x) = \int_0^x \frac{d^{-c}\, x^{c-1} \exp\left(-\frac{u}{d}\right)}{\Gamma(c)} \mathrm{d}u$$

and this distribution has an expected value of cd and a variance of cd^2. Notice that if $d = 1/\delta$ and $c = \delta$, the distribution has a unit expectation and a variance of $1/\delta$.

Let $d = \delta^{-1}$ such that ν^S at $a = 0$ is distributed gamma $\mathcal{G}(\delta, d)$. Given the value of d , ν^S has unit expectation. The distribution of ν^S conditional on being alive at age a is:

$$\begin{aligned} F_{\nu^S}(\nu|T > a, \boldsymbol{x_i}) &= \frac{1}{S^S(a|\boldsymbol{x_i})} \int_0^\nu S^S(a|\boldsymbol{x_i}, u) f(u) \mathrm{d}u \\ &= \frac{1}{S^S(a|\boldsymbol{x_i})} \int_0^\nu \exp(-u\Lambda^S(a|\boldsymbol{x_i})) \frac{d^{-\delta} u^{\delta-1} \exp(-\frac{u}{d})}{\Gamma(\delta)} \mathrm{d}u \\ &= \frac{d^{-\delta}}{S^S(a|\boldsymbol{x_i})} \int_0^\nu u^{\delta-1} \frac{\exp(-u(\Lambda^S(a|\boldsymbol{x_i}) + \delta))}{\Gamma(\delta)} \mathrm{d}u. \end{aligned}$$

For the sake of exposition, let $k = 1/(\Lambda^S(a|\boldsymbol{x_i}) + \delta)$, and remember that $S^S(a|\boldsymbol{x_i}) = (d/k)^{-\delta}$

$$\begin{aligned} F_{\nu^S}(\nu|T > a, \boldsymbol{x_i}) &= \frac{d^{-\delta}}{S^S(a|\boldsymbol{x_i})} \int_0^\nu u^{\delta-1} \frac{\exp(-\frac{u}{k})}{\Gamma(\delta)} \mathrm{d}u \\ &= \frac{1}{S^S(a|\boldsymbol{x_i})} \frac{d^{-\delta}}{k^{-\delta}} \int_0^\nu k^{-\delta} u^{\delta-1} \frac{\exp(-\frac{u}{k})}{\Gamma(\delta)} \mathrm{d}u \\ &= \int_0^\nu \frac{k^{-\delta} u^{\delta-1} \exp(-\frac{u}{k})}{\Gamma(\delta)} \mathrm{d}u. \end{aligned}$$

Then it follows directly that $F_{\nu^S}(v|T > a, \boldsymbol{x_i})$ is distributed gamma $\mathcal{G}(\delta, k)$, leading to an expected value of $\frac{\delta}{\Lambda^S(a|\boldsymbol{x_i})+\delta}$ and a variance of $\frac{\delta^2}{(\Lambda^S(a|\boldsymbol{x_i})+\delta)^2}$.

This makes explicit that the mean and variance of the frailty term decreases as age increases, as the integrated hazard is expected to increase with age, leading to a decrease in k.