Modeling biphasic environmental decay of pathogens and implications for risk analysis: Supporting information

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Appendix A. Theoretical Proofs

Proposition 1. *Except for degenerate parameter combinations, the model given in Eq. (4) displays biphasic behavior in $E_1(t)$ and in $E(t) = E_1(t) + E_2(t)$. That is, for some parameter combinations a, b, c, d, and h, $E(t)$ and $E_1(t)$ have the form*

$$cde^{-at} + (1-c)de^{-bt} + F(t) \tag{5}$$

where

$$F(t) = \int_0^t h\upsilon(s)e^{-a(t-s)} + (1-h)\upsilon(s)e^{-b(t-s)}\,ds \tag{6}$$

is a forcing function with $\upsilon(t) = u(t)$ for $E(t)$ and $\upsilon(t) = \eta u(t)$ for $E_1(t)$.

Proof. We solve the inhomogeneous, linear system of differential equations in Eq. (4) by the method of variation of parameters, although other methods, such as Laplace transformation, may also be used. We use dot notation to indicate derivative and drop the explicit dependence of E, E_1, and E_2 on t. The system may be written in matrix form as follows.

$$\begin{bmatrix} \dot{E}_1 \\ \dot{E}_2 \end{bmatrix} = \begin{bmatrix} -(\theta_1+\delta_1) & \delta_2 \\ \delta_1 & -(\theta_2+\delta_2) \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} + \begin{bmatrix} \eta u(t) \\ (1-\eta)u(t) \end{bmatrix}. \tag{S1}$$

The eigenvalues of the corresponding homogeneous system are

$$\lambda_1 = \frac{-(\theta_1+\theta_2+\delta_1+\delta_2) - \sqrt{(\theta_1+\theta_2+\delta_1+\delta_2)^2 - 4\left((\theta_1+\delta_1)(\theta_2+\delta_2)-\delta_1\delta_2\right)}}{2}, \tag{S2}$$

$$\lambda_2 = \frac{-(\theta_1+\theta_2+\delta_1+\delta_2) + \sqrt{(\theta_1+\theta_2+\delta_1+\delta_2)^2 - 4\left((\theta_1+\delta_1)(\theta_2+\delta_2)-\delta_1\delta_2\right)}}{2}. \tag{S3}$$

with eigenvectors

$$\nu_1 = \begin{bmatrix} \delta_2 \\ \\ \theta_1 + \delta_1 + \lambda_1 \end{bmatrix}$$

and

$$\nu_2 = \begin{bmatrix} \theta_2 + \delta_2 + \lambda_2 \\ \\ \delta_1 \end{bmatrix}.$$

Then, a fundamental solution for the homogeneous system is

$$\Phi(t) = \begin{bmatrix} \delta_2 e^{\lambda_1 t} & (\theta_2 + \delta_2 + \lambda_2) e^{\lambda_2 t} \\ \\ (\theta_1 + \delta_1 + \lambda_1) e^{\lambda_1 t} & \delta_1 e^{\lambda_2 t} \end{bmatrix}. \tag{S4}$$

Then

$$\Phi^{-1}(t) = \frac{1}{(\theta_2 + \delta_2 + \lambda_2)(\theta_1 + \delta_1 + \lambda_1) - \delta_1 \delta_2} \begin{bmatrix} -\delta_1 e^{-\lambda_1 t} & (\theta_2 + \delta_2 + \lambda_2) e^{-\lambda_1 t} \\ \\ (\theta_1 + \delta_1 + \lambda_1) e^{-\lambda_2 t} & -\delta_2 e^{-\lambda_2 t} \end{bmatrix}, \tag{S5}$$

$$\Phi^{-1}(t) \begin{bmatrix} \eta u(t) \\ \\ (1-\eta) u(t) \end{bmatrix} = \begin{bmatrix} \frac{(\theta_2+\delta_2+\lambda_2)(1-\eta)-\delta_1\eta}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{-\lambda_1 t} \\ \\ \frac{(\theta_1+\delta_1+\lambda_1)\eta-\delta_2(1-\eta)}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{-\lambda_2 t} \end{bmatrix} u(t), \tag{S6}$$

$$\int_0^t \Phi^{-1}(s) \begin{bmatrix} \eta u(s) \\ \\ (1-\eta) u(t) \end{bmatrix} ds = \int_0^t \begin{bmatrix} \frac{(\theta_2+\delta_2+\lambda_2)(1-\eta)-\delta_1\eta}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{-\lambda_1 s} \\ \\ \frac{(\theta_1+\delta_1+\lambda_1)\eta-\delta_2(1-\eta)}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{-\lambda_2 s} \end{bmatrix} u(s)\, ds, \tag{S7}$$

$$
\Phi(t)\int_0^t \Phi^{-1}(s)\begin{bmatrix} \eta u(s) \\ \\ (1-\eta)u(t) \end{bmatrix} ds
$$

$$
= \int_0^t \begin{bmatrix} \frac{\delta_2((\theta_2+\delta_2+\lambda_2)(1-\eta)-\delta_1\eta)}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{\lambda_1(t-s)} + \frac{(\theta_2+\delta_2+\lambda_2)((\theta_1+\delta_1+\lambda_1)\eta-\delta_2(1-\eta))}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2)} e^{\lambda_2(t-s)} \\ \frac{(\theta_1+\delta_1+\lambda_1)((\theta_2+\delta_2+\lambda_2)(1-\eta)-\delta_1\eta)}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{\lambda_1(t-s)} + \frac{\delta_1((\theta_1+\delta_1+\lambda_1\eta)-\delta_2(1-\eta))}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{\lambda_2(t-s)} \end{bmatrix} u(s)\,ds, \quad \text{(S8)}
$$

Suppose that the initial condition is

$$
\begin{bmatrix} \rho\omega \\ \\ (1-\rho)\omega \end{bmatrix}
$$

where ω is the initial total pathogen population and ρ is the fraction of the initial pathogen population of the first type. Then,

$$
\begin{bmatrix} \rho\omega \\ \\ (1-\rho)\omega \end{bmatrix} = \begin{bmatrix} \delta_2 & \theta_2+\delta_2+\lambda_2 \\ \\ \theta_1+\delta_1+\lambda_1 & \delta_1 \end{bmatrix} \begin{bmatrix} x_1 \\ \\ x_2 \end{bmatrix}, \quad \text{(S9)}
$$

so that we find

$$
\begin{bmatrix} x_1 \\ \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{(\delta_2+\theta_2+\lambda_2)(1-\rho)\omega-\delta_1\rho\omega}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} \\ \\ \frac{(\delta_1+\theta_1+\lambda_1)\rho\omega-\delta_2(1-\rho)\omega}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} \end{bmatrix}. \quad \text{(S10)}
$$

Hence, we have the solution

$$
\begin{aligned}
\begin{bmatrix} E_1(t) \\ E_2(t) \end{bmatrix} &= \Phi(t) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \Phi(t) \int_0^t \Phi^{-1}(s) \begin{bmatrix} \eta u(s) \\ (1-\eta)u(t) \end{bmatrix} ds \\
&= \begin{bmatrix} \delta_2 e^{\lambda_1 t} & (\theta_2+\delta_2+\lambda_2)e^{\lambda_2 t} \\ (\theta_1+\delta_1+\lambda_1)e^{\lambda_1 t} & \delta_1 e^{\lambda_2 t} \end{bmatrix} \begin{bmatrix} \frac{(\delta_2+\theta_2+\lambda_2)(1-\rho)\omega-\delta_1\rho\omega}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} \\ \frac{(\delta_1+\theta_1+\lambda_1)\rho\omega-\delta_2(1-\rho)\omega}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} \end{bmatrix} \\
&+ \int_0^t \begin{bmatrix} \frac{\delta_2((\theta_2+\delta_2+\lambda_2)(1-\eta)-\delta_1\eta)}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{\lambda_1(t-s)} + \frac{(\theta_2+\delta_2+\lambda_2)((\theta_1+\delta_1+\lambda_1)\eta-\delta_2(1-\eta))}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2)} e^{\lambda_2(t-s)} \\ \frac{(\theta_1+\delta_1+\lambda_1)((\theta_2+\delta_2+\lambda_2)(1-\eta)-\delta_1\eta)}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{\lambda_1(t-s)} + \frac{\delta_1((\theta_1+\delta_1+\lambda_1\eta)-\delta_2(1-\eta))}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{\lambda_2(t-s)} \end{bmatrix} u(s)\, ds, \\
&= \begin{bmatrix} \frac{\delta_2((\delta_2+\theta_2+\lambda_2)(1-\rho)\omega-\delta_1\rho\omega)}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{\lambda_1 t} + \frac{(\theta_2+\delta_2+\lambda_2)((\delta_1+\theta_1+\lambda_1)\rho\omega-\delta_2(1-\rho)\omega)}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{\lambda_2 t} \\ \frac{(\theta_1+\delta_1+\lambda_1)((\delta_2+\theta_2+\lambda_2)(1-\rho)\omega-\delta_1\rho\omega)}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{\lambda_1 t} + \frac{\delta_1((\delta_1+\theta_1+\lambda_1)\rho\omega-\delta_2(1-\rho)\omega)}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{\lambda_2 t} \end{bmatrix} \\
&+ \int_0^t \begin{bmatrix} \frac{\delta_2((\theta_2+\delta_2+\lambda_2)(1-\eta)-\delta_1\eta)}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{\lambda_1(t-s)} + \frac{(\theta_2+\delta_2+\lambda_2)((\theta_1+\delta_1+\lambda_1)\eta-\delta_2(1-\eta))}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2)} e^{\lambda_2(t-s)} \\ \frac{(\theta_1+\delta_1+\lambda_1)((\theta_2+\delta_2+\lambda_2)(1-\eta)-\delta_1\eta)}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{\lambda_1(t-s)} + \frac{\delta_1((\theta_1+\delta_1+\lambda_1\eta)-\delta_2(1-\eta))}{(\theta_2+\delta_2+\lambda_2)(\theta_1+\delta_1+\lambda_1)-\delta_1\delta_2} e^{\lambda_2(t-s)} \end{bmatrix} u(s)\, ds,
\end{aligned}
\tag{S11}
$$

which may be written as

$$
\begin{aligned}
&= \begin{bmatrix} \left(\frac{\delta_2/\rho+\theta_2+\lambda_1}{\lambda_1-\lambda_2}\right)\rho\omega e^{\lambda_1 t} + \left(1-\frac{\delta_2/\rho+\theta_2+\lambda_1}{\lambda_1-\lambda_2}\right)\rho\omega e^{\lambda_2 t} \\ \left(\frac{\delta_1/(1-\rho)+\theta_1+\lambda_1}{\lambda_1-\lambda_2}\right)(1-\rho)\omega e^{\lambda_1 t} + \left(1-\frac{\delta_1/(1-\rho)+\theta_1+\lambda_1}{\lambda_1-\lambda_2}\right)(1-\rho)\omega e^{\lambda_2 t} \end{bmatrix} \\
&+ \int_0^t \begin{bmatrix} \left(\frac{\delta_2/\eta+\theta_2+\lambda_1}{\lambda_1-\lambda_2}\right)\eta e^{\lambda_1(t-s)} + \left(1-\frac{\delta_2/\eta+\theta_2+\lambda_1}{\lambda_1-\lambda_2}\right)\eta e^{\lambda_2(t-s)} \\ \left(\frac{\delta_1/(1-\eta)+\theta_1+\lambda_1}{\lambda_1-\lambda_2}\right)(1-\eta)e^{\lambda_1(t-s)} + \left(1-\frac{\delta_1/(1-\eta)+\theta_1+\lambda_1}{\lambda_1-\lambda_2}\right)(1-\eta)e^{\lambda_2(t-s)} \end{bmatrix} u(s)\, ds.
\end{aligned}
\tag{S12}
$$

Thus, we may reparameterize $E_1(t)$ to the form

$$
cde^{-at} + (1-c)de^{-bt} + \int_0^t h\upsilon(s)e^{-a(t-s)} + (1-h)\upsilon(s)e^{-b(t-s)}\, ds \tag{S13}
$$

by setting

$$
\begin{aligned}
a &= -\lambda_1, \\
b &= -\lambda_2, \\
c &= \frac{\delta_2/\rho + \theta_2 + \lambda_1}{\lambda_1 - \lambda_2} \\
d &= \rho\omega, \\
h &= \frac{\delta_2/\eta + \theta_2 + \lambda_1}{\lambda_1 - \lambda_2}, \\
v(t) &= \eta u(t).
\end{aligned} \tag{S14}
$$

We note that $c = 1$ when $\delta_2 = 0$, so that no biphasic behavior is observed.

Now, we see

$$
\begin{aligned}
&\left(\frac{\delta_2/\rho + \theta_2 + \lambda_1}{\lambda_1 - \lambda_2}\right)\rho + \left(\frac{\delta_1/(1-\rho) + \theta_1 + \lambda_1}{\lambda_1 - \lambda_2}\right)(1-\rho), \\
&= \frac{\delta_1 + \delta_2 + \theta_1(1-\rho) + \rho\theta_2 + \lambda_1}{\lambda_1 - \lambda_2} \\
&= \frac{\rho\theta_1 + (1-\rho)\theta_2 + \lambda_2}{\lambda_2 - \lambda_1}.
\end{aligned} \tag{S15}
$$

Then, we may reparameterize $E(t) = E_1(t) + E_2(t)$ to this biphasic form by setting

$$
\begin{aligned}
a &= -\lambda_1, \\
b &= -\lambda_2, \\
c &= \frac{\lambda_2 + \rho\theta_1 + (1-\rho)\theta_2}{\lambda_2 - \lambda_1}, \\
d &= \omega, \\
h &= \frac{\lambda_2 + \eta\theta_1 + (1-\eta)\theta_2}{\lambda_2 - \lambda_1}, \\
v(t) &= u(t).
\end{aligned} \tag{S16}
$$

Here, $c = 0$ when $\theta_1 = \theta_2$, so that biphasic behavior is not observed.

□

Proposition 2. *The parameter combinations a, b, c, d, and h in Proposition 1 are identifiable.*

Proof. First, consider the case where the data is $E = E_1 + E_2$. We find an input–output equation for the model in terms of E, assuming $\theta_1 \neq \theta_2$.

$$\begin{aligned}
E_1 &= E - E_2 \\
\dot{E}_1 &= \dot{E} - \dot{E}_2 \\
\eta u + \delta_2 E_2 - (\theta_1 + \delta_1) E_1 &= \dot{E} - (1-\eta)u - \delta_1 E_1 + (\theta_2 + \delta_2) E_2 \\
\eta u - \theta_1 (E - E_2) &= \dot{E} - (1-\eta)u(t) + \theta_2 E_2 \\
-\theta_1 E + \theta_1 E_2 &= \dot{E} - u + \theta_2 E_2 \\
E_2 &= \frac{\dot{E} + \theta_1 E - u}{\theta_1 - \theta_2} \\
\dot{E}_2 &= \frac{\ddot{E} + \theta_1 \dot{E} - \dot{u}}{\theta_1 - \theta_2} \\
(1-\eta)u + \delta_1 E_1 - (\theta_2 + \delta_2) E_2 &= \frac{\ddot{E} + \theta_1 \dot{E} - \dot{u}}{\theta_1 - \theta_2} \\
(1-\eta)u + \delta_1 E - (\theta_2 + \delta_1 + \delta_2) E_2 &= \frac{\ddot{E} + \theta_1 \dot{E} - \dot{u}}{\theta_1 - \theta_2} \\
(1-\eta)u + \delta_1 E - (\theta_2 + \delta_1 + \delta_2)\left(\frac{\dot{E} + \theta_1 E - u}{\theta_1 - \theta_2}\right) &= \frac{\ddot{E} + \theta_1 \dot{E} - \dot{u}}{\theta_1 - \theta_2} \\
((1-\eta)u + \delta_1 E)(\theta_1 - \theta_2) - (\theta_2 + \delta_1 + \delta_2)(\dot{E} + \theta_1 E - u) &= \ddot{E} + \theta_1 \dot{E} - \dot{u} \\
(\delta_1 + \delta_2 + (1-\eta)\theta_1 + \eta\theta_2)u - (\delta_1\theta_2 + \delta_2\theta_1 + \theta_1\theta_2)E &= \ddot{E} + (\theta_1 + \theta_2 + \delta_1 + \delta_2)\dot{E}
\end{aligned}$$

$$\ddot{E} + (\theta_1 + \theta_2 + \delta_1 + \delta_2)\dot{E} + ((\theta_1 + \delta_1)(\theta_2 + \delta_2) - \delta_1\delta_2) E = \dot{u} + (\delta_1 + \delta_2 + (1-\eta)\theta_1 + \eta\theta_2)u \tag{S17}$$

The coefficients of the input–output equation are the identifiable combinations. Additionally, we have $E(0) = E_1(0) + E_2(0) = \omega$ and $\dot{E}(0) = u(0) - \theta_1\rho\omega - \theta_2(1-\rho)\omega$ identifiable from the initial conditions.

Under the degenerate condition $\theta_1 = \theta_2 = \theta$, biphasic behavior is not observed, and the input–output equation is

$$\dot{E} + 2\theta E = u. \tag{S18}$$

Now, consider the case where the data is E_1, assuming $\delta_2 \neq 0$.

$$\begin{aligned}
E_2 &= \frac{1}{\delta_2}\dot{E}_1 + \frac{\theta_1+\delta_1}{\delta_2}E_1 - \frac{\eta}{\delta_2}u \\
\dot{E}_2 &= \frac{1}{\delta_2}\ddot{E}_1 + \frac{\theta_1+\delta_1}{\delta_2}\dot{E}_1 - \frac{\eta}{\delta_2}\dot{u} \\
(1-\eta)u + \delta_1 E_1 - (\theta_2+\delta_2)\,E_2 &= \frac{1}{\delta_2}\ddot{E}_1 + \frac{\theta_1+\delta_1}{\delta_2}\dot{E}_1 - \frac{\eta}{\delta_2}\dot{u} \\
(1-\eta)u + \delta_1 E_1 - (\theta_2+\delta_2)\left(\frac{1}{\delta_2}\dot{E}_1 + \frac{\theta_1+\delta_1}{\delta_2}E_1 - \frac{\eta}{\delta_2}u\right) &= \frac{1}{\delta_2}\ddot{E}_1 + \frac{\theta_1+\delta_1}{\delta_2}\dot{E}_1 - \frac{\eta}{\delta_2}\dot{u} \\
\delta_2(1-\eta)u + \delta_1\delta_2 E_1 - (\theta_2+\delta_2)\left(\dot{E}_1 + (\theta_1+\delta_1)E_1 - \eta u\right) &= \ddot{E}_1 + (\theta_1+\delta_1)\dot{E}_1 - \eta\dot{u} \\
\ddot{E}_1 + (\theta_1+\delta_1)\dot{E}_1 - \delta_1\delta_2 E_1 + (\theta_2+\delta_2)\left(\dot{E}_1 + (\theta_1+\delta_1)E_1\right) &= \eta\dot{u} + \delta_2(1-\eta)u + (\theta_2+\delta_2)\,\eta u \\
\ddot{E}_1 + (\theta_1+\theta_2+\delta_1+\delta_2)\dot{E}_1 + \left((\theta_2+\delta_2)(\theta_1+\delta_1) - \delta_1\delta_2\right)E_1 &= \eta\dot{u} + (\theta_2+\delta_2/\eta)\,\eta u
\end{aligned} \tag{S19}$$

Additionally, from the initial conditions, we have $E_1(0) = \rho\omega$ and $\dot{E}_1(0) = \eta u(0) + \delta_2(1-\rho)\omega - (\theta_1+\delta_1)\,\rho\omega$.

Under the degenerate condition $\delta_2 = 0$, the input–output equation is

$$\dot{E}_1 + (\theta_1+\delta_1)E_1 = \eta u. \tag{S20}$$

We now show that a, b, c, d, and h can be written in terms of the coefficients of the input–output equation under either data regime. From the proof of Proposition 1, one calculates

$$\begin{aligned} a &= \frac{1}{2}\left((\theta_1+\theta_2+\delta_1+\delta_2)+\sqrt{(\theta_1+\theta_2+\delta_1+\delta_2)^2-4\left((\theta_1+\delta_1)(\theta_2+\delta_2)-\delta_1\delta_2\right)}\right), \\ b &= \frac{1}{2}\left((\theta_1+\theta_2+\delta_1+\delta_2)-\sqrt{(\theta_1+\theta_2+\delta_1+\delta_2)^2-4\left((\theta_1+\delta_1)(\theta_2+\delta_2)-\delta_1\delta_2\right)}\right). \end{aligned} \tag{S21}$$

We see that a and b are combinations of identifiable combinations (coefficients of the input–output equation) and are thus identifiable. How the other three parameters are identified depends on whether one assumes that the measured environmental compartment is all pathogens $E(t)$ or only labile pathogens $E_1(t)$. If the environmental compartment is assumed to be $E(t)$, then one calculates

$$\begin{aligned} c &= \frac{\rho\theta_1+(1-\rho)\theta_2-b}{a-b}, \\ d &= \omega, \\ h &= \frac{(\eta\theta_1+(1-\eta)\theta_2)-b}{a-b}. \end{aligned} \tag{S22}$$

So, we see that c and d are identifiable if $u \equiv 0$ or if u is known. If $u \not\equiv 0$ and is known, then $\eta\theta_1+(1-\eta)\theta_2$ is identifiable, and so h is identifiable.

If the environmental compartment is assumed to be $E_1(t)$, then one calculates

$$\begin{aligned} c &= \frac{\delta_2/\rho+\theta_2-a}{b-a} \\ d &= \rho\omega, \\ h &= \frac{\delta_2/\eta+\theta_2-a}{b-a}. \end{aligned} \tag{S23}$$

So, again, c and d are identifiable if $u \equiv 0$ or if u is known. If $u \not\equiv 0$ and $\eta u(t)$ is known, then $\theta_2+\delta_2/\eta$ is identifiable, and so h is identifiable.

□

Appendix B. Additional details and figures of the hydrological fate and transport model

Assuming, per Robles-Morua et al., an initial *E. coli* concentration of 24.63 times the EPA standard for bathwater, which is 126 cfu/100 mL (EPA, 1986), we compare the estimated population of *E. coli* at a distance x downstream of the waterwater treatment outfall by generating a posterior distribution from 100,000 Markov chain Monte Carlo simulations in WinBUGS in three scenarios:

1. Biphasic decay

$$P(x) = 24.63 \cdot 126 \left(ce^{-ax/16.75} + (1-c)e^{-bx/16.75} \right) e^{-k_s x/16.75}$$

2. Monophasic decay fit to the rapid-decaying regime

$$P(x) = 24.63 \cdot 126 e^{-(a+k_s)x/16.75}$$

3. Monophasic decay fit to entire data

$$P(x) = 24.63 \cdot 126 e^{-(a^*+k_s)x/16.75}$$

where a, b, and c are described as in the text, and a^* was assumed to be normally distributed with mean μ_{a^*}=0.79 and standard deviation σ_{a^*}=0.044. This distribution was estimated from the biphasic data presented in Hellweger et al. by generating a posterior distribution from 100,000 Markov chain Monte Carlo simulations in WinBUGS using the following prior distributions, where $C_{i,j}$ is the concentration at day i for the jth experiment, $C_{0,j}$ is the initial condition of the jth experiment, and $C_{0,j}^-$ and $C_{0,j}^+$ are the 95% confidence bounds for the initial condition.

$$\ln(C_{i,j}) \sim N(\mu_{C_{i,j}}, \tau_{C_{i,j}})$$

$$\mu_{C_{i,j}} = \ln(C_{0,j}) - a^* t_i$$

$$\ln(C_{0,j}) \sim U(C^-_{0,j}, C^+_{0,j})$$

$$\tau_c \sim \Gamma(10^{-5}, 10^{-5})$$

$$a^* \sim N(\mu_{a^*}, \tau_{a^*})$$

$$\mu_{a^*} \sim N(0, 10^{-6})$$

$$\tau_{a^*} \sim \Gamma(10^{-5}, 10^{-5})$$

Because the removal rate calculated by Robles-Morua et al. represents both pathogen decay and removal by sedimentation, we first perform Monte Carlo simulations to estimate total *E. coli* removal according to their normally distributed removal rate coefficient k (μ_k=2.15, σ_k=0.53). We then estimate a distribution of values for the sedimentation rate coefficient k_s by subtracting the vector of biphasic decay coefficients at the last sampling time point (~1.24 days; Figure 6 of Robles-Morua *et al.* from the vector of removal rate coefficients, each sorted in ascending order. This method of estimation results in some negative values of k_s, which can be interpreted as resuspension of previously sedimented *E. coli*.

In Figure S1, we extend the simulations up to 120 km, assuming that river conditions remain constant beyond 60 km. Here, we see that Scenario 3 becomes an underestimate of the bacterial concentration at around 76 km.

In Figure S2, we plot *E. coli* concentrations simulated for Scenario 1 by distance and simulation percentile. Simulations in percentiles above the 25th percentile exhibit regrowth of bacteria within 60 km, although this regrowth is not substantial until the 50th percentile.

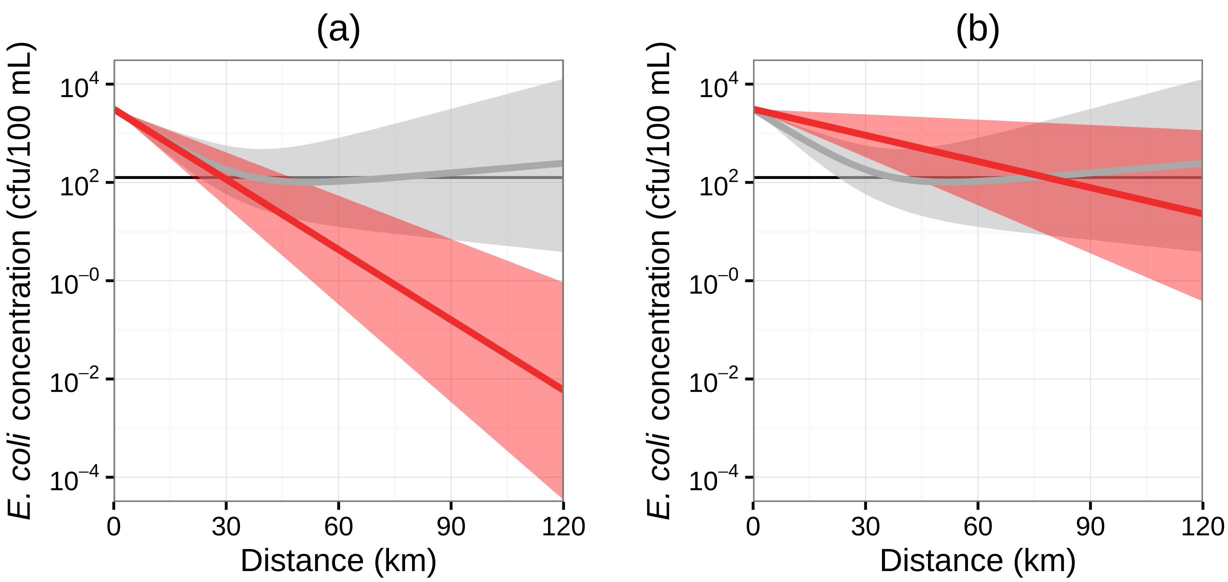

Figure S1: Comparative simulation results for (a) Scenarios 1 (biphasic, in grey) and 2 (monophasic fit to the labile regime, in red) and (b) Scenarios 1 (biphasic, in grey) and 3 (monophasic fit to entire data, in red). Each scenario shows the median and 95% CI simulated *E. coli* concentrations over 60 km of hydrologic transport. The black line gives the EPA regulatory compliance threshold of 126 cfu/100 mL.

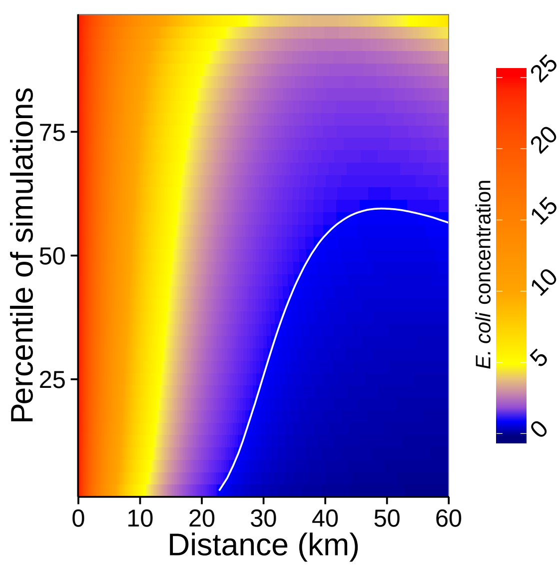

Figure S2: *E. coli* concentrations simulated for Scenario 1 by percentile and distance. The white line is the compliance concentration (126 cfu/100 mL).