Evolution of reproductive parasites with direct fitness benefits

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Supplementary Information

Supplementary Figure S1:

Direct fitness benefits have a strong effect on CI equilibrium frequency

Supplementary Information 1:

Recursion equations and derivation of invasion conditions for infection dynamics with two symbionts, but without doubly infected hosts

Supplementary Information 2:

Recursion equations for infection dynamics with two symbionts and doubly infected hosts

Supplementary Figure S1:

Direct fitness benefits have a strong effect on CI equilibrium frequency

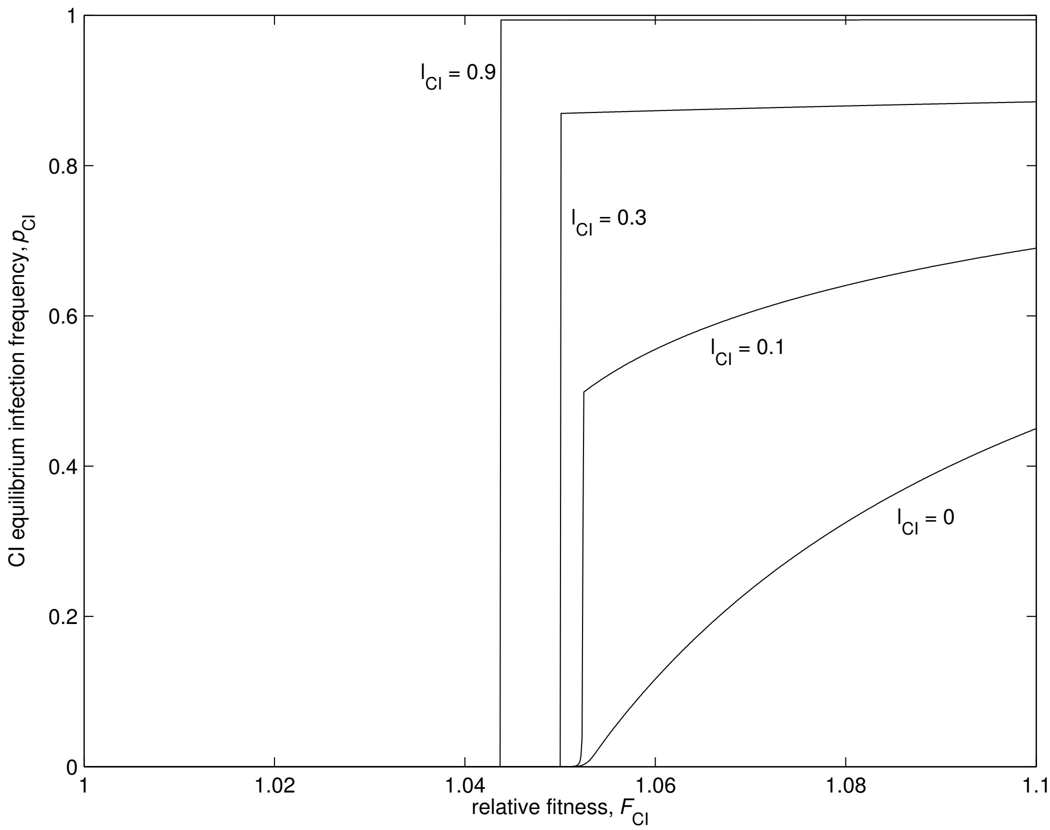

Figure 1: The effect of direct fitness benefits on CI equilibrium frequency $\hat{p}_{\mathrm{CI}}$ for different l_{CI} values, given a low initial frequency. A small fitness benefit is unable to raise the infection above the threshold. For most l_{CI} values, a switch-like increase in $\hat{p}_{\mathrm{CI}}$ occurs where $p_{\mathrm{CI}}^{\mathrm{ini}} = p_{\mathrm{CI}}^{\mathrm{thr}}$. Above this point, the positive effect of F_{CI} is small for larger l_{CI} values, but large for small l_{CI} values. Parameters take the values $p_{\mathrm{CI}}^{\mathrm{ini}} = 0.01$, and $t_{\mathrm{CI}} = 0.95$.

Supplementary Information 1:

Recursion equations and derivation of invasion conditions for infection dynamics with two symbionts, but without doubly infected hosts

Here we describe in detail how invasion conditions can be derived from the recursion equations, following an approach taken by Kriesner *et al.* (2013). The same approach is used in all further sections.

Invasion of a beneficial symbiont into a CI population

We consider a population to be infected with a CI strain at equilibrium frequency. We are interested in the conditions for an initially rare beneficial strain to increase in frequency. Individuals can be infected with a CI strain, with a beneficial strain, or uninfected. Thus we assume that individuals cannot be doubly infected with both strains. We denote the frequency of each female type by p_{CI}, $p_{\oplus}$, and p_{U}, respectively. The recursions for the frequencies of the three female types are

$$p'_{\mathrm{CI}} = \frac{(p_{\mathrm{CI}} F_{\mathrm{CI}} t_{\mathrm{CI}})\left[p_{\mathrm{CI}} + p_{\oplus} + p_{\mathrm{U}}\right]}{\overline{w}}, \tag{A1a}$$

$$p'_{\oplus} = \frac{(p_{\oplus} F_{\oplus} t_{\oplus})\left[(1 - l_{\mathrm{CI}})\, p_{\mathrm{CI}} + p_{\oplus} + p_{\mathrm{U}}\right]}{\overline{w}}, \tag{A1b}$$

$$p'_{\mathrm{U}} = \frac{(p_{\mathrm{U}} + p_{\mathrm{CI}} F_{\mathrm{CI}}\,(1 - t_{\mathrm{CI}}) + p_{\oplus} F_{\oplus}\,(1 - t_{\oplus}))\left[(1 - l_{\mathrm{CI}})\, p_{\mathrm{CI}} + p_{\oplus} + p_{\mathrm{U}}\right]}{\overline{w}}, \tag{A1c}$$

where

$$\overline{w} = 2\left(p_{\mathrm{CI}} F_{\mathrm{CI}}\left((1 - l_{\mathrm{CI}}\,(1 - t_{\mathrm{CI}}))\, p_{\mathrm{CI}} + p_{\oplus} + p_{\mathrm{U}}\right) + (p_{\oplus} F_{\oplus} + p_{\mathrm{U}})\left((1 - l_{\mathrm{CI}})\, p_{\mathrm{CI}} + p_{\oplus} + p_{\mathrm{U}}\right)\right).$$

The first term in the numerator (in round brackets) denotes the maternal contribution, and the second term (in square brackets) denotes the paternal contribution. The frequencies of each type do not differ between the sexes, therefore the paternal contribution can be expressed in terms of female frequencies.

Now consider the condition for a beneficial strain to increase when it is extremely rare in a population infected at equilibrium with a CI strain. With $p_{\oplus} \approx 0$, this situation corresponds to the scenario with one-symbiont CI dynamics. When the CI strain is at equilibrium ($p'_{\mathrm{CI}} = p_{\mathrm{CI}} = \hat{p}_{\mathrm{CI}}$), we see from equation (A1a) that $\overline{w} = F_{\mathrm{CI}} t_{\mathrm{CI}} \left[p_{\mathrm{CI}} + p_{\oplus} + p_{\mathrm{U}}\right]$. The condition for the beneficial strain to increase when rare is $p'_{\oplus} > p_{\oplus}$ and thus, from equation (A1b), $F_{\oplus} t_{\oplus} \left[(1 - l_{\mathrm{CI}})\, p_{\mathrm{CI}} + p_{\oplus} + p_{\mathrm{U}}\right] > \overline{w}$. Combining both equations, we get $F_{\oplus} t_{\oplus} \left[(1 - l_{\mathrm{CI}})\, p_{\mathrm{CI}} + p_{\oplus} + p_{\mathrm{U}}\right] > F_{\mathrm{CI}} t_{\mathrm{CI}} \left[p_{\mathrm{CI}} + p_{\oplus} + p_{\mathrm{U}}\right]$. Considering that $p_{\mathrm{CI}} + p_{\oplus} + p_{\mathrm{U}} = 0.5$, we get $F_{\oplus} t_{\oplus} \left(0.5 - l_{\mathrm{CI}} \hat{p}_{\mathrm{CI}}\right) > F_{\mathrm{CI}} t_{\mathrm{CI}}/2$ and hence Condition (11) in the main text:

$$F_{\oplus} t_{\oplus} (1 - 2 l_{\mathrm{CI}} \hat{p}_{\mathrm{CI}}) > F_{\mathrm{CI}} t_{\mathrm{CI}}.$$

If the beneficial strain is able to rescue CI (i.e. if it is a *mod*$^-$*resc*$^+$ strain), equation (A1b) simplifies to $p'_{\oplus} = (p_{\oplus} F_{\oplus} t_{\oplus}) \left[p_{\mathrm{CI}} + p_{\oplus} + p_{\mathrm{U}}\right] / \overline{w}$. Hence, for the invasion condition for a beneficial *resc*$^+$ strain, we get Condition (12) in the main text:

$$F_{\oplus} t_{\oplus} > F_{\mathrm{CI}} t_{\mathrm{CI}}.$$

Invasion of a beneficial symbiont into a MK population

The recursions for the frequencies of the three female types are

$$p'_{\mathrm{MK}} = \frac{p_{\mathrm{MK}} R F_{\mathrm{MK}} t_{\mathrm{MK}}}{\overline{w}}, \tag{A2a}$$

$$p'_{\oplus} = \frac{p_{\oplus} F_{\oplus} t_{\oplus}}{\overline{w}}, \tag{A2b}$$

$$p'_{\mathrm{U}} = \frac{p_{\mathrm{U}} + p_{\mathrm{MK}} R F_{\mathrm{MK}} (1 - t_{\mathrm{MK}}) + p_{\oplus} F_{\oplus} (1 - t_{\oplus})}{\overline{w}}, \tag{A2c}$$

where

$$\overline{w} = p_{\mathrm{MK}} R F_{\mathrm{MK}} \left(2 - (1 - v)\, t_{\mathrm{MK}}\right) + 2 \left(p_{\oplus} F_{\oplus} + p_{\mathrm{U}}\right).$$

Using the same reasoning as above, we derive at Condition (14) in the main text:

$$F_{\oplus} t_{\oplus} > R F_{\mathrm{MK}} t_{\mathrm{MK}}.$$

Invasion of a MK symbiont into a CI population

The recursions for the frequencies of the three female types are

$$p'_{\mathrm{CI}} = \frac{\left(p_{\mathrm{CI}} F_{\mathrm{CI}} t_{\mathrm{CI}}\right)\left[p_{\mathrm{CI}} + p_{\mathrm{MK}} v + p_{\mathrm{U}}\right]}{\overline{w}}, \tag{A3a}$$

$$p'_{\mathrm{MK}} = \frac{\left(p_{\mathrm{MK}} R F_{\mathrm{MK}} t_{\mathrm{MK}}\right)\left[\left(1 - l_{\mathrm{CI}}\right) p_{\mathrm{CI}} + p_{\mathrm{MK}} v + p_{\mathrm{U}}\right]}{\overline{w}}, \tag{A3b}$$

$$p'_{\mathrm{U}} = \frac{\left(p_{\mathrm{U}} + p_{\mathrm{CI}} F_{\mathrm{CI}}\left(1 - t_{\mathrm{CI}}\right) + p_{\mathrm{MK}} R F_{\mathrm{MK}}\left(1 - t_{\mathrm{MK}}\right)\right)\left[\left(1 - l_{\mathrm{CI}}\right) p_{\mathrm{CI}} + p_{\mathrm{MK}} v + p_{\mathrm{U}}\right]}{\overline{w}}, \tag{A3c}$$

where

$$\overline{w} = 2\left(p_{\mathrm{CI}} F_{\mathrm{CI}} t_{\mathrm{CI}}\right)\left[p_{\mathrm{CI}} + p_{\mathrm{MK}} v + p_{\mathrm{U}}\right] + (1 + v)\left(p_{\mathrm{MK}} R F_{\mathrm{MK}} t_{\mathrm{MK}}\right)\left[\left(1 - l_{\mathrm{CI}}\right) p_{\mathrm{CI}} + p_{\mathrm{MK}} v + p_{\mathrm{U}}\right]$$
$$+ 2\left(p_{\mathrm{U}} + p_{\mathrm{CI}} F_{\mathrm{CI}}\left(1 - t_{\mathrm{CI}}\right) + p_{\mathrm{MK}} R F_{\mathrm{MK}}\left(1 - t_{\mathrm{MK}}\right)\right)\left[\left(1 - l_{\mathrm{CI}}\right) p_{\mathrm{CI}} + p_{\mathrm{MK}} v + p_{\mathrm{U}}\right].$$

Note that for the frequency of MK-infected males in the paternal contribution (in square brackets) we use the term $p_{\mathrm{MK}} v$ because it is the only male frequency that does not equal the female one. Using the same reasoning as above, we derive at Condition (16) in the main text:

$$R F_{\mathrm{MK}} t_{\mathrm{MK}}\left(1 - 2 l_{\mathrm{CI}} \hat{p}_{\mathrm{CI}}\right) > F_{\mathrm{CI}} t_{\mathrm{CI}}.$$

Supplementary Information 2:

Recursion equations for infection dynamics with two symbionts and doubly infected hosts

The recursion equations for the frequencies of the four female types are:

$$p'_{\mathrm{CI+MK}} = \frac{\left(p_{\mathrm{CI+MK}} R F_{\mathrm{CI}} F_{\mathrm{MK}} t_{\mathrm{CI}} t_{\mathrm{MK}}\right)\left[\left(p_{\mathrm{CI+MK}} + p_{\mathrm{MK}}\right) v + p_{\mathrm{CI}} + p_{\mathrm{U}}\right]}{\overline{w}}, \tag{A4a}$$

$$p'_{\mathrm{CI}} = \frac{\left(\left(p_{\mathrm{CI}} + p_{\mathrm{CI+MK}} R F_{\mathrm{MK}} \left(1 - t_{\mathrm{MK}}\right)\right) F_{\mathrm{CI}} t_{\mathrm{CI}}\right)\left[\left(p_{\mathrm{CI+MK}} + p_{\mathrm{MK}}\right) v + p_{\mathrm{CI}} + p_{\mathrm{U}}\right]}{\overline{w}}, \tag{A4b}$$

$$\begin{aligned} p'_{\mathrm{MK}} &= \left(\left(p_{\mathrm{MK}} + p_{\mathrm{CI+MK}} F_{\mathrm{CI}} \left(1 - t_{\mathrm{CI}}\right)\right) R F_{\mathrm{MK}} t_{\mathrm{MK}}\right) \\ &\quad \times \left[\left(1 - l_{\mathrm{CI}}\right)\left(p_{\mathrm{CI+MK}} v + p_{\mathrm{CI}}\right) + p_{\mathrm{MK}} v + p_{\mathrm{U}}\right] \frac{1}{\overline{w}}, \end{aligned} \tag{A4c}$$

$$\begin{aligned} p'_{\mathrm{U}} &= \left(p_{\mathrm{U}} + p_{\mathrm{CI+MK}} R F_{\mathrm{CI}} F_{\mathrm{MK}} \left(1 - t_{\mathrm{CI}}\right)\left(1 - t_{\mathrm{MK}}\right) + p_{\mathrm{CI}} F_{\mathrm{CI}} \left(1 - t_{\mathrm{CI}}\right) + p_{\mathrm{MK}} R F_{\mathrm{MK}} \left(1 - t_{\mathrm{MK}}\right)\right) \\ &\quad \times \left[\left(1 - l_{\mathrm{CI}}\right)\left(p_{\mathrm{CI+MK}} v + p_{\mathrm{CI}}\right) + p_{\mathrm{MK}} v + p_{\mathrm{U}}\right] \frac{1}{\overline{w}}, \end{aligned} \tag{A4d}$$

where

$$\begin{aligned} \overline{w} &= (1 + v)\left(p_{\mathrm{CI+MK}} R F_{\mathrm{CI}} F_{\mathrm{MK}} t_{\mathrm{CI}} t_{\mathrm{MK}}\right)\left[\left(p_{\mathrm{CI+MK}} + p_{\mathrm{MK}}\right) v + p_{\mathrm{CI}} + p_{\mathrm{U}}\right] \\ &\quad + 2\left(\left(p_{\mathrm{CI}} + p_{\mathrm{CI+MK}} R F_{\mathrm{MK}} \left(1 - t_{\mathrm{MK}}\right)\right) F_{\mathrm{CI}} t_{\mathrm{CI}}\right)\left[\left(p_{\mathrm{CI+MK}} + p_{\mathrm{MK}}\right) v + p_{\mathrm{CI}} + p_{\mathrm{U}}\right] \\ &\quad + (1 + v)\left(\left(p_{\mathrm{MK}} + p_{\mathrm{CI+MK}} F_{\mathrm{CI}} \left(1 - t_{\mathrm{CI}}\right)\right) R F_{\mathrm{MK}} t_{\mathrm{MK}}\right) \\ &\quad \times \left[\left(1 - l_{\mathrm{CI}}\right)\left(p_{\mathrm{CI+MK}} v + p_{\mathrm{CI}}\right) + p_{\mathrm{MK}} v + p_{\mathrm{U}}\right] \\ &\quad + 2\left(p_{\mathrm{U}} + p_{\mathrm{CI+MK}} R F_{\mathrm{CI}} F_{\mathrm{MK}} \left(1 - t_{\mathrm{CI}}\right)\left(1 - t_{\mathrm{MK}}\right) + p_{\mathrm{CI}} F_{\mathrm{CI}} \left(1 - t_{\mathrm{CI}}\right) + p_{\mathrm{MK}} R F_{\mathrm{MK}} \left(1 - t_{\mathrm{MK}}\right)\right) \\ &\quad \times \left[\left(1 - l_{\mathrm{CI}}\right)\left(p_{\mathrm{CI+MK}} v + p_{\mathrm{CI}}\right) + p_{\mathrm{MK}} v + p_{\mathrm{U}}\right]. \end{aligned}$$