

S3 Appendix: Model details

The full network equations for the neural tube application in Sec. *Application to neural tube network* are of the form (1, 2) and taken from [24]:

$$\begin{aligned}\partial_t[\text{P}] &= \alpha_{\text{P}} \frac{w_{\text{P}}}{w_{\text{P}} + (1 + w_{\text{PO}}[\text{O}])^2(1 + w_{\text{PN}}[\text{N}])^2} - \beta_{\text{P}}[\text{P}] \\ \partial_t[\text{O}] &= \alpha_{\text{O}} \frac{w_{\text{O}}(1 + w_{\text{OS}}[\text{S}])}{w_{\text{O}}(1 + w_{\text{OS}}[\text{S}]) + (1 + w_{\text{ON}}[\text{N}])^2(1 + w_{\text{OI}}[\text{I}])^2} - \beta_{\text{O}}[\text{O}] \\ \partial_t[\text{N}] &= \alpha_{\text{N}} \frac{w_{\text{N}}(1 + w_{\text{NS}}[\text{S}])}{w_{\text{N}}(1 + w_{\text{NS}}[\text{S}]) + (1 + w_{\text{NP}}[\text{P}])^2(1 + w_{\text{NO}}[\text{O}])^2(1 + w_{\text{NI}}[\text{I}])^2} - \beta_{\text{N}}[\text{N}] \\ \partial_t[\text{I}] &= \alpha_{\text{I}} \frac{w_{\text{I}}}{w_{\text{I}} + (1 + w_{\text{IO}}[\text{O}])^2(1 + w_{\text{IN}}[\text{N}])^2} - \beta_{\text{I}}[\text{I}]\end{aligned}$$

Here [S] is the level of Sonic Hedgehog signaling - the net amount of Gli activity, see [24] for details. This is taken to be an exponential gradient, $[\text{S}] = e^{-s/0.15}$ where $s \in [0, 1.2]$ labels the neural tube position. The remaining symbols in brackets indicate the concentration of the TFs. Parameters used are $\alpha_{\text{P}} = 2$, $\alpha_{\text{O}} = 2$, $\alpha_{\text{N}} = 2$, $\alpha_{\text{I}} = 2$, $\beta_{\text{P}} = 2$, $\beta_{\text{O}} = 2$, $\beta_{\text{N}} = 2$, $\beta_{\text{I}} = 2$, $w_{\text{PO}} = 1.9$, $w_{\text{PN}} = 26.7$, $w_{\text{P}} = 3.84$, $w_{\text{ON}} = 60.6$, $w_{\text{OI}} = 28.4$, $w_{\text{OS}} = 180$, $w_{\text{O}} = 38.24$, $w_{\text{NP}} = 4.8$, $w_{\text{NO}} = 27.1$, $w_{\text{NI}} = 47.1$, $w_{\text{NS}} = 373$, $w_{\text{N}} = 21.92$, $w_{\text{IO}} = 58.8$, $w_{\text{IN}} = 76.2$, $w_{\text{I}} = 18.72$.