Optomechanical measurement of the stiffness of single adherent cells

Kidong Park[1, a)], Ali Mehrnezhad[1], Elise A. Corbin[2], and Rashid Bashir[2,3]

[1] *Division of Electrical and Computer Engineering, Louisiana State University, Baton Rouge, LA, 70803*
[2] *Department of Bioengineering, University of Illinois at Urbana-Champaign, Urbana, IL 61802, USA*
[3] *Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61802, USA*
[a)] *Author to whom correspondence should be addressed. Electronic mail: kidongp@lsu.edu.*

One-dimensional analytical model

As shown in Fig. 2a, a cell attached on a vibrating substrate is modelled with one dimensional array of mass-spring-damper system. The height of the culture media and the height of the cell are approximated to 5 mm and 10 μm, respectively. The media region is divided with 500 layers and the cell region is divided with 10 layers. Mass, spring constant, and damping coefficient of each layer are obtained from the mechanical properties reported in literature.

Dynamical equation for each layer in the model can be described as below.

$$m_i x_i'' = k_i(x_i - x_{i-1}) + c_i(x_i' - x_{i-1}') + k_{i+1}(x_i - x_{i+1}) + c_{i+1}(x_i' - x_{i+1}') \qquad 1 \le i \le n \qquad \text{(S1)}$$

where m_i, k_i, c_i, and x_i are mass, spring constant, damping coefficient, and displacement of each layer, respectively. The top ceiling is assumed to be stationary and the displacement of the top ceiling, x_{n+1} is fixed to 0 as a boundary condition ($x_{n+1} = 0$). The displacement of the substrate at x_0, which is an external loading in the model, is assumed to be sinusoidally varying with amplitude A_s and the phase of the substrate vibration is set to 0 ($x_0 = A_s \sin \omega t$).

To solve the analytical model, eq. (S1) is re-written in a frequency domain, as shown in eq. (S2).

$$(-m_i\omega^2 + k_i + k_{i+1} + c_i j\omega + c_{i+1} j\omega)X_i - (k_i + c_i j\omega)X_{i-1} - (k_{i+1} + c_{i+1} j\omega)X_{i+1} = 0 \quad 1 \le i \le n \qquad \text{(S2)}$$

where ω, X_i are angular velocity and phasor representation of the displacement at *i*-th layer, x_i, respectively.

To obtain the solution of the model, eq. (S2) is converted in a matrix form and solved for *X*, as shown below.

$$AX = B$$

where,

$$A = \begin{pmatrix} \begin{matrix}(-m_1\omega^2 + k_1 + k_2 \\ + c_1 j\omega + c_2 j\omega)\end{matrix} & -(k_2 + c_2 j\omega) & 0 & \cdots & 0 \\ -(k_2 + c_2 j\omega) & \begin{matrix}(-m_2\omega^2 + k_2 + k_3 \\ + c_2 j\omega + c_3 j\omega)\end{matrix} & -(k_3 + c_3 j\omega) & \cdots & 0 \\ 0 & -(k_3 + c_3 j\omega) & \begin{matrix}(-m_3\omega^2 + k_3 + k_4 \\ + c_3 j\omega + c_4 j\omega)\end{matrix} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & -(k_n + c_n j\omega) & \begin{matrix}(-m_n\omega^2 + k_n + k_{n+1} \\ + c_n j\omega + c_{n+1} j\omega)\end{matrix} \end{pmatrix}$$

$$B = \begin{pmatrix} (k_1 + c_1 j\omega)X_0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{pmatrix} = A^{-1}B$$

As the matrix *A* is non-singular, solution *X* can be readily obtained. After obtaining the solution, the VIPS was calculated using Eq. (2) in the main text, where A_c and ϕ are $|X_k|$ and $\angle X_k$ respectively (X_k is the displacement of the top layer of the cell region.).

The above process was repeated with varying viscosity and elasticity of the cell, to produce the plots in Fig. 2b.