SUPPORTING INFORMATION

Hydrodynamics in cell studies

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S1. Velocity profile and shear in a rectangular channel

Solving Navier-Stokes for a square microchannel with no slip boundary conditions at the walls the velocity profile is obtained as:

$$u_x(y,z) = \frac{48Q}{\pi^3 hw} \frac{\sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^3}\left[1-\frac{\cosh(n\pi\frac{y}{h})}{\cosh(n\pi\frac{w}{2h})}\right]\sin\left(n\pi\frac{z}{h}\right)}{1-\sum_{n=1,3,5,\ldots}^{\infty} \frac{192h}{n^5\pi^5 w}\tanh\left(n\pi\frac{w}{2h}\right)} \tag{s1}$$

Where Q is the volumetric flow rate and h and w are the channel height and width. The series in above solution converge after summation of the first two terms and can be simplified as follows with negligible loss of accuracy:

$$u_x(y,z) = \frac{48Q}{\pi^3 hw} \frac{\left(\left[1-\frac{\cosh(\pi\frac{y}{h})}{\cosh(\pi\frac{w}{2h})}\right]\sin\left(\pi\frac{z}{h}\right)+\frac{1}{27}\left[1-\frac{\cosh(3\pi\frac{y}{h})}{\cosh(3\pi\frac{w}{2h})}\right]\sin\left(3\pi\frac{z}{h}\right)\right)}{1-\left(\frac{192h}{\pi^5 w}\tanh\left(\pi\frac{w}{2h}\right)+\frac{192h}{243\pi^5 w}\tanh\left(3\pi\frac{w}{2h}\right)\right)} \tag{s2}$$

The wall shear stress in a microchannel is critically important in context of microfluidic cell culture as the shear can alter the cells behavior or even damage them if the magnitude is too large. To obtain the shear distribution we differentiate u in equation (s2) with respect to z resulting in wall strain rate:

$$\gamma_{wall} = \frac{du_x}{dz}\Big|_{z=0} = \frac{48Q}{\pi^2 h^2 w} \frac{\left(\left[1-\frac{\cosh(\pi\frac{y}{h})}{\cosh(\pi\frac{w}{2h})}\right]+\frac{1}{9}\left[1-\frac{\cosh(3\pi\frac{y}{h})}{\cosh(3\pi\frac{w}{2h})}\right]\right)}{1-\left(\frac{192h}{\pi^5 w}\tanh\left(\pi\frac{w}{2h}\right)+\frac{192h}{243\pi^5 w}\tanh\left(3\pi\frac{w}{2h}\right)\right)} \tag{s3}$$

With channel aspect ratio $r = w/h$ equation (s3) can be rearranged in terms of r and h, the aspect ratio and height of the channel:

$$\frac{\gamma_{wall}}{Q}\Big|_{y=0} = \frac{48}{\pi^2 h^3 r} \frac{\left(\left[1-\frac{1}{\cosh(\pi\frac{r}{2})}\right]+\frac{1}{9}\left[1-\frac{1}{\cosh(3\pi\frac{r}{2})}\right]\right)}{1-\left(\frac{192}{\pi^5 r}\tanh\left(\pi\frac{r}{2}\right)+\frac{192}{243\pi^5 r}\tanh\left(3\pi\frac{r}{2}\right)\right)} \tag{s4}$$

S2. Solution for Stagnation flow

Often when processing an open surface the fluid flow is brought to the surface from the top rather than parallel to the surface. This flow pattern can be modeled by a stagnation flow, a classical boundary layer problem that has been described as follows. For the inviscid region outside of the boundary layer (y>δ) the stagnation flow is described by the stream function:

$$\psi = Bxy, \; B = \frac{V}{H} \qquad \text{(s5)}$$

Which gives:

$$u = \frac{\partial \psi}{\partial y} = Bx \quad \text{And} \quad v = -\frac{\partial \psi}{\partial x} = -By \qquad \text{(s6)}$$

For the region inside of the boundary layer (y <δ):

$$\psi = xF(\eta)\sqrt{B\nu}, = y\sqrt{\frac{B}{\nu}}, B = \frac{V}{H} \qquad \text{(s7)}$$

Which gives:

$$u = \frac{\partial \psi}{\partial y} = BxF'(\eta) \text{ And } v = -F(\eta)\sqrt{B\nu} \qquad \text{(s8)}$$

The ratio of x-velocity inside and outside of the boundary layer is given with:

$$\frac{u}{U} = F'(\eta) \qquad \text{(s9)}$$

Solving Navier-Stokes equations using the above relationships for the velocity field and numerical solution from [1, 2] the momentum boundary layer thickness is given by:

$$\delta_{99} \approx 2\sqrt{\frac{\nu}{B}} = 2\sqrt{\frac{\nu H}{V}} \qquad \text{(s10)}$$

Wall shear stress for stagnation flow can be obtained as follows:

$$\tau_{wall} = \mu \frac{\partial u}{\partial y}|_{y=0} \approx \mu BxF''(0)\sqrt{\frac{B}{\nu}} = 1.31\mu U\sqrt{\frac{V}{\nu H}} \qquad \text{(s11)}$$

Showing that shear stress in axisymmetric stagnation flow increases with an increase in velocity and viscosity of the fluid while an increase in the distance between the jet and the surface reduces the shear.

S3. Hydrodynamic force on a solid particle moving perpendicular to open surface

The drag force acting on the particle with diameter *d* moving towards a solid surface at a velocity of *U* is given by:

$$F = 6\pi\mu dU\lambda \tag{s12}$$

$$\lambda = \frac{4}{3}\sinh\alpha \sum_{n=1}^{\infty} \frac{n(n+1)}{(2n-1)(2n+3)}\left[\frac{2\sinh(2n+1)\alpha + (2n+1)\sinh 2\alpha}{4\sinh^2\left(n+\frac{1}{2}\right)\alpha - (2n+1)^2\sinh^2\alpha} - 1\right] \tag{s13}$$

Where $\alpha = \frac{d}{h}$ is the ratio of the particle diameter, *d*, to the distance from the surface, *h*.

S4. References

[1] H. Schlichting, K. Gersten, E. Krause and H. Oertel, *Boundary-layer theory* vol. 8: Springer Berlin Heidelberg, 2000.

[2] F. M. White and I. Corfield, *Viscous fluid flow* vol. 3: McGraw-Hill New York, 2006.