A Multinomial Regression Approach to Model Outcome Heterogeneity

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Web Appendix

Web Appendix 1. Implementation of Constrained Bayesian Estimation in Simulation

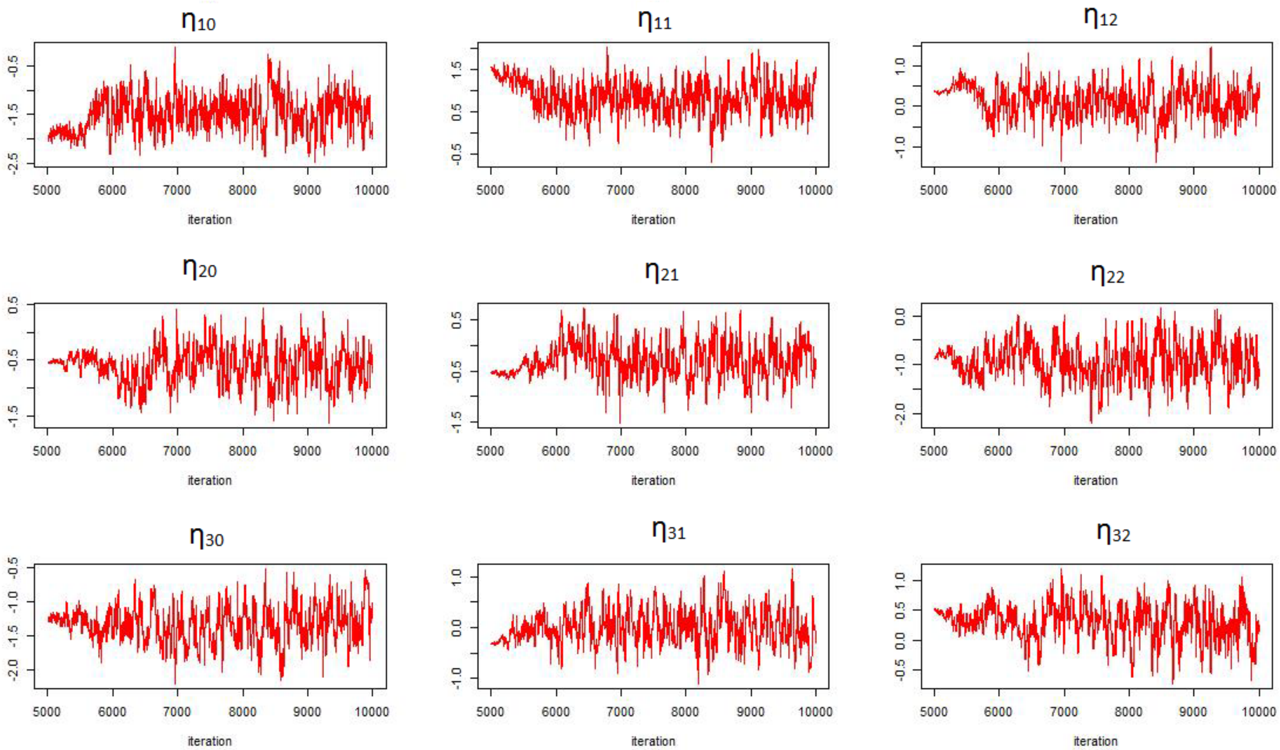
The OpenBUGS code for posterior computation in the simulation study is shown below. For the i^{th} individual with outcome $Y = k$, the encoding follows that $Y_k[i] = 1$ and $Y_j[i] = 0$ for $j \neq k$. The first part of the code describes each individual's contribution to the observed data likelihood, where the $N \times 1$ dummy vector Z of all 1's is assumed to be the results of Bernoulli trials with probabilities $p[i]$. By making each p[i] equal to the probability of observing the outcome for the i^{th} individual, the required likelihood term is provided. The input covariates are the $N \times 1$ vectors X_1 and X_2.

Prior distributions for the parameters in the model are specified as independent $N(0, 10^2)$. The final part of the code imposes constraints on the sampling space so that $0 < \Pr\{Y = 0|X_i; \hat{\eta}\} < 1$ for all individuals $i = 1, 2, ..., n$. Posterior mean, median and 95% credible intervals can be obtained directly from Markov-chain Monte-Carlo sampling in R through BRugs, an interface to the OpenBUGS software.

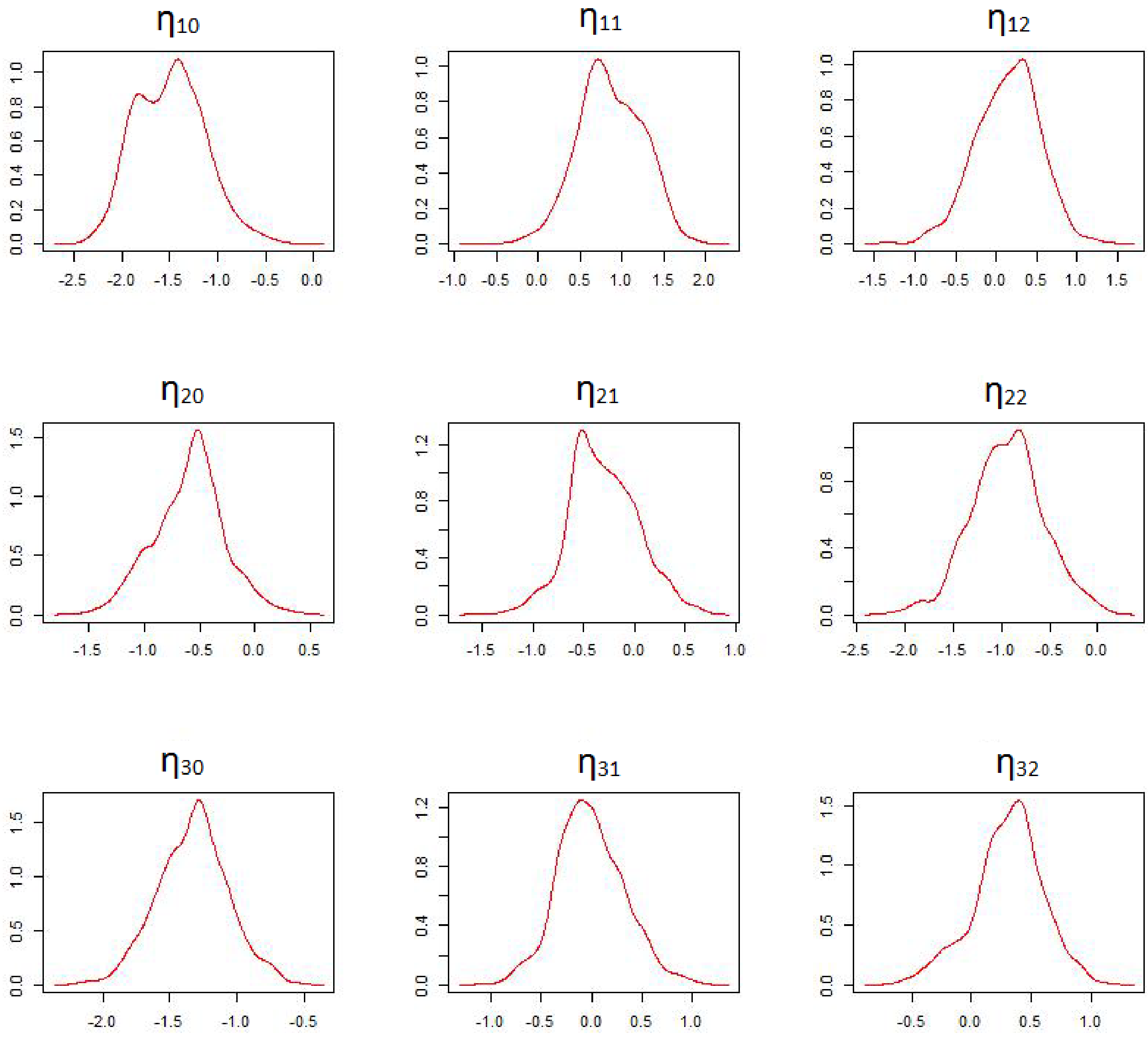
```
Model <- function() {

    for (i in 1:N){
        z[i]   <- 1
        z[i]    ~ dbern(p[i])
        p[i]   <- L[i]
        L[i]   <-  Y0[i]*pi0[i]+Y1[i]*pi1[i]+Y2[i]*pi2[i]+Y3[i]*pi3[i]

        #Probability for each of the outcomes Y=1,2,3.

        logit(pi1[i])<-eta[1]+eta[2]*X1[i]+eta[3]*X2[i]
        logit(pi2[i])<-eta[4]+eta[5]*X1[i]+eta[6]*X2[i]
```

```
        logit(pi3[i])<-eta[7]+eta[8]*X1[i]+eta[9]*X2[i]
        pi0[i]  <- 1-pi1[i]-pi2[i]-pi3[i]
    }
    #Priors for parameters in missing data model
    for (j in 1:9) {
        eta[j] ~ dnorm(0, 0.01)
    }

    # implementing the constraints
    for (k in 1:N){
        ones[k] <- 1
        ones[k] ~ dbern(C[k])
        C[k] <- step(pi0[k])
    }
}
```

Trace Plots and Posterior Densities of η in a Typical Simulation Replicate



Web Figure 1: Trace plots for posterior sampling of η in a typical simulation replicate ($n = 500$), after an initial burn-in of 5000 iterations.



Web Figure 2: Smoothed kernel estimated posterior densities of η in a typical simulation replicate ($n = 500$).

Web Appendix 2. Implementation of Constrained Bayesian Estimation in Application Data-set

The OpenBUGS code for posterior computation in the application data-set is shown below. The first part of the code describes each individual's contribution to the observed data likelihood, where the $N \times 1$ dummy vector Z of all 1's is assumed to be the results of Bernoulli trials with probabilities $p[i]$. By making each p[i] equal to the probability of observing the outcome for the i^{th} individual, the required likelihood term is provided. *surv* is the $N \times 1$ vector with 1 for survivals and 0 for deaths. Similarly, *chd* is the $N \times 1$ vector with 1 for deaths from CHD and 0 otherwise, *str* is the $N \times 1$ vector with 1 for deaths from stroke and 0 otherwise, *can* is the $N \times 1$ vector with 1 for deaths from cancer and 0 otherwise. The covariates are $N \times 1$ vectors *age* for age, *sex* for gender, *sc* for serum cholesterol, *bmi* for BMI and *hbp* for high blood pressure.

```
CBmodel <- function() {

for (i in 1:N){
z[i]   <- 1
z[i]    ~ dbern(p[i])
p[i]   <- L[i]
L[i]   <-   step(surv[i]-0.5)*(pi1[i])
    + step(chd[i]-0.5)*(pi2[i])
    + step(str[i]-0.5)*(pi3[i])
    + step(can[i]-0.5)*(pi4[i])

logit(pi2[i])<- g[1]+g[2]*age[i]+g[3]*sex[i]+g[4]*sc[i]+g[5]*bmi[i]+g[6]*hbp[i]
logit(pi3[i])<- g[7]+g[8]*age[i]+g[9]*sex[i]+g[10]*sc[i]+g[11]*bmi[i]+g[12]*hbp[i]
logit(pi4[i])<- g[13]+g[14]*age[i]+g[15]*sex[i]+g[16]*sc[i]+g[17]*bmi[i]+g[18]*hbp[i]
pi1[i]       <- 1-pi2[i]-pi3[i]-pi4[i]
}
for (k in 1:18) {
g[k] ~ dnorm(0, 0.001)
}
for (j in 1:N) {
ones[j] <- 1
ones[j] ~ dbern(CONSTR[j])
CONSTR[j] <- step(pi1[j])
}
}
```