Web-based Supplementary Materials for Empirical Null Estimation using Zero-inflated Discrete Mixture Distributions and its Application to Protein Domain Data by Iris Ivy Gauran, Junyong Park, Johan Lim, DoHwan Park, John Zylstra, Thomas Peterson, Maricel Kann and John Spouge

Web Appendix A: EM Algorithm

Suppose the sample space of X, denoted by $\mathcal{X}$, is partitioned into $K+1$ mutually exclusive subsets $\mathcal{X}_j = \{j\}$, $j \in \mathcal{A}$, where independent observations are made on X. Since the information contained in $\mathbf{x}_j$ is analogous to knowing n_j, for any $j \leq C$, then another, equivalent format of the data set $\mathbf{x}'_n = (\mathbf{x}'_0, \mathbf{x}'_1, \ldots, \mathbf{x}'_C)$ is $\mathbf{y}_n = (n_0, n_1, \ldots, n_C)'$. After choosing a suitable value for C, the null sample $\mathbf{x}'_n$ and the corresponding vector of mutation counts $\mathbf{y}_n$ are available for the estimation of the parameters of f_0. However, the problem that arises is that the number of observations n_j falling in $\mathcal{X}_j$, $j > C$ are not available for the subsequent estimation of the parameters of f_0.

For the n observations in $\mathbf{x}'_n$, it is assumed that $\mathbf{y}_n = (n_0, n_1, \ldots, n_C)'$ has a Multinomial distribution consisting of n draws on $C+1$ categories with probabilities p_j

$$p_j = \frac{f_0(j;\Theta)}{\sum_{j=0}^{C} f_0(j;\Theta)} \tag{1}$$

where $\Theta = (\eta, \lambda, \theta)$, $\sum_{j=0}^{C} p_j = 1$ and $\sum_{j=0}^{C} n_j = n$. This gives the likelihood function

$$L_0(\Theta; \mathbf{y}_n) = \frac{n!}{n_0! n_1! \ldots n_C!} \prod_{j=0}^{C} p_j^{n_j} \tag{2}$$

From (2), we can solve the likelihood equation $\partial L_0(\Theta; \mathbf{y}_n)/\partial\Theta = \mathbf{0}$. The EM machinery is invoked by defining $\mathbf{w}_N = (\mathbf{y}'_n, \mathbf{y}'_{N-n})'$ as the complete-data vector where $\mathbf{y}_{N-n} = (n_{C+1}, n_{C+2}, \ldots n_K)'$. Then, instead of looking at the log likelihood for $\mathbf{y}_n$, we consider the log likelihood function of the complete data, $\ell(\eta, \lambda, \theta \mid \mathbf{w}_N)$. In order to find the estimates, it is important to note that each entry of $\mathbf{y}_{N-n}$ is a realization of a hidden random variable. However, since these realizations do not exist in reality, we have to consider each entry of $\mathbf{y}_{N-n}$ as a random variable itself.

Furthermore, McLachlan and Jones (1988) proposed an extension of the complete-data vector $\mathbf{w}_N$ for mixture densities to include the zero-one indicator variables $\mathbf{z}_{jk} = (z_{0jk}, z_{1jk})' \;\; j = 0, 1, \ldots, K; \;\; k = 1, 2 \ldots, n_j$ where $z_{0jk} + z_{1jk} = 1$ and given the number of mutations j, $\mathbf{z}_{jk}$ are conditionally independent. Conditional on the value of j, the probability of membership to a component can be computed using Bayes' Theorem as

$$\tau_{0j}(\Theta) = P(z_{0jk} = 1 \mid j) = \frac{\eta I_{\{0\}}(j)}{f_0(j)}$$

and $\tau_{1j}(\Theta) = P(z_{1jk} = 1 \mid j) = 1 - P(z_{0jk} = 1 \mid j)$. The indicator function $I_{\mathcal{S}}(j)$ is equal to 1 if $j \in \mathcal{S}$ and 0 otherwise. Using these indicator variables in the complete-data specification, the log likelihood becomes

$$\sum_{j=0}^{K} \sum_{k=1}^{n_j} z_{0jk} \log \eta I_{\{0\}}(j) + \sum_{j=0}^{K} \sum_{k=1}^{n_j} z_{1jk} \log \left[(1-\eta) g(j)\right] \tag{3}$$

E- Step:

At the $(p+1)th$ stage, the expectation $Q(\Theta; \Theta^{(p)})$ of the log-likelihood of the complete data specified in (3) can be computed conditional on the observed data $\mathbf{y}_n$ and the current fit $\Theta^{(p)}$ for Θ.

$$\begin{aligned} Q(\Theta; \Theta^{(p)}) \;\; = \;\; & n_0 \tau_{00}(\Theta^{(p)}) \log \eta + \sum_{j=0}^{C} n_j \tau_{1j}(\Theta^{(p)}) \log(1-\eta) + \sum_{j=0}^{C} n p_j(\Theta^{(p)}) \tau_{1j}(\Theta^{(p)}) \log(1-\eta) \\ & + (\log \lambda - \lambda) \sum_{j=0}^{C} n_j \tau_{1j}(\Theta^{(p)}) + (\log \lambda - \lambda) \sum_{j=C+1}^{K} n p_j(\Theta^{(p)}) \tau_{1j}(\Theta^{(p)}) \\ & + \sum_{j=0}^{C} n_j (j-1) \tau_{1j}(\Theta^{(p)}) \log(\lambda + \theta j) + \sum_{j=C+1}^{K} n(j-1) p_j(\Theta^{(p)}) \tau_{1j}(\Theta^{(p)}) \log(\lambda + \theta j) \\ & - \left[\theta \sum_{j=0}^{C} j n_j \tau_{1j}(\Theta^{(p)}) + \theta \sum_{j=C+1}^{K} j n p_j(\Theta^{(p)}) \tau_{1j}(\Theta^{(p)}) + constant \right] \end{aligned}$$

M- Step:

In order to arrive at an estimate of $\Theta^{(p+1)}$ at the $(p+1)th$ stage, the goal is to maximize $Q(\Theta;\Theta^{(p)})$ with respect to Θ. The estimates of η, λ and θ obtained at the $(p+1)th$ stage are as follows:

$$\eta^{(p+1)} = \frac{n_0\tau_{00}(\Theta^{(p)})}{n_0\tau_{00}(\Theta^{(p)}) + \sum_{j=0}^{C} n_j\tau_{1j}(\Theta^{(p)}) + \sum_{j=C+1}^{K} np_j(\Theta^{(p)})\tau_{1j}(\Theta^{(p)})}$$

$$\lambda^{(p+1)} = \frac{\sum_{j=0}^{C} n_j[\tau_{1j}(\Theta^{(p)}) + (j-1)\tau_{2j}(\Theta^{(p)})] + \sum_{j=C+1}^{K} np_j(\Theta^{(p)})[\tau_{1j}(\Theta^{(p)}) + (j-1)\tau_{2j}(\Theta^{(p)})]}{\sum_{j=0}^{C} n_j\tau_{1j}(\Theta^{(p)}) + \sum_{j=C+1}^{K} np_j(\Theta^{(p)})\tau_{1j}(\Theta^{(p)})}$$

$$\theta^{(p+1)} = \frac{\sum_{j=0}^{C} n_j(j-1)\tau_{3j}(\Theta^{(p)}) + \sum_{j=C+1}^{K} np_j(j-1)\tau_{3j}(\Theta^{(p)})}{\sum_{j=0}^{C} jn_j\tau_{1j}(\Theta^{(p)}) + \sum_{j=C+1}^{K} jnp_j(\Theta^{(p)})\tau_{1j}(\Theta^{(p)})}$$

where $\tau_{2j}(\Theta^{(p)}) = \dfrac{\lambda^{(p)}}{\lambda^{(p)} + \theta^{(p)}j}$ and $\tau_{3j}(\Theta^{(p)}) = \dfrac{\theta^{(p)}j}{\lambda^{(p)} + \theta^{(p)}j}$.

If the null distribution is modeled using Zero-Inflated Poisson distribution then the log likelihood $\ell(\eta, \lambda \mid \mathbf{x}_N)$ of the entire data vector is

$$n_0 \log\left(\eta + (1-\eta)e^{-\lambda}\right) + \sum_{j=1}^{C} n_j \log(1-\eta)\ \frac{\lambda^j e^{-\lambda}}{j!} + \sum_{j=C+1}^{K} n_j \log f(j;\cdot)$$

Following the same procedure, the $E-$Step at the $(p+1)th$ stage would yield

$$\begin{aligned} Q(\Theta;\Theta^{(p)}) = {} & n_0\tau_{00}(\Theta^{(p)})\log\eta + \sum_{j=0}^{C} n_j\tau_{1j}(\Theta^{(p)})\log(1-\eta) + \sum_{j=0}^{C} np_j(\Theta^{(p)})\tau_{1j}(\Theta^{(p)})\log(1-\eta) \\ & + \log\lambda \sum_{j=0}^{C} jn_j\tau_{1j}(\Theta^{(p)}) + \log\lambda \sum_{j=C+1}^{K} jnp_j(\Theta^{(p)})\tau_{1j}(\Theta^{(p)}) \\ & - \left[\lambda \sum_{j=0}^{C} n_j\tau_{1j}(\Theta^{(p)}) + \lambda \sum_{j=C+1}^{K} np_j(\Theta^{(p)})\tau_{1j}(\Theta^{(p)}) + constant\right] \end{aligned}$$

For the $M-$Step, the estimates of η and λ obtained at the $(p+1)th$ stage are as follows:

$$\eta^{(p+1)} = \frac{n_0\tau_{00}(\Theta^{(p)})}{n_0\tau_{00}(\Theta^{(p)}) + \sum_{j=0}^{C} n_j\tau_{1j}(\Theta^{(p)}) + \sum_{j=C+1}^{K} np_j(\Theta^{(p)})\tau_{1j}(\Theta^{(p)})}$$

$$\lambda^{(p+1)} = \frac{\sum_{j=0}^{C} jn_j\tau_{1j}(\Theta^{(p)}) + \sum_{j=C+1}^{K} jnp_j(\Theta^{(p)})\tau_{1j}(\Theta^{(p)})}{\sum_{j=0}^{C} n_j\tau_{1j}(\Theta^{(p)}) + \sum_{j=C+1}^{K} np_j(\Theta^{(p)})\tau_{1j}(\Theta^{(p)})}$$

If the null distribution is modeled using Generalized Poisson distribution then the log likelihood $\ell(\lambda, \theta \mid \mathbf{x}_N)$ of the entire data is

$$\sum_{j=0}^{C} n_j \log\left(\frac{\lambda(\lambda+\theta j)^{j-1}e^{-\lambda-\theta j}}{j!}\right) + \sum_{j=C+1}^{K} n_j \log f(j;\cdot)$$

Unlike ZIGP, this model is not a mixture density so the procedure does not require the inclusion of latent variables. The $E-$Step would yield

$$\begin{aligned} Q(\Theta;\Theta^{(p)}) &= (\log\lambda-\lambda)\sum_{j=0}^{C} n_j + (\log\lambda-\lambda)\sum_{j=C+1}^{K} np_j(\Theta^{(p)}) + \sum_{j=0}^{C} n_j(j-1)\log(\lambda+\theta j) \\ &\quad + \sum_{j=C+1}^{K} n(j-1)p_j(\Theta^{(p)})\log(\lambda+\theta j) - \left[\theta\sum_{j=0}^{C} jn_j + \theta\sum_{j=C+1}^{K} jnp_j(\Theta^{(p)}) + constant\right] \end{aligned}$$

At the $(p+1)th$ stage, the $M-$Step would yield the estimates of λ and θ as follows:

$$\lambda^{(p+1)} = \frac{\sum_{j=0}^{C} n_j[1+(j-1)\tau_{2j}(\Theta^{(p)})] + \sum_{j=C+1}^{K} np_j(\Theta^{(p)})[1+(j-1)\tau_{2j}(\Theta^{(p)})]}{\sum_{j=0}^{C} n_j + \sum_{j=C+1}^{K} np_j(\Theta^{(p)})}$$

$$\theta^{(p+1)} = \frac{\sum_{j=0}^{C} n_j(j-1)\tau_{3j}(\Theta^{(p)}) + \sum_{j=C+1}^{K} n(j-1)p_j(\Theta^{(p)})\tau_{3j}(\Theta^{(p)})}{\sum_{j=0}^{C} jn_j + \sum_{j=C+1}^{K} jnp_j(\Theta^{(p)})}$$

Lastly, if f_0 is modeled using Poisson distribution then the log likelihood $\ell(\lambda \mid \mathbf{x}_N)$ of the entire data vector is

$$\sum_{j=0}^{C} n_j \ \log\left(\frac{\lambda^j e^{-\lambda}}{j!}\right) + \sum_{j=C+1}^{K} n_j \log f(j;\cdot)$$

Since this model is also not a mixture density then the procedure does not require the inclusion of zero-one indicator variables. The $E-$Step would yield

$$\begin{aligned} Q(\Theta;\Theta^{(p)}) \quad &= \quad \log\lambda \sum_{j=0}^{C} jn_j + \log\lambda \sum_{j=C+1}^{K} jnp_j(\Theta^{(p)}) \\ &\quad - \left[\lambda \sum_{j=0}^{C} n_j + \lambda \sum_{j=C+1}^{K} np_j + constant\right] \end{aligned}$$

The $M-$Step would yield the estimate of λ at the $(p+1)th$ stage as follows:

$$\lambda^{(p+1)} \quad = \quad \frac{\displaystyle\sum_{j=0}^{C} jn_j + \sum_{j=C+1}^{K} jnp_j(\Theta^{(p)})}{\displaystyle\sum_{j=0}^{C} n_j + \sum_{j=C+1}^{K} np_j(\Theta^{(p)})}$$

Initial Values:

The starting values for η, λ and θ are based on the moments of the data $\mathbf{a} = (a_1, a_2, \ldots, a_N)$. Suppose the mean of the entire data set is $\mu_a = \dfrac{1}{N}\displaystyle\sum_{i=1}^{N} a_i$ and the variance is $\sigma_a^2 = \dfrac{1}{N-1}\displaystyle\sum_{i=1}^{N}(a_i - \mu_a)^2$. Also, define $\mathcal{C}_0 = \{a_i \in \mathbf{a} : a_i = 0\}, \ \ \mathcal{C}_1 = \{a_i \in \mathbf{a} : 0 < a_i \leq Q_2\}, \ \mathcal{C}_2 = \{a_i \in \mathbf{a} : 0 \leq a_i \leq Q_3\}$, where Q_2 and Q_3 refers to the second and the third quartile, respectively. Furthermore, define $\mu_a^{(p)}$ and $\sigma_a^{(p)}$ as the mean and standard deviation of the set $\mathcal{C}_p, \ p = 0, 1, 2.$ The initial values for each of the model for f_0 are presented as follows:

1. **Poisson**: $\lambda^{(0)} = \dfrac{1}{N}\displaystyle\sum_{i=1}^{N} a_i I(a_i > 0)$

2. **Generalized Poisson**: $\theta^{(0)} = 1 - \dfrac{\sqrt{\mu_a^{(1)}}}{\sigma_a^{(1)}}$ and $\lambda^{(0)} = \mu_a^{(1)} \dfrac{\sqrt{\mu_a^{(1)}}}{\sigma_a^{(1)}}$

3. **Zero-inflated Poisson**: $\eta^{(0)} = \mu_a^{(0)}$ and $\lambda^{(0)} = \dfrac{\mu_a}{1 - \mu_a^{(0)}}$

4. **Zero-inflated Generalized Poisson**: $\eta^{(0)} = \mu_a^{(0)}, \ \ \theta^{(0)} = 1 - \dfrac{1}{\sqrt{\dfrac{\left[\sigma_a^{(2)}\right]^2}{\mu_a^{(2)}} - \left(\dfrac{\mu_a^{(0)}}{1-\mu_a^{(0)}}\right)\mu_a^{(2)}}}$ and

$$\lambda^{(0)} = \frac{\mu_a^{(2)}}{(1-\mu_a^{(0)})\sqrt{\dfrac{\left[\sigma_a^{(2)}\right]^2}{\mu_a^{(2)}} - \left(\dfrac{\mu_a^{(0)}}{1-\mu_a^{(0)}}\right)\mu_a^{(2)}}}$$

When f_0 is assumed to be ZIGP, the EM Algorithm takes around 2 seconds, for a given data set. This is the slowest among the four models since it requires estimation of three parameters. The procedure stops when

$$\max\{|\eta^{(k+1)} - \eta^{(k)}|, |\lambda^{(k+1)} - \lambda^{(k)}|, |\theta^{(k+1)} - \theta^{(k)}|\} < 0.0001 = \epsilon$$

Web Appendix B: Calculation of D_N

When we have observations a_i for $1 \leq i \leq N$ generated from the null distribution, we are interested in figuring out D_N such that

$$P\left(\max_{1\leq i\leq N} a_i < D_N\right) \to 1 \tag{4}$$

as $N \to \infty$. Once a sequence D_N is identified, $a_i(\geq D_N)$ is hardly observed under the null hypothesis, so the corresponding null hypothesis is rejected directly rather than making decision based on local FDR procedure. There are many choices of D_N, but a smaller sequence of D_N satisfying (4) is of our interest since any sequence B_N satisfying $B_N > D_N$ also satisfies the property.

When a_i is observed from Generalized Poisson distribution, Klar (2000) showed that the tail probabilities satisfy the following inequality:

$$P(a_i \geq D_N) < \left[1 - e^{1-\theta}\left(\theta + \frac{\lambda}{D_N+1}\right)\right]^{-1} \frac{\lambda(\lambda+\theta D_N)^{D_N-1}}{(D_N)^{D_N+1/2}} e^{-\lambda-(\theta-1)D_N} \tag{5}$$

where $D_N \geq \dfrac{\lambda}{e^{\theta-1}-\theta}, \quad \theta \in (0,1), \quad \lambda > 0$. Using (5), we can compute for

$$\begin{aligned} P\left(\max_{1\leq i\leq N} a_i \geq D_N\right) &= 1-[1-P(a_i \geq D_N)]^N \\ &\leq 1-\left[1-(\delta_{D_N})^{-1}\frac{\lambda(\lambda+\theta D_N)^{D_N-1}}{(D_N)^{D_N+1/2}}e^{-\lambda-(\theta-1)D_N}\right]^N \end{aligned}$$

where $\delta_{D_N} = 1-e^{1-\theta}\left(\theta+\dfrac{\lambda}{D_N+1}\right)$. For (4) to hold,

$$\log N - \log \delta_{D_N} + \log\left(\frac{\lambda(\lambda+\theta D_N)^{D_N-1}}{(D_N)^{D_N+1/2}}e^{-\lambda-(\theta-1)D_N}\right) \to -\infty \tag{6}$$

and (6) can be simplified in terms of N and D_N as

$$\mathcal{G}_N \equiv \log N - 0.5\log D_N - \log(D_N+1) + D_N \log\left(\theta+\frac{\lambda}{D_N}\right) - (\theta-1)D_N$$

leading to

$$\mathcal{G}_N \asymp \log N + (\log\theta - \theta + 1)D_N. \tag{7}$$

To assure that $\mathcal{G}_N \to -\infty$, we can take $D_N = \zeta \log N$ for some constant ζ satisfying

$$\zeta > \frac{1}{\theta-1-\log\theta}$$

Since $\log\theta \leq \theta - 1$, then $\zeta > 0, \quad \theta \in (0,1)$ as desired. Hence, we take

$$D_N = \left\lceil \max\left(\frac{\lambda}{e^{\theta-1}-\theta}, \ \frac{\log N}{\theta-1-\log\theta}\right)\right\rceil \tag{8}$$

where $\lceil x \rceil$ is the smallest integer greater than or equal to $x (x > 0)$. Meanwhile, if a_i is observed from Poisson distribution, Mitzenmacher and Upfal (2005) derived the bounds for the tail probabilities using the Chernoff bound argument:

$$P(a_i \geq D_N) < \frac{e^{-\lambda}(e\lambda)^{D_N}}{(D_N)^{D_N}} \tag{9}$$

where $0 < \lambda < D_N$. Using the inequality in (9),

$$P\left(\max_{1\leq i\leq N} a_i \geq D_N\right) \leq 1-\left(1-\frac{e^{-\lambda}(e\lambda)^{D_N}}{(D_N)^{D_N}}\right)^N$$

and in order to satisfy the condition in $P\left(\max_{1\leq i\leq N} a_i < D_N\right) \to 1$, $\mathcal{P}_N \to -\infty$ where

$$\mathcal{P}_N \asymp \log N - D_N \log D_N$$

Therefore, we take

$$D_N = \lceil \max\left(\lambda, \ \ \log N\right) \rceil \tag{10}$$

When a_i is observed from ZIGP, D_N can be calculated exactly as shown in (8) since the derivation will eventually yield the leading terms in (7) which does not involve η. Similarly, if a_i is observed from ZIP, D_N can be computed using (10).

Web Tables

Using the model specifications in terms of the true f_0 and f_1, there are 15 mixture models considered for data generation. Following the key assumption on f_0, the support of f_1 does not contain values in $[0, C]$. Hence, f_1 can be expressed as $f_1 = C + 1 + W$ where W follows another count model. The model specifications are presented in Table 1.

Table 1: Model Specification of f_0 and f_1 for data generation and numerical comparison

			Parameters for f_0			
True f_0	W	Label	η	λ	θ	π_0
ZIGP	Geometric	ZIGP_1	0.8	1.5	0.3	0.8
		ZIGP_2	0.8	3	0.3	0.8
ZIP		ZIP_1	0.8	1.5	0	0.8
ZIGP	Binomial	ZIGP_3	0.4	1	0.3	0.35
		ZIGP_4	0.4	4	0.3	0.35
ZIP		ZIP_2	0.4	1.5	0	0.35
ZIGP	Geometric	ZIGP_5	0.4	1	0.15	0.85
		ZIGP_6	0.4	3	0.15	0.85
ZIP		ZIP_3	0.4	1.5	0	0.85
ZIGP	Binomial	ZIGP_7	0.4	1	0.2	0.8
		ZIGP_8	0.4	3	0.2	0.8
ZIP		ZIP_4	0.4	1.5	0	0.8

For each of the model specification of f_0, $\hat{C}$ is calculated and the corresponding set of parameter estimates $\widehat{\Theta}^{\star}_{\hat{C}} = (\widehat{\eta}_{\hat{C}}, \widehat{\lambda}_{\hat{C}}, \widehat{\theta}_{\hat{C}}, \widehat{\pi}_{0,\hat{C}})$ from EM Algorithm are obtained. The bias of the parameter estimates for each of these specifications are presented in Tables 2 to 6.

It can be seen that regardless of the true null distribution, if f_0 is modeled using the Zero-inflated Generalized Poisson (ZIGP) distribution, then $\hat{C}$ produces the most accurate estimate for C in terms of the bias and standard error. This validates the robustness of ZIGP as a model for f_0, that is, even if the true null distribution is Zero-inflated Poisson (ZIP), the most accurate estimate of $\hat{C}$ can still be observed when f_0 is modeled using ZIGP.

Table 2: Bias of Parameter Estimates when f_1 is shifted Geometric($p = 0.08$), $\pi_0 = 0.80$ and $C = 5$. ZIP$_1$($\eta = 0.80, \lambda = 1.5$) represents the well-separated case, ZIGP$_1$($\eta = 0.80, \lambda = 1.5, \theta = 0.3$) represents the moderately mixed case, and ZIGP$_2$($\eta = 0.80, \lambda = 3, \theta = 0.3$) represents the heavily mixed case. The number in $(\cdot)$ represents the standard error.

True	Choice	Model	Bias of Parameter Estimates				
f_0	of C	for f_0	$\widehat{\eta}_{\hat{C}}$	$\widehat{\lambda}_{\hat{C}}$	$\widehat{\theta}_{\hat{C}}$	$\hat{\pi}_{0,\hat{C}}$	$\hat{C}$
ZIP$_1$	$\hat{C}_1$	ZIGP	0.0135	0.3006	0.0005	0.0008	-0.07
			(0.0156)	(0.1040)	(0.0004)	(0.0129)	(0.27)
		ZIP	-0.0010	-0.1063	NA	-0.0046	-1.53
			(0.0193)	(0.2044)		(0.0146)	(1.17)
		GP	NA	-0.1420	0.8775	0.1998	10.54
				(0.0316)	(0.0307)	(0.0049)	(3.75)
		P	NA	-1.3666	NA	-0.0466	-3.42
				(0.0459)		(0.0209)	(0.49)
	$\hat{C}_2$	ZIGP	0.0058	0.5220	0.0033	0.0132	-1.90
			(0.0176)	(0.1665)	(0.0070)	(0.0178)	(0.45)
		ZIP	-0.0035	0.0134	NA	0.0020	-2.99
			(0.0197)	(0.3089)		(0.0194)	(0.09)
		GP	NA	-0.0317	0.8679	0.1998	1.64
				(0.0458)	(0.0306)	(0.0048)	0.83
		P	NA	-1.3666	NA	-0.0466	-3.42
				(0.0459)		(0.0209)	0.49
ZIGP$_1$	$\hat{C}_1$	ZIGP	0.0332	0.6883	-0.2633	-0.0057	-0.57
			(0.0262)	(0.1876)	(0.1252)	(0.0254)	0.65
		ZIP	0.0408	0.0296	NA	-0.0290	-2.45
			(0.0175)	(0.3177)		(0.0163)	(0.72)
		GP	NA	-0.1168	0.5774	0.2000	10.65
				(0.0319)	(0.0130)	(0.0000)	(3.62)
		P	NA	-1.3823	NA	-0.0603	-3.23
				(0.0328)		(0.0172)	(0.42)
	$\hat{C}_2$	ZIGP	0.0245	0.7423	-0.2361	0.0039	-1.13
			(0.0445)	(0.2565)	(0.1764)	(0.0470)	(0.66)
		ZIP	0.0388	0.0806	NA	-0.0262	-3.00
			(0.0189)	(0.3802)		(0.0198)	(0.06)
		GP	NA	0.0248	0.5647	0.2000	0.97
				(0.0507)	(0.0141)		(0.81)
		P	NA	-1.3823	NA	-0.0603	-3.23
				(0.0328)		(0.0172)	(0.42)
ZIGP$_2$	$\hat{C}_1$	ZIGP	-0.1006	0.6798	0.1591	0.1076	0.25
			(0.0982)	(0.5178)	(0.2674)	(0.0915)	(0.93)
		ZIP	0.0710	-0.4510	NA	-0.0640	-2.22
			(0.0274)	(0.5938)		(0.0275)	(0.91)
		GP	NA	-1.5446	0.5106	0.1821	8.64
				(0.0753)	(0.2194)	(0.0663)	(4.67)
		P	NA	-2.9138	NA	-0.1130	-3.11
				(0.0247)		(0.0160)	(0.32)
	$\hat{C}_2$	ZIGP	-0.1249	0.8171	0.1803	0.1260	-1.00
			(0.1065)	(0.5603)	(0.2557)	(0.0916)	(0.88)
		ZIP	0.0634	-0.3429	NA	-0.0564	-2.90
			(0.0372)	(0.6975)		(0.0373)	(0.31)
		GP	NA	-1.3985	0.4980	0.1821	-0.09
				(0.1097)	(0.2160)	(0.0663)	(0.58)
		P	NA	-2.9138	NA	-0.1130	-3.11
				(0.0247)		(0.0160)	(0.32)

Table 3: Bias of Parameter Estimates when f_1 is shifted Binomial($n = 250, p = 0.20$), $\pi_0 = 0.35$ and $C = 5$. ZIP$_2$($\eta = 0.40, \lambda = 1.5$) is the well-separated case, ZIGP$_3$($\eta = 0.40, \lambda = 1, \theta = 0.30$) represents the moderately mixed case, and ZIGP$_4$($\eta = 0.40, \lambda = 4, \theta = 0.30$) represents the heavily mixed case. The number in $(\cdot)$ represents the standard error.

True f_0	Choice of C	Model for f_0	Bias of Parameter Estimates $\widehat{\eta}_{\hat{C}}$	$\widehat{\lambda}_{\hat{C}}$	$\widehat{\theta}_{\hat{C}}$	$\hat{\pi}_{0,\hat{C}}$	$\hat{C}$
ZIP$_2$	$\hat{C}_1$	ZIGP	0.0400	0.3046	0.0006	0.0090	0.01
			(0.0320)	(0.0904)	(0.0031)	(0.0154)	(0.11)
		ZIP	-0.0015	-0.0015	NA	0.0077	-0.02
			(0.0393)	(0.1242)		(0.0154)	(0.16)
		GP	NA	0.0887	0.7257	0.1516	3.24
				(0.0412)	(0.0143)	(0.0220)	(0.60)
		P	NA	-0.7645	NA	-0.0109	-2.08
				(0.1092)		(0.0165)	(0.83)
	$\hat{C}_2$	ZIGP	0.0319	0.3385	0.0320	0.0187	-0.25
			(0.0464)	(0.1573)	(0.1301)	(0.0433)	(0.46)
		ZIP	-0.0065	0.0294	NA	0.0112	-3.00
			(0.0412)	(0.2733)		(0.0237)	(0.00)
		GP	NA	0.1751	0.7222	0.1861	0.00
				(0.0718)	(0.0144)	(0.0358)	(0.32)
		P	NA	-0.8083	NA	-0.0169	-2.63
				(0.0836)		(0.0151)	(0.48)
ZIGP$_3$	$\hat{C}_1$	ZIGP	0.1576	0.8973	-0.2661	0.0066	0.08
			(0.0371)	(0.1168)	(0.1174)	(0.0205)	(0.33)
		ZIP	0.1318	0.5296	NA	-0.0009	-0.50
			(0.0400)	(0.2266)		(0.0155)	(0.67)
		GP	NA	0.5653	0.4559	0.1608	3.03
				(0.0403)	(0.0121)	(0.0219)	(0.65)
		P	NA	-0.4767	NA	-0.0196	-2.30
				(0.0758)		(0.0159)	(0.47)
	$\hat{C}_2$	ZIGP	0.1025	1.0264	-0.0639	0.0498	-0.47
			(0.0803)	(0.2468)	(0.2847)	(0.0631)	(0.62)
		ZIP	0.1074	0.2151	NA	-0.0134	-3.00
			(0.0524)	(0.2745)		(0.0180)	(0.00)
		GP	NA	0.6497	0.4479	0.1959	-0.28
				(0.0946)	(0.0155)	(0.0431)	(0.54)
		P	NA	-0.4772	NA	-0.0197	-2.31
				(0.0755)		(0.0159)	(0.46)
ZIGP$_4$	$\hat{C}_1$	ZIGP	-0.0799	0.5839	0.0639	0.1396	0.75
			(0.1043)	(0.6536)	(0.1776)	(0.1633)	(1.73)
		ZIP	0.0172	1.1610	NA	0.0015	2.36
			(0.0563)	(0.8908)		(0.0561)	(1.57)
		GP	NA	-1.7787	0.4293	0.1350	2.30
				(0.1203)	(0.0142)	(0.0332)	(1.35)
		P	NA	-3.3416	NA	-0.1453	-1.96
				(0.1089)		(0.0131)	(0.20)
	$\hat{C}_2$	ZIGP	-0.1610	1.0876	0.1257	0.3081	-2.36
			(0.0961)	(0.7925)	(0.1084)	(0.2153)	(0.72)
		ZIP	0.1346	-0.2583	NA	-0.0768	-1.97
			(0.0751)	(0.8000)		(0.0555)	(0.17)
		GP	NA	-1.4689	0.3949	0.2825	-2.90
				(0.1630)	(0.0192)	(0.0762)	(0.30)
		P	NA	-3.3416	NA	-0.1453	-1.96
				(0.1089)		(0.0131)	(0.20)

Table 4: Bias of Parameter Estimates when f_1 is shifted Geometric($p = 0.08$), $\pi_0 = 0.85$ and $C = 5$. ZIP$_3$($\eta = 0.40, \lambda = 1.5$) represents the well-separated case, ZIGP$_5$($\eta = 0.40, \lambda = 1, \theta = 0.15$) represents the moderately mixed case, and ZIGP$_6$($\eta = 0.40, \lambda = 3, \theta = 0.15$) represents the heavily mixed case. The number in $(\cdot)$ represents the standard error.

True f_0	Choice of C	Model for f_0	Bias of Parameter Estimates $\widehat{\eta}_{\hat{C}}$	$\widehat{\lambda}_{\hat{C}}$	$\widehat{\theta}_{\hat{C}}$	$\hat{\pi}_{0,\hat{C}}$	$\hat{C}$
ZIP$_3$	$\hat{C}_1$	ZIGP	0.0399	0.2986	0.0000	0.0023	0.06
			(0.0210)	(0.0553)	(0.0000)	(0.0117)	(0.26)
		ZIP	-0.0051	-0.0583	NA	-0.0079	-1.33
			(0.0274)	(0.1132)		(0.0168)	(1.12)
		GP	NA	-0.0644	0.0000	0.0003	-0.06
				(0.0288)	(0.0000)	(0.0117)	(0.27)
		P	NA	-1.1227	NA	-0.1880	-4.00
				(0.0340)		(0.0174)	(0.00)
	$\hat{C}_2$	ZIGP	0.0360	0.5099	0.0000	0.0371	-1.90
			(0.0202)	(0.0968)	(0.0000)	(0.0175)	(0.31)
		ZIP	-0.0034	0.0071	NA	0.0031	-3.00
			(0.0269)	(0.1749)		(0.0285)	(0.03)
		GP	NA	-0.0062	0.0000	0.0175	-1.68
				(0.0474)	(0.0000)	(0.0149)	(0.52)
		P	NA	-1.1227	NA	-0.1880	-4.00
				(0.0340)		(0.0174)	(0.00)
ZIGP$_5$	$\hat{C}_1$	ZIGP	0.1358	0.6784	-0.1500	-0.0020	-0.17
			(0.0205)	(0.0592)	(0.0000)	(0.0118)	(0.40)
		ZIP	0.0602	0.1168	NA	-0.0244	-2.46
			(0.0360)	(0.1364)		(0.0167)	(0.71)
		GP	NA	0.3066	-0.1500	-0.0034	-0.35
				(0.0248)	(0.0000)	(0.0118)	(0.48)
		P	NA	-0.6936	NA	-0.1329	-4.00
				(0.0269)		(0.0166)	(0.00)
	$\hat{C}_2$	ZIGP	0.1331	0.8139	-0.1500	0.0164	-1.99
			(0.0201)	(0.0882)	(0.0000)	(0.0158)	(0.18)
		ZIP	0.0607	0.1268	NA	-0.0230	-3.00
			(0.0362)	(0.1527)		(0.0188)	(0.00)
		GP	NA	0.3407	-0.1500	0.0065	-1.62
				(0.0391)	(0.0000)	(0.0137)	(0.51)
		P	NA	-0.6936	NA	-0.1329	-4.00
				(0.0269)		(0.0166)	(0.00)
ZIGP$_6$	$\hat{C}_1$	ZIGP	0.0141	0.2415	-0.0878	-0.0212	-0.04
			(0.0428)	(0.1963)	(0.1014)	(0.0384)	(0.91)
		ZIP	0.0650	-0.1889	NA	-0.1118	-2.01
			(0.0388)	(0.3495)		(0.0621)	(0.96)
		GP	NA	-1.2251	-0.1500	-0.1562	-1.99
				(0.0544)	(0.0000)	(0.0186)	(0.11)
		P	NA	-2.8201	NA	-0.4126	-4.00
				(0.0233)		(0.0156)	(0.00)
	$\hat{C}_2$	ZIGP	-0.0364	0.5027	0.0006	0.0303	-1.60
			(0.0962)	(0.4628)	(0.2183)	(0.0710)	(0.51)
		ZIP	0.0563	-0.1229	NA	-0.0927	-2.98
			(0.0511)	(0.4417)		(0.0921)	(0.14)
		GP	NA	-1.0759	-0.1500	-0.0975	-3.00
				(0.0599)	(0.0000)	(0.0278)	(0.06)
		P	NA	-2.8201	NA	-0.4126	-4.00
				(0.0233)		(0.0156)	(0.00)

Table 5: Bias of Parameter Estimates when f_1 is shifted Binomial($n = 250, p = 0.20$), $\pi_0 = 0.80$ and $C = 5$. ZIP$_4$($\eta = 0.40, \lambda = 1.5$) is the well-separated case, ZIGP$_7$($\eta = 0.40, \lambda = 1, \theta = 0.20$) represents the moderately mixed case, and ZIGP$_8$($\eta = 0.40, \lambda = 3, \theta = 0.20$) represents the heavily mixed case. The number in $(\cdot)$ represents the standard error.

True f_0	Choice of C	Model for f_0	Bias of Parameter Estimates $\widehat{\eta}_{\hat{C}}$	$\widehat{\lambda}_{\hat{C}}$	$\widehat{\theta}_{\hat{C}}$	$\hat{\pi}_{0,\hat{C}}$	$\hat{C}$
ZIP$_4$	$\hat{C}_1$	ZIGP	0.0390	0.2949	0.0000	0.0079	0.44
			(0.0211)	(0.0569)	(0.0000)	(0.0128)	(0.54)
		ZIP	-0.0061	-0.0605	NA	-0.0016	-1.12
			(0.0268)	(0.1152)		(0.0188)	(1.19)
		GP	NA	-0.0644	0.0000	0.0063	0.10
				(0.0305)	(0.0000)	(0.0128)	(0.48)
		P	NA	-1.1214	NA	-0.1721	-4.00
				(0.0332)		(0.0173)	(0.00)
	$\hat{C}_2$	ZIGP	0.0343	0.5281	0.0000	0.0452	-1.97
			(0.0205)	(0.1100)	(0.0000)	(0.0230)	(0.25)
		ZIP	-0.0046	0.0028	NA	0.0087	-3.00
			(0.0264)	(0.1823)		(0.0301)	(0.00)
		GP	NA	0.0030	0.0000	0.0253	-1.79
				(0.0466)	(0.0000)	(0.0156)	(0.45)
		P	NA	-1.1214	NA	-0.1721	-4.00
				(0.0332)		(0.0173)	(0.00)
ZIGP$_7$	$\hat{C}_1$	ZIGP	0.1449	0.7476	-0.1999	0.0017	-0.25
			(0.0210)	(0.0701)	(0.0003)	(0.0130)	(0.53)
		ZIP	0.0778	0.1486	NA	-0.0254	-2.46
			(0.0357)	(0.1516)		(0.0183)	(0.71)
		GP	NA	0.3272	-0.2000	-0.0006	-0.58
				(0.0279)	(0.0000)	(0.0131)	(0.50)
		P	NA	-0.7076	NA	-0.1295	-4.00
				(0.0269)		(0.0168)	(0.00)
	$\hat{C}_2$	ZIGP	0.1387	0.8908	-0.1999	0.0228	-2.09
			(0.0227)	(0.1557)	(0.0006)	(0.0284)	(0.29)
		ZIP	0.0775	0.1487	NA	-0.0253	-3.00
			(0.0357)	(0.1603)		(0.0195)	(0.00)
		GP	NA	0.3656	-0.2000	0.0103	-1.90
				(0.0332)	(0.0000)	(0.0145)	(0.30)
		P	NA	-0.7076	NA	-0.1295	-4.00
				(0.0269)		(0.0168)	(0.00)
ZIGP$_8$	$\hat{C}_1$	ZIGP	0.0187	0.2582	-0.1085	-0.0177	0.38
			(0.0399)	(0.1978)	(0.1004)	(0.0554)	(1.11)
		ZIP	0.0800	-0.1840	NA	-0.1206	-1.88
			(0.0411)	(0.3807)		(0.0606)	(1.05)
		GP	NA	-1.2556	-0.2000	-0.1630	-1.98
				(0.0526)	(0.0000)	(0.0182)	(0.13)
		P	NA	-2.8292	NA	-0.3900	-4.00
				(0.0239)		(0.0150)	(0.00)
	$\hat{C}_2$	ZIGP	-0.0633	0.5981	0.0469	0.0780	-1.38
			(0.1009)	(0.4933)	(0.1941)	(0.1143)	(0.60)
		ZIP	0.0752	-0.1559	-0.2000	-0.1096	-2.99
			(0.0525)	(0.4576)		(0.0882)	(0.11)
		GP	NA	-1.1042	-0.2000	-0.1085	-3.00
				(0.0568)	(0.0000)	(0.0269)	(0.04)
		P	NA	-2.8292	NA	-0.3900	-4.00
				(0.0239)		(0.0150)	(0.00)

Table 6: Numerical Comparison when f_1 is shifted Geometric($p = 0.08$), $\pi_0 = 0.85$ and $C = 5$. ZIP$_3$($\eta = 0.40, \lambda = 1.5$) represents the well-separated case, ZIGP$_5$($\eta = 0.40, \lambda = 1, \theta = 0.15$) represents the moderately mixed case, and ZIGP$_6$($\eta = 0.40, \lambda = 3, \theta = 0.15$) represents the heavily mixed case. The number in $(\cdot)$ represents the standard error.

			Two-Stage Procedure			One-Stage Procedure			Storey's FDR		
True f_0	Choice of C	Model for f_0	R	$\widehat{FDR}$	$\widehat{TPR}$	R	$\widehat{FDR}$	$\widehat{TPR}$	R	$\widehat{FDR}$	$\widehat{TPR}$
ZIP$_3$	$\hat{C}_1$	ZIGP	152.53	**0.0146**	**0.999**	135.92	0.0005	0.903	136.04	0.0005	0.904
			(11.78)	(0.0124)	(0.005)	(11.31)	(0.0019)	(0.024)	(11.09)	(0.0019)	(0.023)
		ZIP	144.50	0.0029	0.958	142.64	0.0024	0.945	142.02	0.0022	0.942
			(11.41)	(0.0045)	(0.017)	(12.07)	(0.0043)	(0.022)	(11.85)	(0.0041)	(0.031)
		GP	153.44	0.0190	1.000	139.19	0.0014	0.923	137.11	0.0008	0.910
			(12.13)	(0.0206)	(0.001)	(12.34)	(0.0034)	(0.029)	(11.48)	(0.0025)	(0.029)
		P	196.66	0.2232	1.000	196.66	0.2232	1.000	163.71	0.0783	1.000
			(29.71)	(0.0908)	(0.000)	(29.71)	(0.0908)	(0.000)	(15.17)	(0.0484)	(0.000)
	$\hat{C}_2$	ZIGP	152.81	0.0160	0.999	132.59	0.0003	0.880	131.24	0.0003	0.872
			(11.77)	(0.0140)	(0.007)	(12.09)	(0.0016)	(0.036)	(11.81)	(0.0016)	(0.041)
		ZIP	144.50	0.0029	0.958	141.00	0.0019	0.935	140.44	0.0019	0.931
			(11.41)	(0.0045)	(0.017)	(12.15)	(0.0040)	(0.029)	(12.04)	(0.0039)	(0.034)
		GP	161.41	0.0664	1.000	137.43	0.0009	0.912	136.39	0.0006	0.906
			(13.68)	(0.0377)	(0.000)	(11.76)	(0.0026)	(0.027)	(11.18)	(0.0021)	(0.026)
		P	196.66	0.2232	1.000	196.66	0.2232	1.000	163.71	0.0783	1.000
			(29.71)	(0.0908)	(0.000)	(29.71)	(0.0908)	(0.000)	(15.17)	(0.0484)	(0.000)
ZIGP$_5$	$\hat{C}_1$	ZIGP	156.38	**0.0374**	**1.000**	139.25	0.0064	0.919	135.43	0.0041	0.896
			(12.24)	(0.0270)	(0.001)	(12.75)	(0.0075)	(0.033)	(11.37)	(0.0056)	(0.030)
		ZIP	151.38	0.0213	0.984	151.37	0.0212	0.984	148.69	0.0167	0.971
			(12.60)	(0.0144)	(0.023)	(12.60)	(0.0144)	(0.023)	(12.54)	(0.0128)	(0.029)
		GP	157.94	0.0465	1.000	143.92	0.0094	0.947	144.09	0.0095	0.948
			(12.56)	(0.0312)	(0.000)	(11.48)	(0.0082)	(0.020)	(11.48)	(0.0082)	(0.021)
		P	190.55	0.2025	1.000	190.55	0.2025	1.000	175.68	0.1420	1.000
			(23.90)	(0.0792)	(0.000)	(23.90)	(0.0792)	(0.000)	(14.31)	(0.0439)	(0.000)
	$\hat{C}_2$	ZIGP	158.91	0.0529	1.000	135.50	0.0041	0.897	134.51	0.0036	0.891
			(12.10)	(0.0268)	(0.003)	(11.73)	(0.0058)	(0.030)	(11.00)	(0.0052)	(0.026)
		ZIP	151.31	0.0213	0.984	151.27	0.0213	0.984	148.73	0.0169	0.971
			(12.64)	(0.0142)	(0.023)	(12.68)	(0.0143)	(0.024)	(12.62)	(0.0130)	(0.029)
		GP	173.85	0.1318	1.000	143.25	0.0091	0.943	141.93	0.0083	0.935
			(15.57)	(0.0552)	(0.000)	(11.76)	(0.0083)	(0.024)	(12.08)	(0.0081)	(0.033)
		P	190.55	0.2025	1.000	190.55	0.2025	1.000	175.68	0.1420	1.000
			(23.90)	(0.0792)	(0.000)	(23.90)	(0.0792)	(0.000)	(14.31)	(0.0439)	(0.000)
ZIGP$_6$	$\hat{C}_1$	ZIGP	128.99	0.0527	0.809	113.74	**0.0214**	**0.796**	114.12	0.0208	0.742
			(25.92)	(0.0334)	(0.145)	(23.39)	(0.0188)	(0.141)	(21.36)	(0.0165)	(0.126)
		ZIP	194.97	0.2547	0.966	142.50	0.0829	0.876	136.12	0.0623	0.846
			(12.97)	(0.0326)	(0.015)	(21.61)	(0.0615)	(0.054)	(15.40)	(0.0381)	(0.047)
		GP	290.60	0.4796	1.000	185.19	0.2215	0.953	165.50	0.1571	0.927
			(24.60)	(0.0486)	(0.000)	(19.51)	(0.0576)	(0.023)	(12.40)	(0.0313)	(0.022)
		P	496.39	0.6950	1.000	496.39	0.6950	1.000	390.81	0.6128	1.000
			(43.29)	(0.0310)	(0.000)	(43.29)	(0.0310)	(0.000)	(35.76)	(0.0362)	(0.000)
	$\hat{C}_2$	ZIGP	105.34	0.0355	0.671	93.46	0.0142	0.661	95.43	0.0144	0.625
			(50.16)	(0.0302)	(0.313)	(44.56)	(0.0160)	(0.308)	(41.16)	(0.0150)	(0.265)
		ZIP	194.97	0.2547	0.966	140.94	0.0804	0.866	134.62	0.0603	0.838
			(12.97)	(0.0326)	(0.015)	(23.00)	(0.0637)	(0.067)	(16.69)	(0.0399)	(0.057)
		GP	242.61	0.3778	1.000	168.07	0.1655	0.931	152.31	0.1113	0.898
			(20.34)	(0.0448)	(0.000)	(14.63)	(0.0400)	(0.022)	(14.42)	(0.0390)	(0.031)
		P	496.39	0.6950	1.000	496.39	0.6950	1.000	390.81	0.6128	1.000
			(43.29)	(0.0310)	(0.000)	(43.29)	(0.0310)	(0.000)	(35.76)	(0.0362)	(0.000)

Table 7: Numerical Comparison when f_1 is shifted Binomial($n = 250, p = 0.20$), $\pi_0 = 0.80$ and $C = 5$. ZIP$_4$($\eta = 0.40, \lambda = 1.5$) represents the well-separated case, ZIGP$_7$($\eta = 0.40, \lambda = 1, \theta = 0.20$) represents the moderately mixed case, and ZIGP$_8$($\eta = 0.40, \lambda = 3, \theta = 0.20$) represents the heavily mixed case. The number in $(\cdot)$ represents the standard error.

			Two-Stage Procedure			One-Stage Procedure			Storey's FDR		
True f_0	Choice of C	Model for f_0	R	$\widehat{FDR}$	$\widehat{TPR}$	R	$\widehat{FDR}$	$\widehat{TPR}$	R	$\widehat{FDR}$	$\widehat{TPR}$
ZIP$_4$	$\hat{C}_1$	ZIGP	195.26	0.0084	0.996	186.46	0.0003	0.958	189.34	0.0010	0.973
			(13.05)	(0.0092)	(0.006)	(12.89)	(0.0013)	(0.015)	(13.02)	(0.0025)	(0.016)
		ZIP	191.74	0.0020	0.985	189.08	0.0010	0.973	192.06	0.0027	0.985
			(12.68)	(0.0034)	(0.009)	(12.94)	(0.0026)	(0.012)	(12.81)	(0.0045)	(0.010)
		GP	196.93	0.0137	0.999	187.58	0.0004	0.965	191.57	0.0019	0.984
			(13.21)	(0.0165)	(0.003)	(12.74)	(0.0015)	(0.014)	(12.68)	(0.0032)	(0.010)
		P	237.12	0.1730	1.000	237.12	0.1730	1.000	217.12	0.1024	1.000
			(28.10)	(0.0759)	(0.000)	(28.10)	(0.0759)	(0.000)	(17.44)	(0.0514)	(0.000)
	$\hat{C}_2$	ZIGP	196.41	**0.0110**	**0.999**	183.31	0.0001	0.943	187.51	0.0004	0.964
			(12.81)	(0.0092)	(0.004)	(12.85)	(0.0008)	(0.018)	(12.71)	(0.0015)	(0.015)
		ZIP	191.74	0.0020	0.985	188.55	0.0009	0.970	191.63	0.0026	0.983
			(12.68)	(0.0034)	(0.009)	(12.99)	(0.0025)	(0.015)	(12.96)	(0.0046)	(0.012)
		GP	204.55	0.0492	1.000	187.34	0.0004	0.964	190.94	0.0017	0.981
			(14.35)	(0.0283)	(0.000)	(12.67)	(0.0014)	(0.014)	(12.87)	(0.0030)	(0.013)
		P	237.12	0.1730	1.000	237.12	0.1730	1.000	217.12	0.1024	1.000
			(28.10)	(0.0759)	(0.000)	(28.10)	(0.0759)	(0.000)	(17.44)	(0.0514)	(0.000)
ZIGP$_7$	$\hat{C}_1$	ZIGP	203.78	**0.0474**	**1.000**	188.01	0.0062	0.963	192.48	0.0126	0.979
			(13.86)	(0.0285)	(0.001)	(12.88)	(0.0060)	(0.014)	(12.99)	(0.0087)	(0.013)
		ZIP	197.00	0.0232	0.991	196.85	0.0229	0.991	199.24	0.0296	0.996
			(13.18)	(0.0153)	(0.010)	(13.23)	(0.0155)	(0.010)	(12.91)	(0.0152)	(0.008)
		GP	206.81	0.0611	1.000	191.02	0.0103	0.974	193.70	0.0145	0.984
			(13.99)	(0.0291)	(0.000)	(13.15)	(0.0091)	(0.013)	(12.74)	(0.0086)	(0.009)
		P	250.24	0.2186	1.000	250.24	0.2186	1.000	228.55	0.1509	1.000
			(26.30)	(0.0715)	(0.000)	(26.30)	(0.0715)	(0.000)	(13.59)	(0.0261)	(0.000)
	$\hat{C}_2$	ZIGP	204.14	0.0500	0.999	186.47	0.0052	0.956	190.29	0.0096	0.971
			(13.53)	(0.0272)	(0.005)	(12.94)	(0.0056)	(0.018)	(13.10)	(0.0082)	(0.017)
		ZIP	197.15	0.0236	0.992	196.96	0.0233	0.991	199.26	0.0296	0.996
			(13.25)	(0.0155)	(0.009)	(13.30)	(0.0157)	(0.010)	(12.93)	(0.0150)	(0.008)
		GP	227.63	0.1469	1.000	190.12	0.0091	0.971	193.62	0.0143	0.983
			(14.53)	(0.0338)	(0.000)	(13.10)	(0.0087)	(0.014)	(12.75)	(0.0086)	(0.010)
		P	250.24	0.2186	1.000	250.24	0.2186	1.000	228.55	0.1509	1.000
			(26.30)	(0.0715)	(0.000)	(26.30)	(0.0715)	(0.000)	(13.59)	(0.0261)	(0.000)
ZIGP$_8$	$\hat{C}_1$	ZIGP	178.35	0.0473	0.864	164.97	0.0232	0.851	176.16	**0.0365**	**0.865**
			(40.55)	(0.0335)	(0.181)	(38.30)	(0.0201)	(0.179)	(31.39)	(0.0246)	(0.136)
		ZIP	255.13	0.2422	0.988	205.35	0.0965	0.947	206.65	0.0995	0.948
			(14.48)	(0.0285)	(0.008)	(24.67)	(0.0625)	(0.030)	(19.14)	(0.0462)	(0.025)
		GP	350.89	0.4417	1.000	251.49	0.2316	0.986	231.18	0.1735	0.975
			(18.62)	(0.0331)	(0.000)	(17.52)	(0.0396)	(0.010)	(17.56)	(0.0387)	(0.013)
		P	534.71	0.6323	1.000	534.71	0.6323	1.000	472.62	0.5823	1.000
			(41.15)	(0.0323)	(0.000)	(41.15)	(0.0323)	(0.000)	(45.93)	(0.0457)	(0.000)
	$\hat{C}_2$	ZIGP	105.24	0.0272	0.511	96.12	0.0157	0.503	120.71	0.0206	0.597
			(90.46)	(0.0360)	(0.434)	(84.92)	(0.0201)	(0.427)	(68.90)	(0.0259)	(0.330)
		ZIP	255.20	0.2424	0.988	204.98	0.0965	0.944	206.40	0.0992	0.947
			(14.43)	(0.0282)	(0.008)	(25.90)	(0.0646)	(0.036)	(20.09)	(0.0486)	(0.027)
		GP	306.29	0.3582	1.000	231.62	0.1745	0.975	226.41	0.1596	0.972
			(26.61)	(0.0484)	(0.000)	(18.36)	(0.0417)	(0.012)	(14.01)	(0.0267)	(0.012)
		P	534.71	0.6323	1.000	534.71	0.6323	1.000	472.62	0.5823	1.000
			(41.15)	(0.0323)	(0.000)	(41.15)	(0.0323)	(0.000)	(45.93)	(0.0457)	(0.000)

Table 8: Bias of Parameter Estimates when f_1 is shifted Geometric($p = 0.08$), $\pi_0 = 0.80$ and $C = 1$. The support of f_1 may contain values in $[0, C]$. The degree to which the f_0 is mixed with f_1 is described for values from $[0, K]$. MZIGP$_1$($\eta = 0.80, \lambda = 1.5, \theta = 0.3$) represents the moderately mixed case, MZIGP$_2$($\eta = 0.80, \lambda = 3, \theta = 0.3$) represents the heavily mixed case while MZIP$_1$($\eta = 0.80, \lambda = 1.5$) represents the well-separated case. The number in $(\cdot)$ represents the standard error.

True	Choice	Model	Bias of Parameter Estimates				
f_0	of C	for f_0	$\widehat{\eta}_{\hat{C}}$	$\widehat{\lambda}_{\hat{C}}$	$\widehat{\theta}_{\hat{C}}$	$\hat{\pi}_{0,\hat{C}}$	$\hat{C}$
MZIP$_1$	$\hat{C}_1$	ZIGP	-0.0313	0.7908	0.0046	0.0535	3.15
			(0.0174)	(0.1396)	(0.0208)	(0.0131)	(0.57)
		ZIP	-0.0279	0.3619	NA	0.0367	2.00
			(0.0182)	(0.2376)		(0.0167)	(0.79)
		GP	NA	-0.2453	0.0052	0.0458	3.04
				(0.0247)	(0.0666)	(0.0167)	(1.47)
		P	NA	-1.4209	NA	-0.0681	0.00
				(0.0113)		(0.0142)	(0.00)
	$\hat{C}_2$	ZIGP	-0.0406	0.9192	0.0125	0.0662	2.04
			(0.0204)	(0.1738)	(0.0207)	(0.0204)	(0.36)
		ZIP	-0.0362	0.4980	NA	0.0491	1.04
			(0.0236)	(0.3446)		(0.0280)	(0.20)
		GP	NA	-0.2260	0.0051	0.0513	2.10
				(0.0295)	(0.0663)	(0.0172)	(0.47)
		P	NA	-1.4209	NA	-0.0681	0.00
				(0.0113)		(0.0142)	(0.00)
MZIGP$_1$	$\hat{C}_1$	ZIGP	-0.0226	1.1388	-0.2427	0.0520	3.43
			(0.0301)	(0.2064)	(0.1066)	(0.0263)	(0.91)
		ZIP	0.0038	0.6122	NA	0.0166	1.74
			(0.0200)	(0.3320)		(0.0206)	(0.73)
		GP	NA	-0.2354	-0.2765	0.0302	3.04
				(0.0267)	(0.1412)	(0.0309)	(3.10)
		P	NA	-1.4412	NA	-0.0828	0.00
				(0.0101)		(0.0145)	(0.03)
	$\hat{C}_2$	ZIGP	-0.0404	1.2058	-0.1940	0.0661	2.87
			(0.0622)	(0.2831)	(0.1889)	(0.0480)	(0.69)
		ZIP	-0.0043	0.7245	NA	0.0267	1.06
			(0.0278)	(0.4318)		(0.0318)	(0.24)
		GP	NA	-0.2253	-0.2767	0.0318	2.11
				(0.0477)	(0.1398)	(0.0310)	(0.71)
		P	NA	-1.4412	NA	-0.0828	0.00
				(0.0101)		(0.0145)	(0.03)
MZIGP$_2$	$\hat{C}_1$	ZIGP	-0.0637	0.6712	-0.1755	0.0621	4.26
			(0.0817)	(0.3869)	(0.1826)	(0.0555)	(1.09)
		ZIP	-0.0039	0.2749	NA	0.0075	2.51
			(0.0318)	(0.4407)		(0.0328)	(1.00)
		GP	NA	-1.7202	-0.2931	-0.0269	2.26
				(0.0282)	(0.0769)	(0.0249)	(1.96)
		P	NA	-2.9721	NA	-0.1333	0.00
				(0.0088)		(0.0152)	(0.05)
	$\hat{C}_2$	ZIGP	-0.1066	0.8277	-0.1015	0.0850	3.44
			(0.1266)	(0.5559)	(0.2436)	(0.0732)	(0.69)
		ZIP	-0.0212	0.4601	NA	0.0275	1.52
			(0.0457)	(0.5401)		(0.0494)	(0.54)
		GP	NA	-1.7183	-0.2932	-0.0268	2.02
				(0.0408)	(0.0761)	(0.0249)	(0.25)
		P	NA	-2.9721	NA	-0.1333	0.00
				(0.0088)		(0.0152)	(0.05)

Table 9: Bias of Parameter Estimates when f_1 is shifted Binomial($n = 250, p = 0.20$), $\pi_0 = 0.35$ and $C = 1$. The support of f_1 may contain values in $[0, C]$. The degree to which the f_0 is mixed with f_1 is described for values from $[0, K]$. MZIGP$_3$($\eta = 0.40, \lambda = 1, \theta = 0.3$) represents the moderately mixed case, MZIGP$_4$($\eta = 0.40, \lambda = 4, \theta = 0.3$) represents the heavily mixed case while MZIP$_2$($\eta = 0.40, \lambda = 1.5$) represents the well-separated case. The number in $(\cdot)$ represents the standard error.

True f_0	Choice of C	Model for f_0	Bias of Parameter Estimates $\widehat{\eta}_{\hat{C}}$	$\widehat{\lambda}_{\hat{C}}$	$\widehat{\theta}_{\hat{C}}$	$\hat{\pi}_{0,\hat{C}}$	$\hat{C}$
MZIP$_2$	$\hat{C}_1$	ZIGP	-0.0313	0.8146	0.1342	0.0961	4.16
			(0.0433)	(0.1210)	(0.1611)	(0.0421)	(0.58)
		ZIP	-0.0145	0.5325	NA	0.0586	3.31
			(0.0301)	(0.2276)		(0.0191)	(0.56)
		GP	NA	0.3489	0.6793	0.2740	6.04
				(0.0567)	(0.0156)	(0.0241)	(0.60)
		P	NA	-0.9261	NA	-0.0355	0.72
				(0.2106)		(0.0349)	(0.66)
	$\hat{C}_2$	ZIGP	-0.1172	1.1124	0.3889	0.2454	2.72
			(0.0721)	(0.3754)	(0.2113)	(0.1362)	(0.76)
		ZIP	-0.0197	0.3069	NA	0.0444	1.00
			(0.0343)	(0.3078)		(0.0314)	(0.00)
		GP	NA	0.8247	0.6413	0.4984	1.84
				(0.2719)	(0.0302)	(0.1012)	(0.56)
		P	NA	-1.0845	NA	-0.0645	0.12
				(0.1098)		(0.0220)	(0.33)
MZIGP$_3$	$\hat{C}_1$	ZIGP	-0.0213	1.5176	0.1573	0.1723	4.74
			(0.0482)	(0.1618)	(0.1506)	(0.0469)	(0.72)
		ZIP	0.0963	0.9915	NA	0.0407	2.79
			(0.0323)	(0.3795)		(0.0234)	(0.72)
		GP	NA	0.8144	0.4057	0.2820	5.77
				(0.0548)	(0.0144)	(0.0244)	(0.64)
		P	NA	-0.5978	NA	-0.0361	0.63
				(0.1188)		(0.0222)	(0.49)
	$\hat{C}_2$	ZIGP	-0.0900	1.9875	0.2011	0.3099	1.90
			(0.0523)	(0.3843)	(0.0681)	(0.1175)	(0.53)
		ZIP	0.1013	0.5689	NA	0.0148	1.00
			(0.0386)	(0.3252)		(0.0237)	(0.00)
		GP	NA	1.1286	0.3751	0.4428	1.90
				(0.1914)	(0.0244)	(0.0926)	(0.42)
		P	NA	-0.6189	NA	-0.0399	0.52
				(0.1140)		(0.0223)	(0.50)
MZIGP$_4$	$\hat{C}_1$	ZIGP	-0.1182	0.7706	-0.0268	0.2040	4.55
			(0.0900)	(0.6054)	(0.1636)	(0.1817)	(1.32)
		ZIP	-0.0361	0.9477	NA	0.0522	5.11
			(0.0518)	(0.5440)		(0.0497)	(1.83)
		GP	NA	-1.6401	0.3958	0.1999	4.70
				(0.1262)	(0.0163)	(0.0416)	(1.00)
		P	NA	-3.6318	NA	-0.1646	1.00
				(0.0635)		(0.0123)	(0.04)
	$\hat{C}_2$	ZIGP	-0.2267	1.7636	0.0454	0.4527	1.51
			(0.1063)	(1.2774)	(0.1185)	(0.2162)	(0.84)
		ZIP	0.0516	0.0295	NA	-0.0159	1.51
			(0.1027)	(0.7972)		(0.0886)	(0.53)
		GP	NA	-1.3778	0.3682	0.3337	1.02
				(0.1095)	(0.0160)	(0.0653)	(0.13)
		P	NA	-3.6321	NA	-0.1647	1.00
				(0.0619)		(0.0123)	(0.03)

Table 10: Bias of Parameter Estimates when f_1 is shifted Geometric($p = 0.08$), $\pi_0 = 0.85$ and $C = 1$. The support of f_1 may contain values in $[0, C]$. The degree to which the f_0 is mixed with f_1 is described for values from $[0, K]$. MZIGP$_5$($\eta = 0.40, \lambda = 1, \theta = 0.15$) represents the moderately mixed case, MZIGP$_6$($\eta = 0.40, \lambda = 3, \theta = 0.15$) represents the heavily mixed case while MZIP$_3$($\eta = 0.40, \lambda = 1.5$) represents the well-separated case. The number in $(\cdot)$ represents the standard error.

True f_0	Choice of C	Model for f_0	Bias of Parameter Estimates $\hat{\eta}_{\hat{C}}$	$\widehat{\lambda}_{\hat{C}}$	$\widehat{\theta}_{\hat{C}}$	$\hat{\pi}_{0,\hat{C}}$	$\hat{C}$
MZIP$_3$	$\hat{C}_1$	ZIGP	0.0305	0.4481	0.0000	0.0370	3.74
			(0.0200)	(0.0629)	(0.0000)	(0.0108)	(0.53)
		ZIP	-0.0057	0.0300	NA	0.0121	2.00
			(0.0258)	(0.1290)		(0.0194)	(0.82)
		GP	NA	0.0088	0.0000	0.0306	3.21
				(0.0314)	(0.0000)	(0.0110)	(0.43)
		P	NA	-1.1227	NA	-0.1880	0.00
				(0.0340)		(0.0174)	(0.00)
	$\hat{C}_2$	ZIGP	0.0255	0.5993	0.0000	0.0653	2.03
			(0.0196)	(0.0899)	(0.0000)	(0.0164)	(0.18)
		ZIP	-0.0058	0.0821	NA	0.0223	1.00
			(0.0253)	(0.1810)		(0.0307)	(0.05)
		GP	NA	0.0516	0.0000	0.0447	2.05
				(0.0364)	(0.0000)	(0.0132)	(0.22)
		P	NA	-1.1227	NA	-0.1880	0.00
				(0.0340)		(0.0174)	(0.00)
MZIGP$_5$	$\hat{C}_1$	ZIGP	0.1232	0.8350	-0.1500	0.0307	3.16
			(0.0196)	(0.0688)	(0.0000)	(0.0110)	(0.47)
		ZIP	0.0644	0.2103	NA	-0.0073	1.35
			(0.0326)	(0.1480)		(0.0183)	(0.53)
		GP	NA	0.3755	-0.1500	0.0262	3.01
				(0.0278)	(0.0000)	(0.0109)	(0.28)
		P	NA	-0.6936	NA	-0.1329	0.00
				(0.0269)		(0.0166)	(0.00)
	$\hat{C}_2$	ZIGP	0.1202	0.9150	-0.1500	0.0433	1.98
			(0.0197)	(0.0944)	(0.0000)	(0.0166)	(0.13)
		ZIP	0.0646	0.2194	NA	-0.0060	1.00
			(0.0326)	(0.1625)		(0.0205)	(0.03)
		GP	NA	0.4021	-0.1500	0.0346	2.06
				(0.0303)	(0.0000)	(0.0121)	(0.23)
		P	NA	-0.6936	NA	-0.1329	0.00
				(0.0269)		(0.0166)	(0.00)
MZIGP$_6$	$\hat{C}_1$	ZIGP	0.0070	0.3117	-0.1172	-0.0020	3.60
			(0.0325)	(0.1786)	(0.0769)	(0.0305)	(0.91)
		ZIP	0.0474	-0.0871	NA	-0.0813	2.02
			(0.0369)	(0.3235)		(0.0616)	(0.92)
		GP	NA	-1.1926	-0.1500	-0.1402	2.01
				(0.0548)	(0.0000)	(0.0184)	(0.08)
		P	NA	-2.8201	NA	-0.4126	0.00
				(0.0233)		(0.0156)	(0.00)
	$\hat{C}_2$	ZIGP	-0.0262	0.5512	-0.0952	0.0456	1.95
			(0.0664)	(0.3922)	(0.1364)	(0.0631)	(0.46)
		ZIP	0.0290	0.0566	NA	-0.0491	1.00
			(0.0645)	(0.5305)		(0.1002)	(0.21)
		GP	NA	-1.0474	-0.1500	-0.0814	1.00
				(0.0607)	(0.0000)	(0.0287)	(0.04)
		P	NA	-2.8201	NA	-0.4126	0.00
				(0.0233)		(0.0156)	(0.00)

Table 11: Bias of Parameter Estimates when f_1 is shifted Binomial($n = 250, p = 0.20$), $\pi_0 = 0.80$ and $C = 1$. The support of f_1 may contain values in $[0, C]$. The degree to which the f_0 is mixed with f_1 is described for values from $[0, K]$. MZIGP$_7$($\eta = 0.40, \lambda = 1, \theta = 0.2$) represents the moderately mixed case, MZIGP$_8$($\eta = 0.40, \lambda = 3, \theta = 0.2$) represents the heavily mixed case while MZIP$_4$($\eta = 0.40, \lambda = 1.5$) represents the well-separated case. The number in $(\cdot)$ represents the standard error.

True f_0	Choice of C	Model for f_0	Bias of Parameter Estimates $\widehat{\eta}_{\hat{C}}$	$\widehat{\lambda}_{\hat{C}}$	$\widehat{\theta}_{\hat{C}}$	$\hat{\pi}_{0,\hat{C}}$	$\hat{C}$
MZIP$_4$	$\hat{C}_1$	ZIGP	0.0347	0.3930	0.0000	0.0271	3.94
			(0.0205)	(0.0631)	(0.0000)	(0.0127)	(0.50)
		ZIP	-0.0058	-0.0114	NA	0.0082	2.32
			(0.0261)	(0.1288)		(0.0206)	(0.93)
		GP	NA	-0.0189	0.0000	0.0226	3.43
				(0.0324)	(0.0000)	(0.0128)	(0.51)
		P	NA	-1.1214	NA	-0.1721	0.00
				(0.0332)		(0.0173)	(0.00)
	$\hat{C}_2$	ZIGP	0.0289	0.5738	0.0000	0.0588	2.00
			(0.0203)	(0.1087)	(0.0000)	(0.0234)	(0.18)
		ZIP	-0.0056	0.0395	NA	0.0175	1.00
			(0.0256)	(0.1842)		(0.0311)	(0.03)
		GP	NA	0.0344	0.0000	0.0389	2.05
				(0.0382)	(0.0000)	(0.0151)	(0.22)
		P	NA	-1.1214	NA	-0.1721	0.00
				(0.0332)		(0.0173)	(0.00)
MZIGP$_7$	$\hat{C}_1$	ZIGP	0.1384	0.8343	-0.2000	0.0175	3.20
			(0.0206)	(0.0770)	(0.0004)	(0.0134)	(0.49)
		ZIP	0.0795	0.1919	NA	-0.0181	1.41
			(0.0338)	(0.1561)		(0.0191)	(0.57)
		GP	NA	0.3627	-0.2000	0.0133	3.00
				(0.0297)	(0.0000)	(0.0132)	(0.28)
		P	NA	-0.7076	NA	-0.1295	0.00
				(0.0269)		(0.0168)	(0.00)
	$\hat{C}_2$	ZIGP	0.1313	0.9486	-0.1999	0.0370	1.88
			(0.0234)	(0.1715)	(0.0005)	(0.0321)	(0.33)
		ZIP	0.0793	0.1928	NA	-0.0179	1.00
			(0.0338)	(0.1641)		(0.0201)	(0.00)
		GP	NA	0.3898	-0.2000	0.0213	2.02
				(0.0297)	(0.0000)	(0.0143)	(0.14)
		P	NA	-0.7076	NA	-0.1295	0.00
				(0.0269)		(0.0168)	(0.00)
MZIGP$_8$	$\hat{C}_1$	ZIGP	0.0082	0.3075	-0.1070	0.0027	4.17
			(0.0364)	(0.1839)	(0.0936)	(0.0531)	(1.02)
		ZIP	0.0708	-0.1315	NA	-0.1065	2.08
			(0.0404)	(0.3716)		(0.0621)	(0.99)
		GP	NA	-1.2371	-0.2000	-0.1550	2.01
				(0.0532)	(0.0000)	(0.0185)	(0.11)
		P	NA	-2.8292	NA	-0.3900	0.00
				(0.0239)		(0.0150)	(0.00)
	$\hat{C}_2$	ZIGP	-0.0707	0.6657	0.0296	0.0918	2.46
			(0.0959)	(0.4907)	(0.1742)	(0.1038)	(0.65)
		ZIP	0.0627	-0.0753	NA	-0.0897	1.02
			(0.0549)	(0.4748)		(0.0935)	(0.14)
		GP	NA	-1.0901	-0.2000	-0.1013	1.00
				(0.0568)	(0.0000)	(0.0271)	(0.03)
		P	NA	-2.8292	NA	-0.3900	0.00
				(0.0239)		(0.0150)	(0.00)

Table 12: Numerical Comparison when f_1 is shifted Geometric($p = 0.08$), $\pi_0 = 0.80$ and $C = 1$. The support of f_1 may contain values in $[0, C]$. The degree to which the f_0 is mixed with f_1 is described for values from $[0, K]$. MZIGP$_1$($\eta = 0.80, \lambda = 1.5, \theta = 0.3$) represents the moderately mixed case, MZIGP$_2$($\eta = 0.80, \lambda = 3, \theta = 0.3$) represents the heavily mixed case while MZIP$_1$($\eta = 0.80, \lambda = 1.5$) represents the well-separated case. The number in $(\cdot)$ represents the standard error.

			Two-Stage Procedure			One-Stage Procedure			Storey's FDR		
True f_0	Choice of C	Model for f_0	R	$\widehat{FDR}$	$\widehat{TPR}$	R	$\widehat{FDR}$	$\widehat{TPR}$	R	$\widehat{FDR}$	$\widehat{TPR}$
MZIP$_1$	C_1	ZIGP	138.74	**0.0018**	**0.693**	122.42	0.0002	0.676	120.32	0.0001	0.602
			(12.84)	(0.0038)	(0.052)	(11.98)	(0.0014)	(0.043)	(11.67)	(0.0010)	(0.049)
		ZIP	136.74	0.0012	0.683	132.48	0.0009	0.683	129.17	0.0006	0.646
			(11.37)	(0.0032)	(0.037)	(12.99)	(0.0030)	(0.037)	(12.10)	(0.0022)	(0.048)
		GP	163.90	0.0214	0.801	134.83	0.0008	0.722	134.90	0.0009	0.674
			(19.08)	(0.0180)	(0.071)	(14.97)	(0.0024)	(0.065)	(14.91)	(0.0025)	(0.062)
		P	259.70	0.2358	0.986	259.70	0.2358	0.986	217.01	0.1402	0.933
			(26.43)	(0.0565)	(0.027)	(26.43)	(0.0565)	(0.027)	(12.92)	(0.0243)	(0.018)
	C_2	ZIGP	133.69	0.0012	0.668	118.19	0.0001	0.659	116.59	0.0001	0.583
			(13.54)	(0.0032)	(0.055)	(12.69)	(0.0010)	(0.048)	(11.57)	(0.0009)	(0.048)
		ZIP	136.74	0.0012	0.683	129.38	0.0009	0.680	126.57	0.0005	0.633
			(11.36)	(0.0032)	(0.037)	(14.20)	(0.0030)	(0.040)	(13.10)	(0.0021)	(0.055)
		GP	181.36	0.0547	0.857	134.75	0.0008	0.693	134.89	0.0009	0.674
			(19.33)	(0.0194)	(0.072)	(14.97)	(0.0024)	(0.066)	(14.92)	(0.0025)	(0.062)
		P	259.70	0.2358	0.986	259.70	0.2358	0.986	217.01	0.1402	0.933
			(26.43)	(0.0565)	(0.027)	(26.43)	(0.0565)	(0.027)	(12.92)	(0.0243)	(0.018)
MZIGP$_1$	C_1	ZIGP	124.44	**0.0280**	**0.604**	112.31	0.0170	0.603	110.42	0.0151	0.544
			(25.52)	(0.0197)	(0.116)	(23.52)	(0.0148)	(0.115)	(20.66)	(0.0133)	(0.096)
		ZIP	145.47	0.0486	0.692	137.26	0.0387	0.689	131.51	0.0316	0.636
			(12.45)	(0.0201)	(0.038)	(16.15)	(0.0231)	(0.040)	(13.78)	(0.0189)	(0.053)
		GP	188.23	0.1345	0.813	139.82	0.0465	0.713	139.48	0.0459	0.666
			(36.78)	(0.0385)	(0.141)	(25.81)	(0.0182)	(0.123)	(25.54)	(0.0172)	(0.115)
		P	283.16	0.2942	0.999	283.16	0.2942	0.999	240.01	0.2226	0.933
			(15.95)	(0.0294)	(0.009)	(15.95)	(0.0294)	(0.009)	(13.47)	(0.0275)	(0.018)
	C_2	ZIGP	114.01	0.0250	0.554	102.98	0.0155	0.553	102.09	0.0137	0.503
			(41.00)	(0.0202)	(0.194)	(37.49)	(0.0149)	(0.193)	(33.41)	(0.0132)	(0.161)
		ZIP	145.41	0.0485	0.691	134.07	0.0356	0.681	128.90	0.0292	0.625
			(12.40)	(0.0200)	(0.038)	(17.80)	(0.0239)	(0.049)	(15.19)	(0.0195)	(0.060)
		GP	200.34	0.1594	0.842	139.67	0.0462	0.700	139.48	0.0459	0.666
			(35.69)	(0.0265)	(0.142)	(25.71)	(0.0178)	(0.122)	(25.54)	(0.0172)	(0.115)
		P	283.16	0.2942	0.999	283.16	0.2942	0.999	240.01	0.2226	0.933
			(15.95)	(0.0294)	(0.009)	(15.95)	(0.0294)	(0.009)	(13.47)	(0.0275)	(0.018)
MZIGP$_2$	C_1	ZIGP	94.00	0.0477	0.442	92.41	0.0455	0.442	93.25	**0.0435**	**0.442**
			(38.47)	(0.0371)	(0.173)	(38.05)	(0.0364)	(0.173)	(33.49)	(0.0327)	(0.151)
		ZIP	172.93	0.1832	0.706	129.31	0.0931	0.626	123.30	0.0815	0.565
			(12.11)	(0.0310)	(0.033)	(20.56)	(0.0436)	(0.058)	(16.38)	(0.0359)	(0.058)
		GP	256.15	0.3254	0.863	181.98	0.2032	0.751	171.17	0.1826	0.700
			(28.29)	(0.0322)	(0.082)	(24.76)	(0.0400)	(0.074)	(19.29)	(0.0301)	(0.070)
		P	333.58	0.4004	1.000	333.58	0.4004	1.000	305.15	0.3769	0.950
			(15.19)	(0.0269)	(0.004)	(15.19)	(0.0269)	(0.004)	(21.09)	(0.0302)	(0.031)
	C_2	ZIGP	79.47	0.0413	0.373	78.21	0.0396	0.373	80.20	0.0373	0.380
			(49.74)	(0.0388)	(0.228)	(49.13)	(0.0380)	(0.228)	(43.63)	(0.0346)	(0.201)
		ZIP	172.95	0.1832	0.706	123.31	0.0822	0.605	119.03	0.0737	0.549
			(12.10)	(0.0310)	(0.033)	(22.43)	(0.0464)	(0.067)	(17.89)	(0.0376)	(0.064)
		GP	258.24	0.3286	0.867	181.93	0.2031	0.750	171.17	0.1826	0.700
			(26.93)	(0.0290)	(0.081)	(24.75)	(0.0400)	(0.074)	(19.29)	(0.0301)	(0.070)
		P	333.58	0.4004	1.000	333.58	0.4004	1.000	305.15	0.3769	0.950
			(15.19)	(0.0269)	(0.004)	(15.19)	(0.0269)	(0.004)	(21.09)	(0.0302)	(0.031)

Table 13: Numerical Comparison when f_1 is shifted Binomial($n = 250, p = 0.20$), $\pi_0 = 0.35$ and $C = 1$. The support of f_1 may contain values in $[0, C]$. The degree to which the f_0 is mixed with f_1 is described for values from $[0, K]$. MZIGP$_3$($\eta = 0.40, \lambda = 1, \theta = 0.3$) represents the moderately mixed case, MZIGP$_4$($\eta = 0.40, \lambda = 4, \theta = 0.3$) represents the heavily mixed case while MZIP$_2$($\eta = 0.40, \lambda = 1.5$) represents the well-separated case. The number in $(\cdot)$ represents the standard error.

			Two-Stage Procedure			One-Stage Procedure			Storey's FDR		
True f_0	Choice of C	Model for f_0	R	$\widehat{FDR}$	$\widehat{TPR}$	R	$\widehat{FDR}$	$\widehat{TPR}$	R	$\widehat{FDR}$	$\widehat{TPR}$
MZIP$_2$	C_1	ZIGP	479.53	**0.0003**	**0.747**	462.31	0.0001	0.726	467.90	0.0000	0.729
			(151.29)	(0.0008)	(0.236)	(143.73)	(0.0003)	(0.227)	(110.52)	(0.0003)	(0.173)
		ZIP	560.41	0.0004	0.872	553.81	0.0003	0.869	548.26	0.0002	0.854
			(15.80)	(0.0009)	(0.015)	(20.18)	(0.0009)	(0.018)	(21.69)	(0.0007)	(0.028)
		GP	0.00	0.0000	0.000	0.00	0.0000	0.000	46.94	0.0000	0.073
			(0.00)	(0.0000)	(0.000)	(0.00)	(0.0000)	(0.000)	(19.16)	(0.0000)	(0.029)
		P	637.38	0.0307	0.961	637.38	0.0307	0.961	616.13	0.0126	0.947
			(30.52)	(0.0266)	(0.022)	(30.52)	(0.0266)	(0.022)	(18.40)	(0.0094)	(0.018)
	C_2	ZIGP	162.67	0.0003	0.254	156.45	0.0001	0.247	249.61	0.0000	0.389
			(233.83)	(0.0009)	(0.365)	(223.72)	(0.0003)	(0.353)	(166.16)	(0.0002)	(0.260)
		ZIP	567.74	0.0008	0.883	564.24	0.0007	0.880	558.27	0.0005	0.869
			(19.64)	(0.0013)	(0.022)	(23.70)	(0.0013)	(0.027)	(23.85)	(0.0011)	(0.031)
		GP	0.00	0.0000	0.000	0.00	0.0000	0.000	30.14	0.0000	0.047
			(0.00)	(0.0000)	(0.000)	(0.00)	(0.0000)	(0.000)	(12.91)	(0.0000)	(0.020)
		P	655.62	0.0443	0.975	655.62	0.0443	0.975	629.68	0.0199	0.961
			(25.58)	(0.0237)	(0.014)	(25.58)	(0.0237)	(0.014)	(15.79)	(0.0081)	(0.011)
MZIGP$_3$	C_1	ZIGP	147.88	0.0006	0.230	146.69	0.0006	0.229	233.79	**0.0004**	**0.364**
			(172.11)	(0.0019)	(0.268)	(169.48)	(0.0017)	(0.265)	(120.58)	(0.0014)	(0.188)
		ZIP	569.39	0.0085	0.879	564.47	0.0080	0.874	554.49	0.0066	0.858
			(20.45)	(0.0051)	(0.024)	(24.59)	(0.0053)	(0.030)	(25.72)	(0.0045)	(0.035)
		GP	0.00	0.0000	0.000	0.00	0.0000	0.000	29.76	0.0000	0.046
			(0.00)	(0.0000)	(0.000)	(0.00)	(0.0000)	(0.000)	(12.90)	(0.0000)	(0.020)
		P	654.25	0.0465	0.972	654.25	0.0465	0.972	634.72	0.0325	0.956
			(17.96)	(0.0169)	(0.011)	(17.96)	(0.0169)	(0.011)	(19.84)	(0.0127)	(0.016)
	C_2	ZIGP	24.50	0.0001	0.038	23.71	0.0002	0.038	147.28	0.0001	0.229
			(75.15)	(0.0008)	(0.117)	(73.97)	(0.0009)	(0.115)	(63.21)	(0.0007)	(0.098)
		ZIP	586.98	0.0121	0.903	586.27	0.0121	0.902	575.04	0.0096	0.887
			(23.58)	(0.0064)	(0.026)	(24.64)	(0.0065)	(0.028)	(23.99)	(0.0052)	(0.030)
		GP	0.00	0.0000	0.000	0.00	0.0000	0.000	22.64	0.0000	0.035
			(0.00)	(0.0000)	(0.000)	(0.00)	(0.0000)	(0.000)	(10.47)	(0.0000)	(0.016)
		P	657.72	0.0494	0.974	657.72	0.0494	0.974	636.37	0.0335	0.958
			(18.55)	(0.0176)	(0.011)	(18.55)	(0.0176)	(0.011)	(19.23)	(0.0125)	(0.015)
MZIGP$_4$	C_1	ZIGP	233.79	0.0132	0.353	233.71	0.0132	0.353	292.67	**0.0133**	**0.444**
			(212.14)	(0.0176)	(0.317)	(212.22)	(0.0177)	(0.317)	(156.32)	(0.0160)	(0.231)
		ZIP	631.75	0.0948	0.886	488.20	0.0342	0.732	485.26	0.0330	0.727
			(36.05)	(0.0235)	(0.032)	(42.71)	(0.0148)	(0.055)	(38.03)	(0.0131)	(0.049)
		GP	0.00	0.0000	0.000	0.00	0.0000	0.000	49.38	0.0004	0.076
			(0.00)	(0.0000)	(0.000)	(0.00)	(0.0000)	(0.000)	(17.78)	(0.0031)	(0.027)
		P	811.58	0.2148	0.988	811.58	0.2148	0.988	781.80	0.1946	0.976
			(17.13)	(0.0166)	(0.007)	(17.13)	(0.0166)	(0.007)	(15.02)	(0.0152)	(0.007)
	C_2	ZIGP	70.91	0.0043	0.107	70.80	0.0043	0.107	166.94	0.0048	0.256
			(147.99)	(0.0137)	(0.219)	(148.02)	(0.0138)	(0.219)	(117.35)	(0.0125)	(0.173)
		ZIP	664.30	0.1141	0.912	557.52	0.0607	0.808	540.92	0.0527	0.793
			(17.50)	(0.0140)	(0.013)	(68.39)	(0.0306)	(0.073)	(58.52)	(0.0238)	(0.066)
		GP	0.00	0.0000	0.000	0.00	0.0000	0.000	43.78	0.0004	0.068
			(0.00)	(0.0000)	(0.000)	(0.00)	(0.0000)	(0.000)	(16.71)	(0.0032)	(0.025)
		P	811.62	0.2148	0.988	811.62	0.2148	0.988	781.80	0.1946	0.976
			(17.06)	(0.0165)	(0.007)	(17.06)	(0.0165)	(0.007)	(15.02)	(0.0152)	(0.007)

Table 14: Numerical Comparison when f_1 is shifted Geometric($p = 0.08$), $\pi_0 = 0.85$ and $C = 1$. The support of f_1 may contain values in $[0, C]$. The degree to which the f_0 is mixed with f_1 is described for values from $[0, K]$. MZIGP$_5$($\eta = 0.40, \lambda = 1, \theta = 0.15$) represents the moderately mixed case, MZIGP$_6$($\eta = 0.40, \lambda = 3, \theta = 0.15$) represents the heavily mixed case while MZIP$_3$($\eta = 0.40, \lambda = 1.5$) represents the well-separated case. The number in $(\cdot)$ represents the standard error.

			Two-Stage Procedure			One-Stage Procedure			Storey's FDR		
True f_0	Choice of C	Model for f_0	R	$\widehat{FDR}$	$\widehat{TPR}$	R	$\widehat{FDR}$	$\widehat{TPR}$	R	$\widehat{FDR}$	$\widehat{TPR}$
MZIP$_3$	C_1	ZIGP	119.14	**0.0189**	**0.777**	96.75	0.0005	0.741	97.50	0.0006	0.648
			(10.80)	(0.0149)	(0.037)	(10.71)	(0.0023)	(0.042)	(10.11)	(0.0024)	(0.048)
		ZIP	108.29	0.0039	0.717	104.11	0.0025	0.717	102.88	0.0020	0.683
			(9.98)	(0.0065)	(0.038)	(10.97)	(0.0058)	(0.038)	(10.31)	(0.0045)	(0.047)
		GP	133.26	0.0606	0.831	100.24	0.0009	0.744	99.43	0.0007	0.660
			(12.79)	(0.0300)	(0.037)	(10.26)	(0.0031)	(0.044)	(9.63)	(0.0026)	(0.039)
		P	194.49	0.2682	0.923	194.49	0.2682	0.923	141.34	0.0895	0.851
			(37.58)	(0.1040)	(0.033)	(37.58)	(0.1040)	(0.033)	(16.83)	(0.0513)	(0.036)
	C_2	ZIGP	117.80	0.0167	0.770	92.87	0.0003	0.702	92.68	0.0002	0.616
			(10.86)	(0.0132)	(0.041)	(10.15)	(0.0017)	(0.046)	(9.70)	(0.0015)	(0.046)
		ZIP	108.29	0.0039	0.717	103.03	0.0023	0.717	102.05	0.0018	0.677
			(9.98)	(0.0065)	(0.038)	(11.01)	(0.0057)	(0.038)	(10.43)	(0.0044)	(0.048)
		GP	135.90	0.0688	0.841	99.47	0.0007	0.716	99.40	0.0007	0.660
			(10.94)	(0.0222)	(0.029)	(9.77)	(0.0026)	(0.043)	(9.63)	(0.0026)	(0.039)
		P	194.49	0.2682	0.923	194.49	0.2682	0.923	141.34	0.0895	0.851
			(37.58)	(0.1040)	(0.033)	(37.58)	(0.1040)	(0.033)	(16.83)	(0.0513)	(0.036)
MZIGP$_5$	C_1	ZIGP	126.92	0.0568	0.795	98.11	0.0051	0.724	98.05	0.0048	0.649
			(13.24)	(0.0314)	(0.050)	(9.97)	(0.0077)	(0.042)	(9.54)	(0.0069)	(0.039)
		ZIP	113.85	0.0246	0.737	113.78	0.0245	0.737	109.75	0.0175	0.717
			(12.24)	(0.0186)	(0.049)	(12.31)	(0.0187)	(0.049)	(11.01)	(0.0147)	(0.047)
		GP	136.31	0.0840	0.829	104.85	0.0110	0.739	102.22	0.0085	0.674
			(13.07)	(0.0346)	(0.034)	(10.94)	(0.0111)	(0.042)	(10.68)	(0.0099)	(0.050)
		P	188.89	0.2546	0.922	188.89	0.2546	0.922	157.63	0.1574	0.880
			(29.58)	(0.0843)	(0.036)	(29.58)	(0.0843)	(0.036)	(15.88)	(0.0465)	(0.035)
	C_2	ZIGP	122.06	**0.0433**	**0.776**	96.81	0.0044	0.724	97.33	0.0046	0.644
			(12.07)	(0.0266)	(0.046)	(10.05)	(0.0067)	(0.044)	(9.74)	(0.0068)	(0.043)
		ZIP	113.84	0.0247	0.737	113.62	0.0244	0.737	109.57	0.0173	0.716
			(12.24)	(0.0188)	(0.049)	(12.47)	(0.0189)	(0.049)	(11.18)	(0.0148)	(0.048)
		GP	161.04	0.1691	0.888	103.79	0.0100	0.710	100.44	0.0069	0.663
			(13.75)	(0.0389)	(0.029)	(11.04)	(0.0109)	(0.041)	(10.46)	(0.0090)	(0.047)
		P	188.89	0.2546	0.922	188.89	0.2546	0.922	157.63	0.1574	0.880
			(29.58)	(0.0843)	(0.036)	(29.58)	(0.0843)	(0.036)	(15.88)	(0.0465)	(0.035)
MZIGP$_6$	C_1	ZIGP	101.52	0.0742	0.623	86.03	**0.0304**	**0.623**	87.11	0.0316	0.561
			(16.63)	(0.0387)	(0.090)	(14.76)	(0.0238)	(0.090)	(13.53)	(0.0212)	(0.078)
		ZIP	167.71	0.2957	0.785	106.83	0.0878	0.724	101.44	0.0688	0.626
			(12.06)	(0.0361)	(0.034)	(19.52)	(0.0629)	(0.054)	(13.39)	(0.0400)	(0.054)
		GP	262.12	0.4938	0.872	156.33	0.2576	0.789	133.94	0.1846	0.725
			(33.74)	(0.0581)	(0.036)	(20.06)	(0.0630)	(0.036)	(13.07)	(0.0413)	(0.042)
		P	497.31	0.7032	0.975	497.31	0.7032	0.975	380.75	0.6292	0.933
			(48.04)	(0.0297)	(0.020)	(48.04)	(0.0297)	(0.020)	(37.42)	(0.0340)	(0.023)
	C_2	ZIGP	91.68	0.0544	0.573	78.63	0.0217	0.573	80.53	0.0236	0.522
			(25.93)	(0.0369)	(0.152)	(22.20)	(0.0206)	(0.152)	(20.52)	(0.0199)	(0.126)
		ZIP	167.71	0.2957	0.785	103.44	0.0805	0.703	98.78	0.0637	0.612
			(12.06)	(0.0361)	(0.034)	(22.26)	(0.0671)	(0.079)	(15.41)	(0.0434)	(0.068)
		GP	215.72	0.4144	0.838	138.10	0.1993	0.784	118.89	0.1293	0.687
			(16.49)	(0.0390)	(0.031)	(13.91)	(0.0442)	(0.033)	(12.85)	(0.0421)	(0.044)
		P	497.31	0.7032	0.975	497.31	0.7032	0.975	380.75	0.6292	0.933
			(48.04)	(0.0297)	(0.020)	(48.04)	(0.0297)	(0.020)	(37.42)	(0.0340)	(0.023)

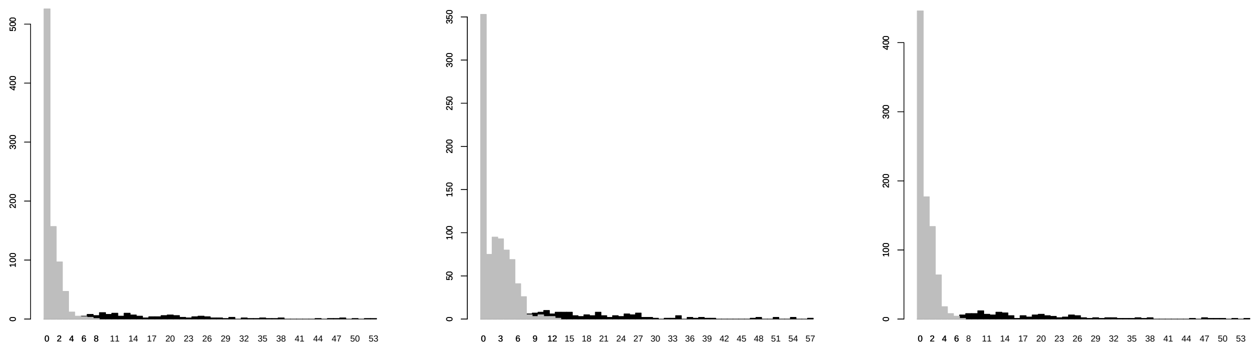
Table 15: Numerical Comparison when f_1 is shifted Binomial($n = 250, p = 0.20$), $\pi_0 = 0.80$ and $C = 1$. The support of f_1 may contain values in $[0, C]$. The degree to which the f_0 is mixed with f_1 is described for values from $[0, K]$. MZIGP$_7$($\eta = 0.40, \lambda = 1, \theta = 0.2$) represents the moderately mixed case, MZIGP$_8$($\eta = 0.40, \lambda = 3, \theta = 0.2$) represents the heavily mixed case while MZIP$_4$($\eta = 0.40, \lambda = 1.5$) represents the well-separated case. The number in $(\cdot)$ represents the standard error.

			Two-Stage Procedure			One-Stage Procedure			Storey's FDR		
True f_0	Choice of C	Model for f_0	R	$\widehat{FDR}$	$\widehat{TPR}$	R	$\widehat{FDR}$	$\widehat{TPR}$	R	$\widehat{FDR}$	$\widehat{TPR}$
MZIP$_4$	C_1	ZIGP	178.59	**0.0128**	**0.907**	160.07	0.0004	0.856	162.04	0.0007	0.833
			(13.15)	(0.0123)	(0.024)	(12.17)	(0.0015)	(0.028)	(12.54)	(0.0021)	(0.032)
		ZIP	169.56	0.0024	0.870	166.66	0.0018	0.870	169.54	0.0027	0.870
			(12.19)	(0.0040)	(0.024)	(13.19)	(0.0037)	(0.024)	(12.53)	(0.0046)	(0.027)
		GP	187.15	0.0349	0.929	163.04	0.0009	0.870	168.38	0.0020	0.865
			(14.46)	(0.0237)	(0.023)	(12.91)	(0.0025)	(0.029)	(12.53)	(0.0035)	(0.029)
		P	235.79	0.1907	0.970	235.79	0.1907	0.970	208.33	0.1061	0.955
			(32.49)	(0.0843)	(0.015)	(32.49)	(0.0843)	(0.015)	(18.94)	(0.0526)	(0.019)
	C_2	ZIGP	177.67	0.0109	0.904	155.66	0.0003	0.821	160.24	0.0004	0.824
			(12.61)	(0.0082)	(0.025)	(13.20)	(0.0013)	(0.034)	(12.17)	(0.0016)	(0.030)
		ZIP	169.56	0.0024	0.870	165.63	0.0016	0.869	168.48	0.0025	0.865
			(12.19)	(0.0040)	(0.024)	(13.32)	(0.0035)	(0.024)	(12.92)	(0.0045)	(0.031)
		GP	191.97	0.0483	0.940	161.48	0.0006	0.845	166.85	0.0017	0.857
			(13.51)	(0.0207)	(0.018)	(12.36)	(0.0021)	(0.028)	(12.89)	(0.0033)	(0.032)
		P	235.79	0.1907	0.970	235.79	0.1907	0.970	208.33	0.1061	0.955
			(32.49)	(0.0843)	(0.015)	(32.49)	(0.0843)	(0.015)	(18.94)	(0.0526)	(0.019)
MZIGP$_7$	C_1	ZIGP	189.21	0.0563	0.919	161.38	0.0080	0.854	165.63	0.0120	0.843
			(15.01)	(0.0280)	(0.027)	(12.63)	(0.0077)	(0.028)	(13.22)	(0.0096)	(0.035)
		ZIP	178.33	0.0304	0.891	178.30	0.0304	0.891	178.56	0.0306	0.892
			(13.88)	(0.0183)	(0.030)	(13.91)	(0.0183)	(0.030)	(13.32)	(0.0161)	(0.029)
		GP	197.88	0.0804	0.937	169.43	0.0153	0.873	170.51	0.0162	0.864
			(14.98)	(0.0290)	(0.019)	(12.71)	(0.0102)	(0.025)	(12.23)	(0.0097)	(0.025)
		P	250.55	0.2380	0.976	250.55	0.2380	0.976	221.29	0.1558	0.963
			(27.26)	(0.0691)	(0.015)	(27.26)	(0.0691)	(0.015)	(13.62)	(0.0269)	(0.014)
	C_2	ZIGP	184.37	**0.0444**	**0.907**	159.29	0.0069	0.844	162.55	0.0095	0.830
			(14.67)	(0.0256)	(0.031)	(12.72)	(0.0070)	(0.035)	(13.09)	(0.0088)	(0.037)
		ZIP	178.35	0.0304	0.891	178.25	0.0303	0.891	178.56	0.0305	0.892
			(13.89)	(0.0181)	(0.030)	(13.97)	(0.0182)	(0.030)	(13.31)	(0.0159)	(0.029)
		GP	221.26	0.1557	0.962	168.61	0.0146	0.862	170.44	0.0162	0.864
			(13.80)	(0.0281)	(0.014)	(13.10)	(0.0103)	(0.025)	(12.33)	(0.0097)	(0.025)
		P	250.55	0.2380	0.976	250.55	0.2380	0.976	221.29	0.1558	0.963
			(27.26)	(0.0691)	(0.015)	(27.26)	(0.0691)	(0.015)	(13.62)	(0.0269)	(0.014)
MZIGP$_8$	C_1	ZIGP	143.50	**0.0500**	**0.693**	129.86	**0.0302**	**0.693**	140.47	0.0406	0.686
			(36.08)	(0.0351)	(0.159)	(34.28)	(0.0249)	(0.159)	(28.91)	(0.0272)	(0.126)
		ZIP	237.23	0.2603	0.897	178.79	0.1103	0.846	177.92	0.1074	0.808
			(13.94)	(0.0291)	(0.022)	(26.68)	(0.0625)	(0.047)	(20.66)	(0.0473)	(0.045)
		GP	338.63	0.4496	0.950	234.15	0.2520	0.901	208.63	0.1871	0.865
			(22.89)	(0.0385)	(0.018)	(17.34)	(0.0387)	(0.023)	(17.32)	(0.0386)	(0.028)
		P	535.43	0.6358	0.991	535.43	0.6358	0.991	468.66	0.5871	0.980
			(43.12)	(0.0321)	(0.008)	(43.12)	(0.0321)	(0.008)	(47.36)	(0.0440)	(0.012)
	C_2	ZIGP	84.53	0.0279	0.410	75.23	0.0219	0.410	93.67	0.0225	0.462
			(73.44)	(0.0360)	(0.351)	(68.05)	(0.0243)	(0.351)	(58.24)	(0.0277)	(0.279)
		ZIP	237.23	0.2603	0.897	177.29	0.1088	0.836	176.54	0.1052	0.803
			(13.94)	(0.0291)	(0.022)	(29.34)	(0.0662)	(0.065)	(22.69)	(0.0507)	(0.053)
		GP	289.10	0.3684	0.930	213.92	0.2000	0.895	204.36	0.1761	0.860
			(24.97)	(0.0429)	(0.021)	(21.10)	(0.0486)	(0.021)	(13.63)	(0.0292)	(0.025)
		P	535.43	0.6358	0.991	535.43	0.6358	0.991	468.66	0.5871	0.980
			(43.12)	(0.0321)	(0.008)	(43.12)	(0.0321)	(0.008)	(47.36)	(0.0440)	(0.012)

Table 16: Comparison of Parameter Estimates for Protein Domain Data

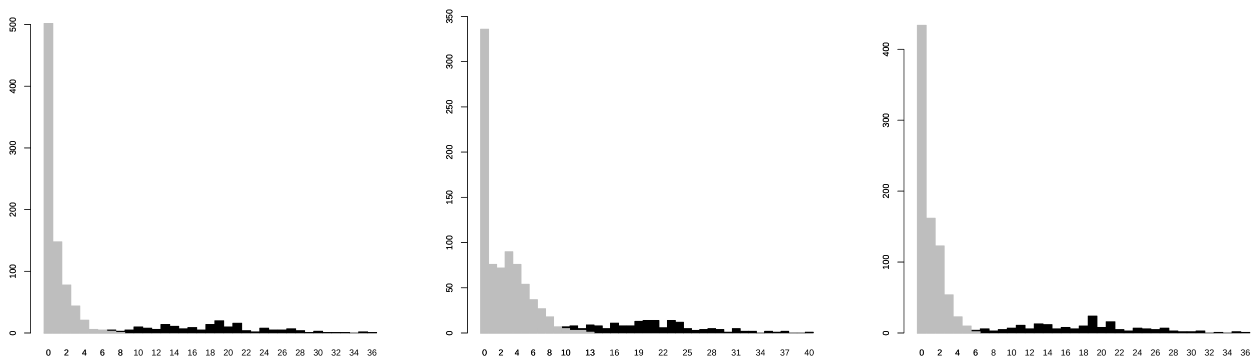
Data	f_0	$\widehat{\eta}_{\hat{C}_1}$	$\widehat{\lambda}_{\hat{C}_1}$	$\widehat{\theta}_{\hat{C}_1}$	$\hat{\pi}_{0,\hat{C}_1}$	$\hat{C}_1$	$D(\hat{C}_1)$	$\widehat{\eta}_{\hat{C}_2}$	$\widehat{\lambda}_{\hat{C}_2}$	$\widehat{\theta}_{\hat{C}_2}$	$\hat{\pi}_{0,\hat{C}_2}$	$\hat{C}_2$	$D(\hat{C}_2)$
cd00031	ZIGP	0.3245	1.9168	0.1416	0.4576	6	7	0.2252	2.1452	0.5741	0.6141	4	36
	ZIP	0.2289	1.0949	NA	0.3985	3	6	0.2760	1.3856	NA	0.4244	2	6
	GP	NA	1.5557	0.6602	0.5537	11	36	NA	2.0943	0.6668	0.8321	3	36
	P	NA	0.8082	NA	0.3944	3	6	NA	0.7994	NA	0.3929	2	6
cd00054	ZIGP	0.1421	1.2959	0.0000	0.5009	5	6	0.1421	1.2959	0.0000	0.5009	5	6
	ZIP	0.0008	0.8689	NA	0.5001	5	6	0.0008	0.8689	NA	0.5001	5	6
	GP	NA	1.3589	0.1970	0.5348	8	9	NA	1.5687	0.7286	0.7164	6	52
	P	NA	0.8682	NA	0.5001	5	6	NA	0.8682	NA	0.5001	5	6
cd00180	ZIGP	0.5966	1.8193	0.0948	0.6954	6	7	0.4573	2.0295	0.7013	0.8710	5	63
	ZIP	0.5507	1.1095	NA	0.6588	3	7	0.5331	0.9701	NA	0.6484	2	7
	GP	NA	1.3096	0.8292	0.8379	17	63	NA	1.5161	0.8286	0.9925	7	63
	P	NA	0.4579	NA	0.6518	3	7	NA	0.4579	NA	0.6518	3	7
cd00204	ZIGP	0.5060	1.2628	0.0002	0.6853	5	6	0.5062	1.2645	0.0001	0.6854	4	5
	ZIP	0.1287	0.5081	NA	0.6784	3	7	0.1403	0.5188	NA	0.6792	2	7
	GP	NA	1.1921	0.7271	0.7799	12	34	NA	1.2046	0.7368	0.7906	10	34
	P	NA	0.4409	NA	0.6780	3	7	NA	0.4358	NA	0.6767	2	7
cd00882	ZIGP	0.6736	1.3970	0.0000	0.8003	4	5	0.6734	1.4406	0.0000	0.8041	3	4
	ZIP	0.5201	0.6786	NA	0.7907	3	7	0.5136	0.6616	NA	0.7896	2	7
	GP	NA	1.1358	0.0000	0.8016	4	5	NA	1.1358	0.0000	0.8016	4	5
	P	NA	0.2174	NA	0.7503	1	7	NA	0.2174	NA	0.7503	1	7
pfam00001	ZIGP	0.0526	2.4020	0.3839	0.4031	13	18	0.0073	5.4512	0.7767	1.0000	2	233
	ZIP	0.0000	44.9641	NA	1.0000	18	45	0.0000	44.9641	NA	1.0000	18	45
	GP	NA	2.2464	0.4164	0.4048	13	21	NA	4.8035	0.7937	1.0000	2	233
	P	NA	46.2118	NA	1.0000	18	47	NA	46.2118	NA	1.0000	18	47

Web Figures:



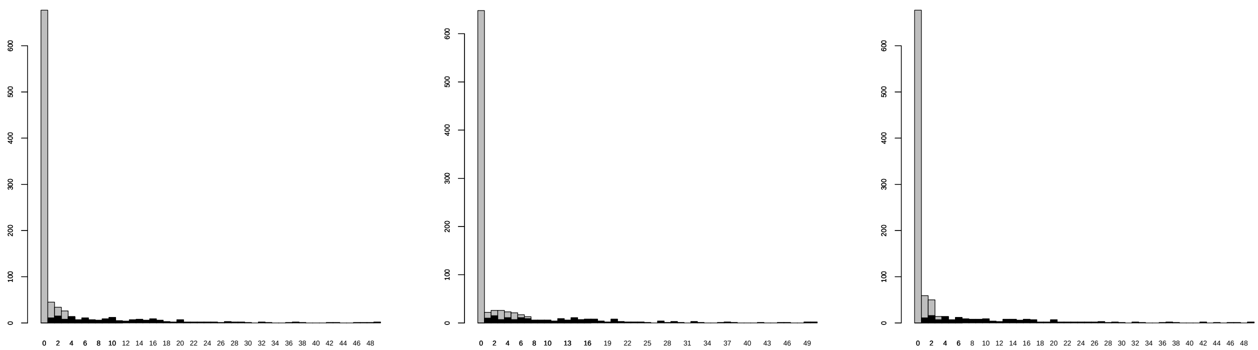
(a) ZIGP_5 (b) ZIGP_6 (c) ZIP_3

Figure 1: Histogram when f_1 is shifted Geometric($p = 0.08$), $\pi_0 = 0.85$ and $C = 5$. (1a) $\text{ZIGP}_5(\eta = 0.40, \lambda = 1, \theta = 0.15)$ represents the moderately mixed case, (1b) $\text{ZIGP}_6(\eta = 0.40, \lambda = 3, \theta = 0.15)$ represents the heavily mixed case while (1c) $\text{ZIP}_3(\eta = 0.40, \lambda = 1.5)$ represents the well-separated case.



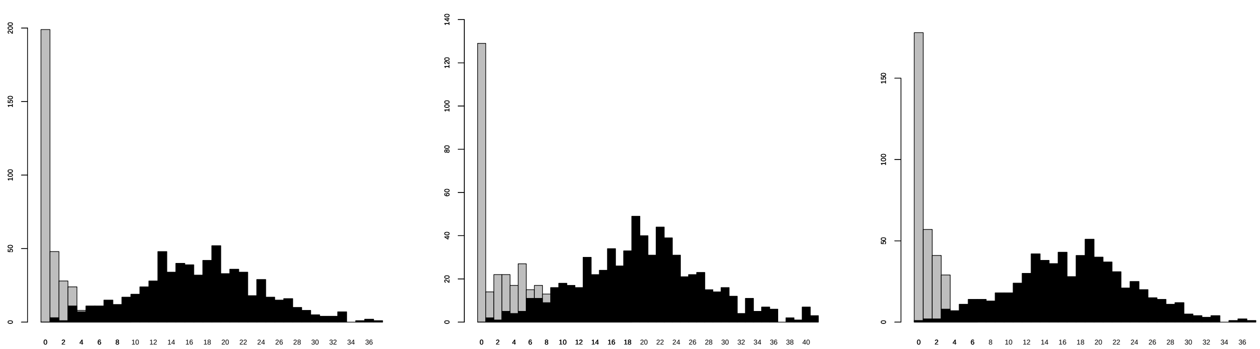
(a) ZIGP_7 (b) ZIGP_8 (c) ZIP_4

Figure 2: Histogram when f_1 is shifted Binomial($n = 250, p = 0.20$), $\pi_0 = 0.80$ and $C = 5$. (2a) $\text{ZIGP}_7(\eta = 0.40, \lambda = 1, \theta = 0.20)$ represents the moderately mixed case, (2b) $\text{ZIGP}_8(\eta = 0.40, \lambda = 3, \theta = 0.20)$ represents the heavily mixed case while (2c) $\text{ZIP}_4(\eta = 0.40, \lambda = 1.5)$ represents the well-separated case.



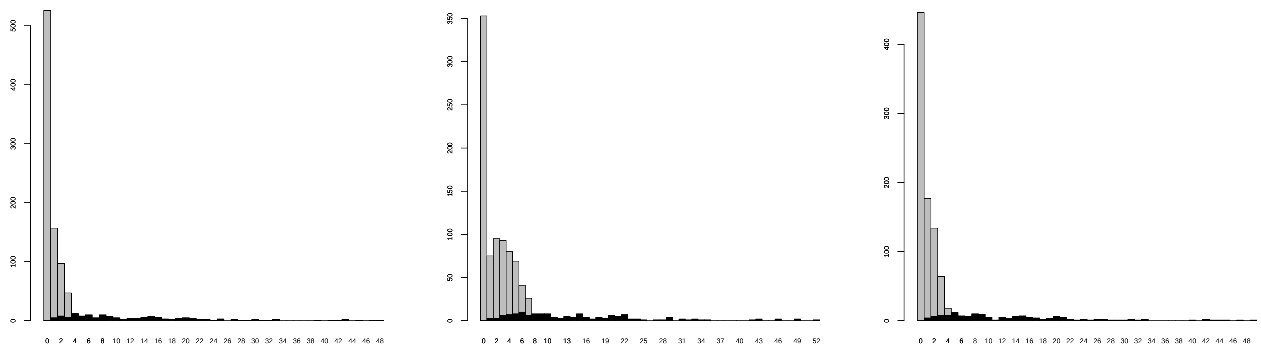
(a) MZIGP_1 (b) MZIGP_2 (c) MZIP_1

Figure 3: Histogram when f_1 is shifted Geometric($p = 0.08$), $\pi_0 = 0.80$ and $C = 5$. The support of f_1 may contain values in $[0, C]$. The degree to which the f_0 is mixed with f_1 is described for values from $[0, K]$. $\text{MZIGP}_1(\eta = 0.80, \lambda = 1.5, \theta = 0.3)$ represents the moderately mixed case, $\text{MZIGP}_2(\eta = 0.80, \lambda = 3, \theta = 0.3)$ represents the heavily mixed case while $\text{MZIP}_1(\eta = 0.80, \lambda = 1.5)$ represents the well-separated case.



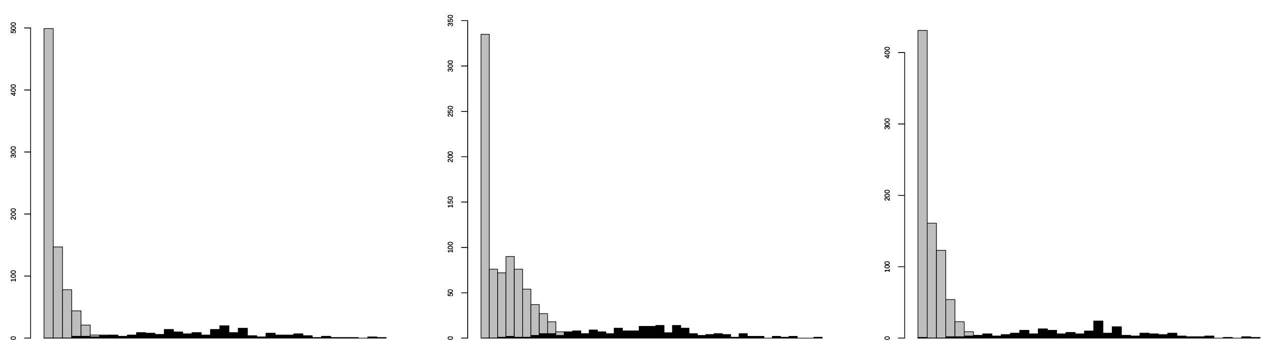
(a) MZIGP_3 (b) MZIGP_4 (c) MZIP_2

Figure 4: Histogram when f_1 is shifted Binomial($n = 250, p = 0.20$), $\pi_0 = 0.35$ and $C = 5$. The support of f_1 may contain values in $[0, C]$. The degree to which the f_0 is mixed with f_1 is described for values from $[0, K]$. $\text{MZIGP}_3(\eta = 0.40, \lambda = 1, \theta = 0.3)$ represents the moderately mixed case, $\text{MZIGP}_4(\eta = 0.40, \lambda = 4, \theta = 0.3)$ represents the heavily mixed case while $\text{MZIP}_2(\eta = 0.40, \lambda = 1.5)$ represents the well-separated case.



(a) MZIGP_5 (b) MZIGP_6 (c) MZIP_3

Figure 5: Histogram when f_1 is shifted Geometric($p = 0.08$), $\pi_0 = 0.85$ and $C = 5$. The support of f_1 may contain values in $[0, C]$. The degree to which the f_0 is mixed with f_1 is described for values from $[0, K]$. $\text{MZIGP}_5(\eta = 0.40, \lambda = 1, \theta = 0.15)$ represents the moderately mixed case, $\text{MZIGP}_6(\eta = 0.40, \lambda = 3, \theta = 0.15)$ represents the heavily mixed case while $\text{MZIP}_3(\eta = 0.40, \lambda = 1.5)$ represents the well-separated case.



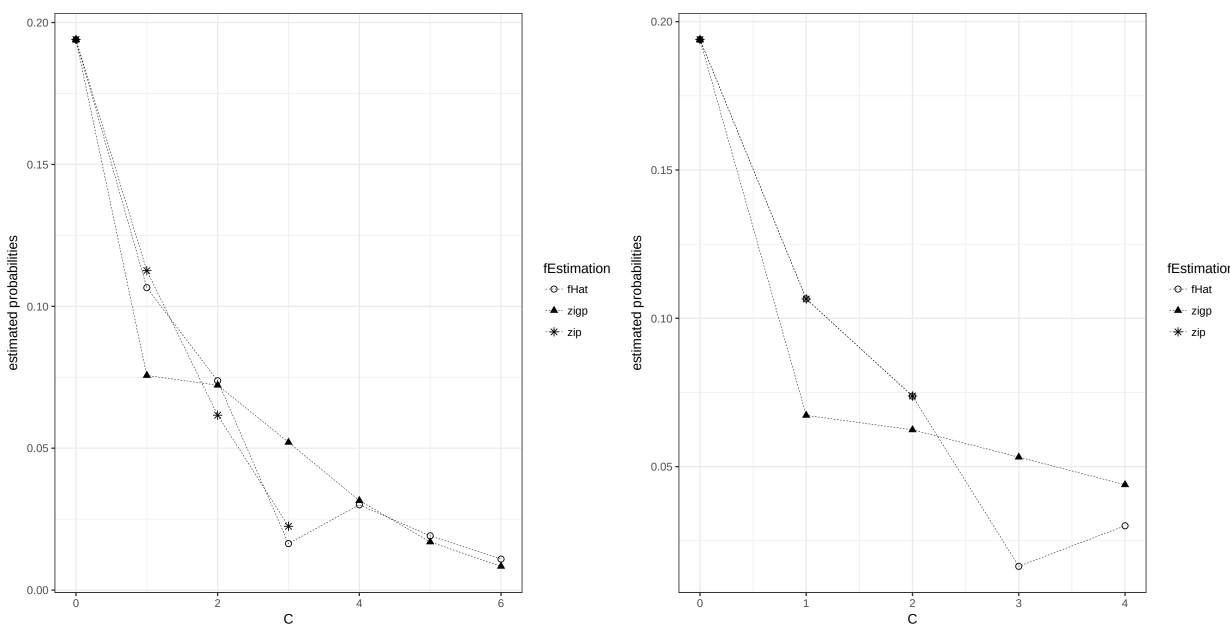
(a) MZIGP_7 (b) MZIGP_8 (c) MZIP_4

Figure 6: Histogram when f_1 is shifted Binomial($n = 250, p = 0.20$), $\pi_0 = 0.80$ and $C = 5$. The support of f_1 may contain values in $[0, C]$. The degree to which the f_0 is mixed with f_1 is described for values from $[0, K]$. $\text{MZIGP}_7(\eta = 0.40, \lambda = 1, \theta = 0.2)$ represents the moderately mixed case, $\text{MZIGP}_8(\eta = 0.40, \lambda = 3, \theta = 0.2)$ represents the heavily mixed case while $\text{MZIP}_4(\eta = 0.40, \lambda = 1.5)$ represents the well-separated case.

Goodness-of-Fit Assessment:

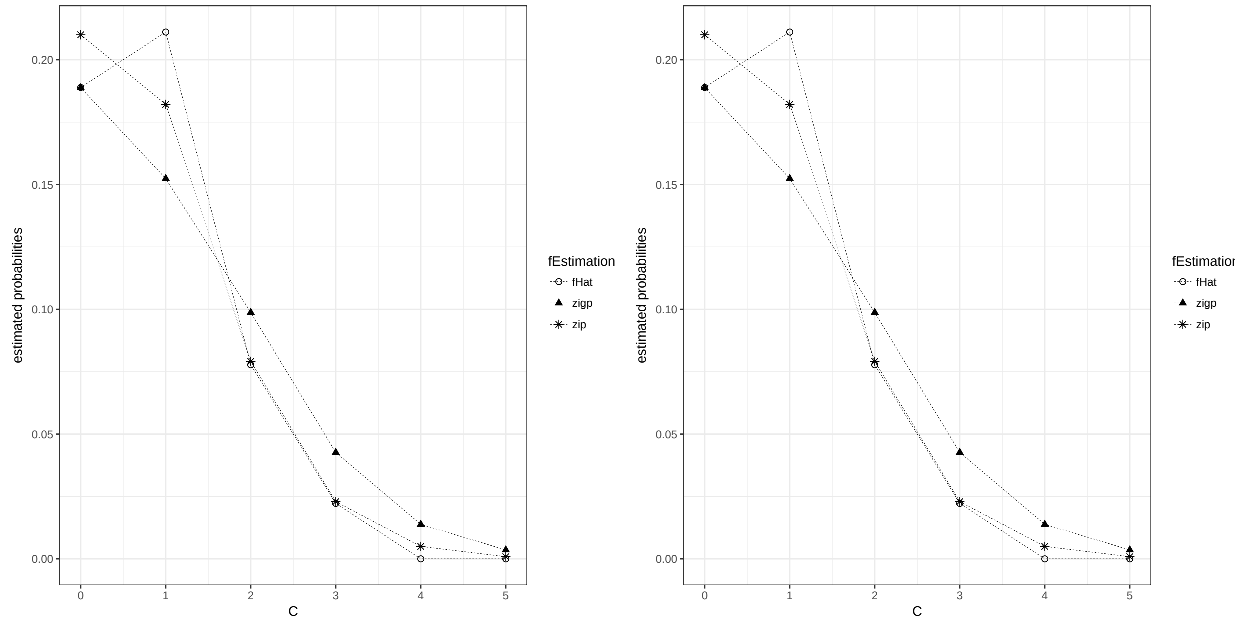
Corresponding to the six protein domains discussed in Section 4.2, Web Figures 7 to 12 represent the graphical comparison of f estimation. For each protein domain, the comparison is based on the choice of the cutoff C and the method of estimation of f for $j \leq C$. Using the assumption on f_0 in (2), $f(j) = \pi_0 f_0(j)$ for $j \leq C$. The three line plots correspond to $\hat{f}(j) = n_j/N$ and $\hat{f}(j) = \hat{\pi}_0 \hat{f}_0(j)$ where f_0 is ZIGP or ZIP.

Since ZIGP is a more flexible model than ZIP, ZIGP tends to fit the data for larger values of $\hat{C}$ while ZIP fits the null distribution for smaller values of $\hat{C}$. In some cases, ZIP seems to show more accurate fitting, however ZIP tends to choose small values of $\hat{C}$ and ZIGP tends to choose large values of $\hat{C}$, so it is not straightforward to compare goodness of fit of ZIGP and ZIP for different values of $\hat{C}$s.



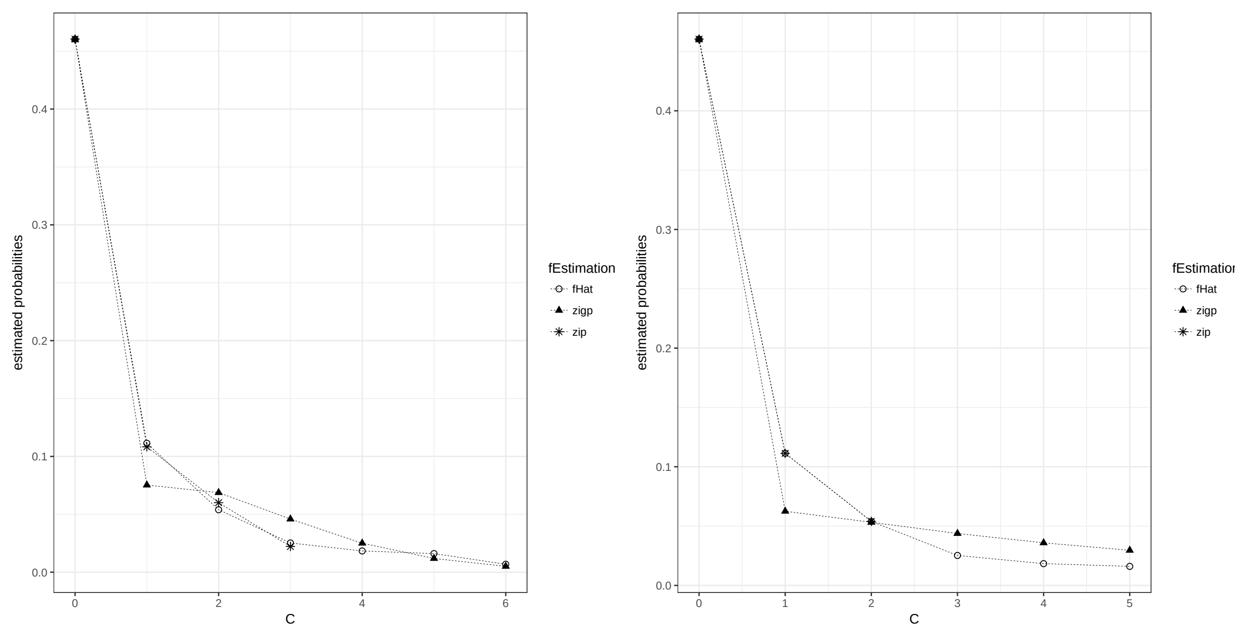
(a) (b)

Figure 7: Graphical comparison of f estimation for cd00031. (7a) C_1 is used for choice of cut-off. (7b) C_2 is used for choice of cut-off.



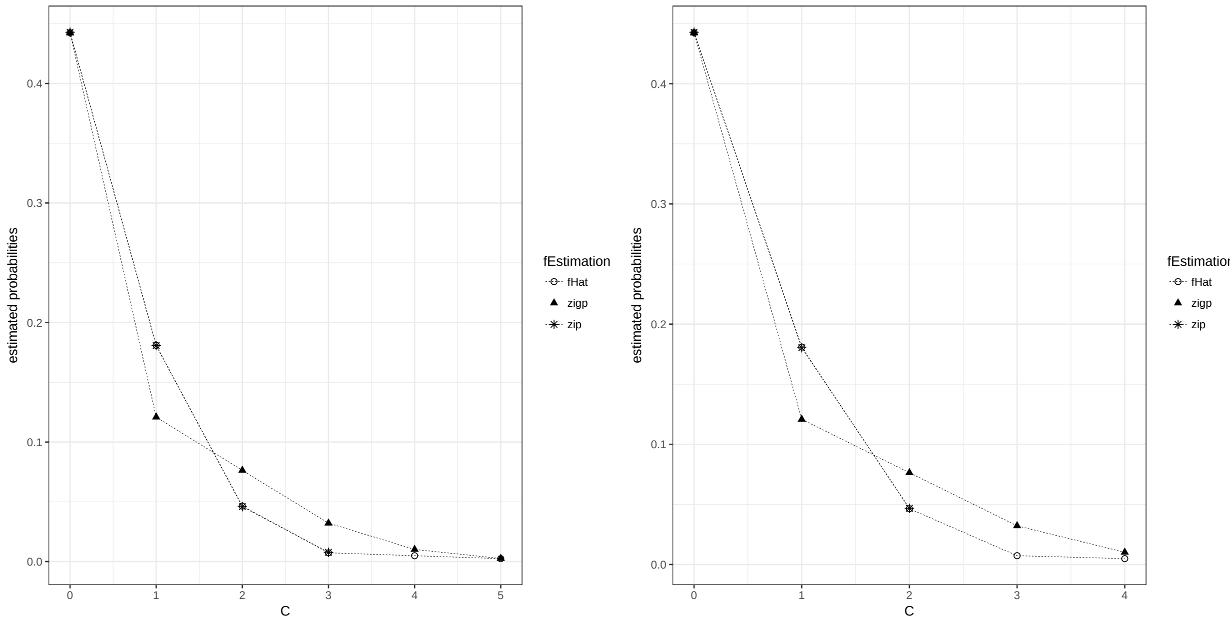
(a) (b)

Figure 8: Graphical comparison of f estimation for cd00054. (8a) C_1 is used for choice of cut-off. (8b) C_2 is used for choice of cut-off.



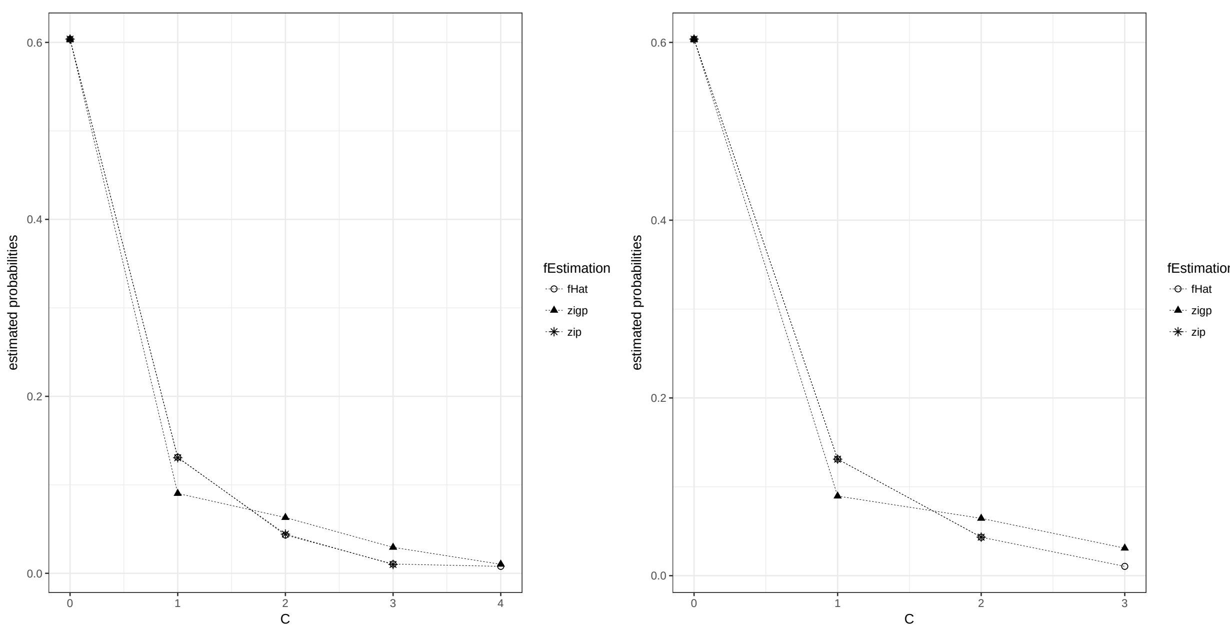
(a) (b)

Figure 9: Graphical comparison of f estimation for cd00180. (9a) C_1 is used for choice of cut-off. (9b) C_2 is used for choice of cut-off.



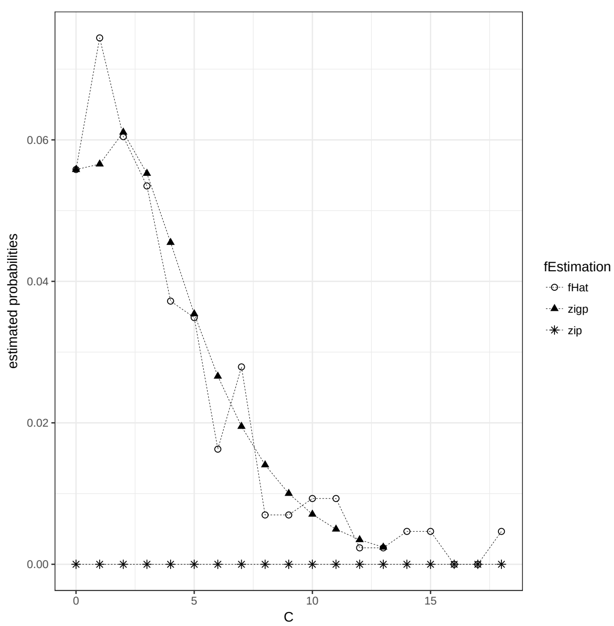
(a) (b)

Figure 10: Graphical comparison of f estimation for cd00204. (10a)C_1 is used for choice of cut-off. (10b) C_2 is used for choice of cut-off.

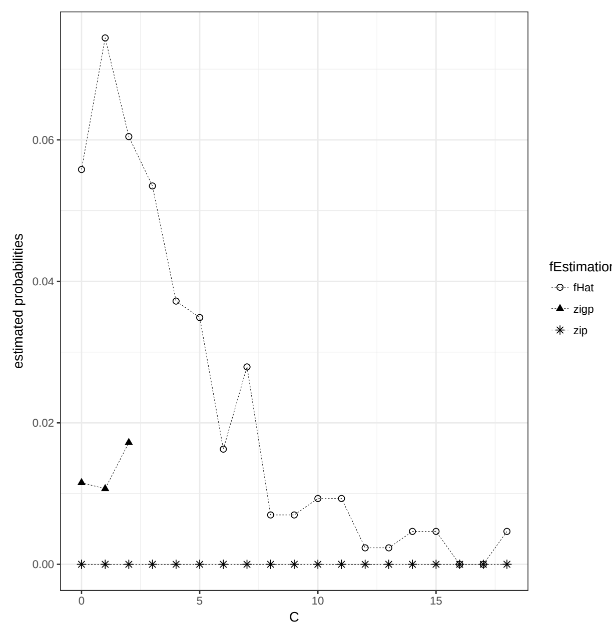


(a) (b)

Figure 11: Graphical comparison of f estimation for cd00882. (11a) C_1 is used for choice of cut-off. (11b) C_2 is used for choice of cut-off.



(a)



(b)

Figure 12: Graphical comparison of f estimation for pfam00001. (12a) C_1 is used for choice of cut-off. (12b) C_2 is used for choice of cut-off.