

# Appendix

**1. Brain parcellation** We consider a brain parcellation that is a more granular version of the AAL regional template; we utilize the existing boundaries but provide a further subdivision within each region of interest. A hierarchical clustering algorithm is applied to each region, where the spatial contiguity of the resulting clusters is enforced using the distance matrix. Connectivity information, averaged across the sample, is utilized to subtly promote functional and structural homogeneity in the grey and white matter, respectively, of each resulting cluster. Finally, we incorporate tissue-type information (grey/white matter) in the distance matrix to encourage balance in the tissue composition of clusters. Based on average linkage, the number of clusters is chosen manually to yield a resulting parcellation consisting of approximately 200 regions (here,  $\sum_{g=1}^G L_g = 282$  regions are selected), using the Hierarchical Clustering tools in MATLAB and Statistics Toolbox Release 2012b (The MathWorks, Inc., Natick, Massachusetts, United States). Independently for each AAL region, let  $d_{ij}$  represent the Manhattan distance, which is the sum of the absolute differences of MNI coordinates, for voxels  $i$  and  $j$ , and define the distance matrix as:

$$D_{ij} = \begin{cases} d_{ij} + (1 - FC_{ij}) & \text{if } i \in \mathcal{G}, j \in \mathcal{G} \\ d_{ij} + (1 - SC_{ij}) & \text{if } i \in \mathcal{W}, j \in \mathcal{W} \\ d_{ij} & \text{otherwise,} \end{cases}$$

where  $FC_{ij}$  represents the average of functional connectivity for voxel pairs in grey matter  $\mathcal{G}$  and  $SC_{ij}$  represents the average of structural connectivity for voxel pairs in white matter  $\mathcal{W}$ . Functional connectivity is quantified by estimating the Pearson correlation coefficient between the fMRI-derived BOLD signals of grey matter voxel pairs for each subject, and averaging across subjects. Structural connectivity is quantified by calculating the correlation between “connectivity profiles” and subsequently averaging across subjects. The “connectivity profile” for a white matter voxel is obtained by using a set of predefined target regions scattered throughout the brain and applying probabilistic tractography tools in FSL (Behrens, 2003) to each subject’s DTI data, yielding values that reflect the strength of structural connectivity between the voxel and each of the target regions. Thus,  $SC_{ij}$  is expected to reflect the average consistency in structural connections between white matter voxel pairs. For tissue-heterogeneous voxel pairs, there is no obvious way to quantify connectivity; by using an unpenalized distance of  $d_{ij}$  for these mismatched voxel pairs, the resulting clusters tend to span both grey and white matter.

## ***2. Posterior distributions***

We present the full conditional posterior distributions for Gibbs sampling. Here, we take the parameters with superscript  $xz$  as examples, parameters with other superscripts can be derived similarly. Let  $\bar{\xi}$  denote all the parameters except  $\xi$ . Denote  $\tilde{Z}_{ilg}(v) = Z_{ilg}(v) - \bar{Z}_{lg}(v)$ . Then the full conditional posterior distributions can be derived as follows:

$$(1) [c_{klg}(v) \mid \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{c}_{klg}(v) ]$$

$$\propto \text{N} \left( \mu_{c_{klg}(v)}, \sigma_{c_{klg}(v)}^2 \right)$$

$$\mu_{c_{klg}(v)} = \frac{\omega_{klg} \sum_{i=1}^n I(D_i = k) \tilde{Z}_{ilg}(v) \{ X_{ilg}(v) - \mathbf{W}_i \boldsymbol{\gamma}_{klg} - \beta_{klg}(v) - \alpha_{ilg} - \eta_{kg} \} + \delta_{lg} \zeta_{klg}}{\omega_{klg} \sum_{i=1}^n I(D_i = k) \tilde{Z}_{ilg}(v)^2 + \delta_{lg}}$$

$$\sigma_{c_{klg}(v)}^2 = \frac{\delta_{lg} \omega_{klg}}{\omega_{klg} \sum_{i=1}^n I(D_i = k) \tilde{Z}_{ilg}(v)^2 + \delta_{lg}}$$

$$(2) [\gamma_{klgq}(v) \mid \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{\gamma}_{klgq}(v) ]$$

$$\propto \text{N} \left( \mu_{\gamma_{klgq}(v)}, \sigma_{\gamma_{klgq}(v)}^2 \right)$$

$$\mu_{\gamma_{klgq}(v)} = \frac{s_{klg} \sum_{i=1}^n I(D_i = k) \left( W_{iq} [X_{ilg}(v) - c_{klg}(v) \tilde{Z}_{ilg}(v) - \beta_{klg}(v) - \alpha_{ilg} - \eta_{kg} - \sum_{q' \neq q} W_{iq'} \gamma_{klgq'}(v)] \right)}{s_{klg} \sum_{i=1}^n I(D_i = k) W_{iq}^2 + \delta_{lg}}$$

$$\sigma_{\gamma_{klgq}(v)}^2 = \frac{\delta_{lg} s_{klg}}{s_{klg} \sum_{i=1}^n I(D_i = k) W_{iq}^2 + \delta_{lg}}$$

$$(3) [\beta_{klg}(v) \mid \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{\beta}_{klg}(v) ]$$

$$\propto \text{N} \left( \mu_{\beta_{klg}(v)}, \sigma_{\beta_{klg}(v)}^2 \right)$$

$$\mu_{\beta_{klg}(v)} = \frac{\lambda_{klg} \sum_{i=1}^n I(D_i = k) [X_{ilg}(v) - c_{klg}(v) \tilde{Z}_{ilg}(v) - \mathbf{W}_i \boldsymbol{\gamma}_{klg} - \alpha_{ilg} - \eta_{kg}] + \beta_{klg} \delta_{lg}}{\lambda_{klg} \sum_{i=1}^n I(D_i = k) + \delta_{lg}}$$

$$\sigma_{\beta_{klg}(v)}^2 = \frac{\delta_{lg} \lambda_{klg}}{\lambda_{klg} \sum_{i=1}^n I(D_i = k) + \delta_{lg}}$$

$$(4) [\alpha_{ilg} \mid X_{ilg}(v), Y_{ilg}(v), Z_{ilg}(v), D_i, \mathbf{W}_i, \bar{\alpha}_{ilg} ]$$

$$\propto \text{N} \left\{ \frac{\tau_{lg} \sum_{v \in l} [X_{ilg}(v) - c_{klg}(v) \tilde{Z}_{ilg}(v) - \mathbf{W}_i \boldsymbol{\gamma}_{klg} - \beta_{klg}(v) - \eta_{kg}]}{\tau_{lg} V_{lg} + \delta_{lg}}, \frac{\delta_{lg} \tau_{lg}}{\tau_{lg} V_{lg} + \delta_{lg}} \right\}$$

$$(5) [\delta_{lg} \mid \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{\delta}_{lg} ]$$

$$\propto \text{InvG}(a, b)$$

$$a = a_\delta + \frac{n V_{lg}}{2}$$

$$b = b_\delta + \frac{\sum_{i=1}^n \sum_{v \in l} \left\{ X_{ilg}(v) - [c_{klg}(v) \tilde{Z}_{ilg}(v) + \mathbf{W}_i \boldsymbol{\gamma}_{klg} + \beta_{klg}(v) + \alpha_{ilg} + \eta_{kg}] \right\}^2}{2}$$

$$(6) [\zeta_{klg} \mid \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{\zeta}_{klg}] \\ \propto \text{N} \left( \frac{b_\zeta \sum_{v \in l} c_{klg}(v) + \omega_{klg} a_\zeta}{V_{lg} b_\zeta + \omega_{klg}}, \frac{\omega_{klg} b_\zeta}{V_{lg} b_\zeta + \omega_{klg}} \right)$$

$$(7) [\omega_{klg} \mid \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{\omega}_{klg}] \\ \propto \text{InvG} \left( a_\omega + \frac{V_{lg}}{2}, b_\omega + \frac{\sum_{v \in l} (c_{klg}(v) - \zeta_{klg})^2}{2} \right)$$

$$(8) [s_{klg} \mid \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{s}_{klg}] \\ \propto \text{InvG} \left( a_s + \frac{Q \times V_{lg}}{2}, b_s + \frac{\sum_{v \in l} \sum_{q=1}^Q \gamma_{klgq}^2(v)}{2} \right)$$

$$(9) [\beta_{klg} \mid \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{\beta}_{klg}] \\ \propto \text{N} \left( \frac{\phi_g \sum_{v \in l} \beta_{klg}(v) + \lambda_{klg} \rho_g \sum_{l' \in N_g(l)} \beta_{kl'g}}{V_{lg} \phi_g + n_{lg} \lambda_{klg}}, \frac{\lambda_{klg} \phi_g}{V_{lg} \phi_g + n_{lg} \lambda_{klg}} \right)$$

$$(10) [\lambda_{klg} \mid \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{\lambda}_{klg}] \\ \propto \text{InvG} \left( a_\lambda + \frac{V_{lg}}{2}, b_\lambda + \frac{\sum_{v \in l} (\beta_{klg}(v) - \beta_{klg})^2}{2} \right)$$

$$(11) [\tau_{lg} \mid \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{\tau}_{lg}] \\ \propto \text{InvG} \left( a_\tau + \frac{n}{2}, b_\tau + \frac{\sum_{i=1}^n \alpha_{ilg}^2}{2} \right)$$

$$(12) [\phi_g | \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{\phi}_g ]$$

$$\propto \text{InvG} \left( a_\phi + L_g, b_\phi + \frac{\sum_{k=0,1} \sum_{l \in g} n_{lg} \left( \beta_{klg} - \frac{\rho_g}{n_{lg}} \sum_{l' \in N_g(l)} \beta_{kl'g} \right)^2}{2} \right)$$

$$(13) [\rho_g | \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{\rho}_g ]$$

$$\propto \exp \left\{ -\frac{1}{2\phi_g} \left( \rho_g^2 \sum_{k=0,1} \sum_{l \in g} \frac{\left( \sum_{l' \in N_g(l)} \beta_{kl'g} \right)^2}{n_{lg}} - 2\rho_g \sum_{k=0,1} \sum_{l \in g} \beta_{klg} \sum_{l' \in N_g(l)} \beta_{kl'g} \right) \right\}$$

$$\propto L(\rho_g)$$

Suppose  $\rho_g$  follows a discrete uniform distribution. In our analysis,  $\rho_g$  can take 36 values  $\{m_1, \dots, m_n\}$ . Denote the posterior probability of  $\rho_g$  as  $\{p_1, \dots, p_n\}$ , then

$$p_j = \frac{L(\rho_g = m_j)}{L(\rho_g = m_1) + \dots + L(\rho_g = m_n)}.$$

(14) Let  $\beta_{kg} = \left( \beta_{k1g}(v_1), \dots, \beta_{k1g}(v_{V_{1g}}), \beta_{k2g}(v_1), \dots, \beta_{kL_gg}(v_{V_{L_gg}}) \right)^T$  denote all the voxels in region  $g$ ; define  $\mathbf{c}_{kg}$ ,  $\gamma_{kg}$ ,  $\mathbf{X}_{ig}$  and  $\mathbf{Z}_{ig}$  in the same way;  $V_g = \sum_{l=1}^{L_g} V_{lg}$  denote the number of voxels in region  $g$ ;  $\alpha_{ig} = (\alpha_{i1g}, \dots, \alpha_{iL_gg})^T$ ;  $\Delta_g = \text{diag}(\Delta_{1g}, \dots, \Delta_{L_gg})$ , where  $\Delta_{lg} = \mathbf{I}_{V_{lg}} \otimes \delta_{lg}$ ;  $\odot$  denote Hadamard product.

$$[\eta_k | \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{\eta}_k ]$$

$$\propto \exp \left\{ -\frac{1}{2} \eta_k^T \Sigma_k^{-1} \eta_k \right\}$$

$$\exp \left\{ -\frac{1}{2} \sum_{i=1}^N \sum_{g=1}^G \left\{ \sum_{k=0,1} \mathbf{1}_{V_g} \eta_{kg} I(D_i = k) - \right. \right.$$

$$\left. \underbrace{\left( \mathbf{X}_{ig} - \sum_{k=0,1} \left\{ \mathbf{c}_{kg} \odot \tilde{\mathbf{Z}}_{ig} + \mathbf{W}_i \gamma_{kg} + \beta_{kg} + \left( \mathbf{1}_{V_{1g}}^T, \dots, \mathbf{1}_{V_{L_gg}}^T \right)^T \otimes \alpha_{ig} \right\} I(D_i = k) \right)}_{=: \mathbf{t}_g} \right\}^T$$

$$\Delta_g^{-1} \left( \sum_{k=0,1} \mathbf{1}_{V_g} \eta_{kg} I(D_i = k) - \mathbf{t}_g \right) \left. \right\}$$

$$\begin{aligned}
& \propto \exp \left\{ -\frac{1}{2} \boldsymbol{\eta}_k^T \boldsymbol{\Sigma}_k^{-1} \boldsymbol{\eta}_k \right\} \\
& \exp \left\{ -\frac{1}{2} \underbrace{\sum_{i=1}^N \sum_{g=1}^G \left\{ \sum_{k=0,1} \mathbf{1}_{V_g} \eta_{kg} I(D_i = k) \right\}^T}_{I} \boldsymbol{\Delta}_g^{-1} \left\{ \sum_{k=0,1} \mathbf{1}_{V_g} \eta_{kg} I(D_i = k) \right\}} \right. \\
& \quad \left. - \underbrace{\sum_{i=1}^N \sum_{g=1}^G \left\{ \sum_{k=0,1} \mathbf{1}_{V_g} \eta_{kg} I(D_i = k) \right\}^T}_{II} \boldsymbol{\Delta}_g^{-1} \mathbf{t}_g \right\} \\
& I = \boldsymbol{\eta}_k^T \text{diag} \left\{ \underbrace{\sum_{i=1}^n I(D_i = k) \sum_{l=1}^{L_1} [\delta_{l1}^{-1} V_{l1}], \dots, \sum_{i=1}^n I(D_i = k) \sum_{l=1}^{L_G} [\delta_{l1}^{-1} V_{lG}]}_{\boldsymbol{\Omega}_{\eta_k}} \right\} \boldsymbol{\eta}_k \\
& II = \boldsymbol{\eta}_k^T \underbrace{(P_{\eta_{k1}}, \dots, P_{\eta_{kG}})}_{\mathbf{P}_{\eta_k}} \\
& P_{\eta_{kg}} = \sum_{i=1}^n I(D_i = k) \sum_{l=1}^{L_g} [\delta_{lg}^{-1} \sum_{v=1}^{V_{lg}} (X_{ilg}(v) - c_{klg}(v) \tilde{Z}_{ilg}(v) - \mathbf{W}_i \boldsymbol{\gamma}_{klg} - \beta_{klg}(v) - \alpha_{ilg})]
\end{aligned}$$

Then

$$\begin{aligned}
& [\boldsymbol{\eta}_k \mid \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{\boldsymbol{\eta}}_k] \\
& \propto \text{N} \left( (\boldsymbol{\Sigma}_k^{-1} + \boldsymbol{\Omega}_{\eta_k})^{-1} \mathbf{P}_{\eta_k}, (\boldsymbol{\Sigma}_k^{-1} + \boldsymbol{\Omega}_{\eta_k})^{-1} \right) \\
(15) \quad & [\boldsymbol{\Sigma}_k \mid \mathbf{X}_{lg}(v), \mathbf{Y}_{lg}(v), \mathbf{Z}_{lg}(v), \mathbf{D}, \mathbf{W}, \bar{\boldsymbol{\Sigma}}_k] \\
& \propto \text{InvW} (\boldsymbol{\Lambda} + \boldsymbol{\eta}_k \boldsymbol{\eta}_k^T, \nu + 1)
\end{aligned}$$

**3. Derivation of voxel-level LOOCV predictive probability** Let  $\mathbf{B}_i(\bar{v})$  denote all the imaging data in the brain except voxel  $v$ . First, we write out the posterior predictive

probability:

$$\begin{aligned}
& P(D_i = k \mid \mathbf{B}_i(v), \mathbf{A}_{-i}) \\
&= \int P(D_i = k \mid \mathbf{B}_i(v), \boldsymbol{\theta}) P(\boldsymbol{\theta} \mid \mathbf{B}_i(v), \mathbf{A}_{-i}) d\boldsymbol{\theta} \\
&= \int P(D_i = k \mid \mathbf{B}_i(v), \boldsymbol{\theta}) \frac{P(\boldsymbol{\theta} \mid \mathbf{B}_i(v), \mathbf{B}_i(\bar{v}), \mathbf{A}_{-i})}{P(\boldsymbol{\theta} \mid \mathbf{B}_i(v), \mathbf{B}_i(\bar{v}), D_i = d_i, \mathbf{A}_{-i})} P(\boldsymbol{\theta} \mid \mathbf{B}_i(v), \mathbf{B}_i(\bar{v}), D_i = d_i, \mathbf{A}_{-i}) d\boldsymbol{\theta},
\end{aligned}$$

in which

$$\frac{P(\boldsymbol{\theta} \mid \mathbf{B}_i(v), \mathbf{B}_i(\bar{v}), \mathbf{A}_{-i})}{P(\boldsymbol{\theta} \mid \mathbf{B}_i, \mathbf{B}_i(\bar{v}), D_i = d_i, \mathbf{A}_{-i})} = \frac{P(\boldsymbol{\theta}, \mathbf{B}_i, \mathbf{A}_{-i}) P(\mathbf{B}_i, D_i = d_i, \mathbf{A}_{-i})}{P(\mathbf{B}_i, \mathbf{A}_{-i}) P(\boldsymbol{\theta}, \mathbf{B}_i, D_i = d_i, \mathbf{A}_{-i})} = \frac{P(D_i = d_i \mid \mathbf{B}_i, \mathbf{A}_{-i})}{P(D_i = d_i \mid \mathbf{B}_i, \boldsymbol{\theta})}.$$

Thus,

$$\begin{aligned}
\frac{P(D_i = k \mid \mathbf{B}_i(v), \mathbf{A}_{-i})}{P(D_i = d_i \mid \mathbf{B}_i, \mathbf{A}_{-i})} &= \int \frac{P(D_i = k \mid \mathbf{B}_i(v), \boldsymbol{\theta})}{P(D_i = d_i \mid \mathbf{B}_i, \boldsymbol{\theta})} P(\boldsymbol{\theta} \mid \mathbf{B}_i, D_i = d_i, \mathbf{A}_{-i}) d\boldsymbol{\theta} \\
&= \int \frac{P(\mathbf{B}_i(v) \mid D_i = k, \boldsymbol{\theta}) P(D_i = k) / P(\mathbf{B}_i(v) \mid \boldsymbol{\theta})}{P(\mathbf{B}_i \mid D_i = d_i, \boldsymbol{\theta}) P(D_i = d_i) / P(\mathbf{B}_i \mid \boldsymbol{\theta})} P(\boldsymbol{\theta} \mid \mathbf{B}_i, D_i = d_i, \mathbf{A}_{-i}) d\boldsymbol{\theta} \\
&:= \frac{P(D_i = k)}{P(D_i = d_i)} Q_{kd_i},
\end{aligned}$$

which leads to (12).

#### 4. Trace plots

Figure S1: Trace plots for selected voxel-level, subregion-level, and region-level parameters from posterior sampling.

