## The phenotype control kernel of a biomolecular regulatory network

Sang-Mok Choo<sup>1</sup>, Byunghyun Ban<sup>2</sup>, Jae II Joo<sup>2</sup>, and Kwang-Hyun Cho<sup>2,\*</sup>

<sup>1</sup>Department of Mathematics, University of Ulsan, Ulsan 44610, Republic of Korea
<sup>2</sup>Department of Bio and Brain Engineering, Korea Advanced Institute of Science and Technology (KAIST), Daejeon 34141, Republic of Korea

## **Supporting information**

Numbers of control nodes and sets for full control methods

<sup>\*</sup>Corresponding author. E-mail: ckh@kaist.ac.kr, Phone: +82-42-350-4325, Fax: +82-42-350-

<sup>4310,</sup> Web: http://sbie.kaist.ac.kr/

The structure-based methods in (8) and (11, 12) are full control methods for finding driver and determining nodes, respectively.

A maximum matching used in (8), the maximum set of directed links that do not share start or end nodes, is the set of red dotted arrows in the upper box in the below figure. Since a node N is said to be matched if a link in the maximum matching points at the node Nand otherwise the node N is said to be unmatched, the 8 unmatched nodes (driver nodes) are marked as red balls in the upper box. Therefore, applying the maximum matching to the simplified MAPK network, we obtain one control set (denoted by number 1 next to the green bar in the first row named Structural Controllability in the below figure (b)) of 8 control nodes (denoted by number 8 before the green bar in the below figure (a)). Note that if the method in (8) is applied multiple times and different maximum matchings are chosen at each time, multiple control sets can be obtained.

There are 6 loops in the simplified MAPK network. The minimum set of nodes required to remove all the loops is the set of the four nodes (determining nodes) marked with red balls in the bottom box in below figure, where the minimum set is referred to as the minimum feedback vertex set (mFVS) in (11, 12). Therefore, applying the idea of FVS to the simplified MAPK network, we obtain one control set (denoted by number 1 next to the orange bar in the second row named Feedback Vertex Set in the below figure (b) of 4 control nodes (denoted by number 4 before the orange bar in the below figure (a)). Note that if the method in (11, 12) is applied multiple times and different determining nodes are chosen at each time, multiple control sets can be obtained.

Therefore we need to simultaneously control 8 driver nodes or 4 determining nodes obtained in the two structure-based methods for their desired full control.

