optica

Extended depth of focus for coherence-based cellular imaging: supplementary material

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Published 9 August 2017

The document provides supplementary information to "Extended depth of focus for coherence-based cellular imaging," https://doi.org/10.1364/optica.4.000959. A detailed theoretical analysis on the formation of the coaxially focused multimode (CAFM) beam is provided.

https://doi.org/10.6084/m9.figshare.5201437

1. Formation of the coaxially focused multimode (CAFM) beam

Assuming the self-imaging wavefront division optical system transmits light with wavelength λ, the cylindrical waveguide of diameter d and a length L generates a maximum of M high order modes, the distance between multimode fiber and focusing lens is s, and the focal length of the lens is f, the direction cosine $\cos\theta_m$ for the emitting beam of order m can be expressed as:

$$\frac{md-d/2}{\sqrt{(md-d/2)^2+L^2}} \le \cos\left(\theta_{m,m>0}\right) \le \frac{md+d/2}{\sqrt{(md+d/2)^2+L^2}}. \quad \textbf{(S1)}$$

Since $d \gg \lambda$, the lens pupil function for order m, P_m can be approximated as the geometric projection of the cylindrical waveguide's distal end aperture on the lens plane. Using the paraxial approximation, a point radiation source at the single-mode fiber's distal end generates a field $U(r,z)$ in image space with a radial coordinate r and axial coordinate z:

$$U(r,z) = -j\frac{2\pi}{\lambda(L+s)z}e^{jk(L+s+z)}e^{j\frac{\pi}{\lambda z}r^2} \int_0^{\infty} P_m(r_0)\left(\sum_{m=0}^{M} e^{j\frac{\pi(r_0+md)^2}{\lambda(L+s)}}\right)e^{-j\frac{\pi r_0^2}{\lambda f}}e^{j\frac{\pi r_0^2}{\lambda z}}J_0(2\pi r r_0/\lambda z)r_0 dr_0. \quad \textbf{(S2)}$$

where J_0 is the 0th order Bessel function of the first kind. The 0th order mode provides a field $U_0(r,z)$ that is identical to the conventional focused field as described in Eq. S3:

$$U_0(r,z) = -j\frac{2\pi}{\lambda(L+s)z}e^{jk(L+s+z)}e^{j\frac{\pi}{\lambda z}r^2} \int_0^{d(L+s)/2L} e^{j\frac{\pi r_0^2}{\lambda}A(z)} J_0(2\pi r r_0/\lambda z)r_0 dr_0, \quad \textbf{(S3)}$$

$$A(z) = \frac{1}{L+s} - \frac{1}{f} + \frac{1}{z}.$$

The high order modes (m>0) introduce fields $U_m(r,z)$, that have ring-shaped pupil functions and phase terms linear with r_0 within Fourier-Bessel integral as in Eq. S4. The total field is the coherent addition of all modes:

$$U_m(r,z) = -j\frac{2\pi}{\lambda(L+s)z}e^{jk(L+s+z)}e^{j\frac{\pi}{\lambda z}r^2}e^{j\frac{\pi m^2 d^2}{\lambda(L+s)}} \int_{r_1(m)}^{r_2(m)} \left(e^{j\frac{\pi r_0^2}{\lambda}A(z)}e^{j\frac{2\pi r_0 md}{\lambda(L+s)}}\right) J_0(2\pi r r_0/\lambda z)r_0 dr_0, \quad \textbf{(S4)}$$

$$r_1(m) = mds/L - d(L+s)/2L,$$
$$r_2(m) = mds/L + d(L+s)/2L.$$

For negative $A(z)$ (the high order modes have a focal length longer than the 0th order mode), Eq. S4 can be approximated as

$$U_m(r,z) \approx$$

$$j\frac{2\pi md}{\lambda(L+s)^2 zA(z)} e^{jk(L+s+z)} e^{j\frac{\pi}{\lambda z}r^2} e^{j\frac{\pi m^2 d^2}{\lambda(L+s)}} e^{-j\frac{\pi(md)^2}{\lambda(L+s)^2 A(z)}} \tag{S5}$$

$$\int_{r_1(m)}^{r_2(m)} e^{-j\frac{\pi}{\lambda}\left(r_0\sqrt{-A(z)}-\frac{md}{(L+s)\sqrt{-A(z)}}\right)^2} J_0\left(2\pi r r_0/\lambda z\right) dr_0.$$

Therefore, for $r_0=r_p$ that satisfies Eq. S6, the phase term within the Fourier-Bessel integral is canceled.

$$r_p(m) = \frac{zfmd}{(L+s-f)z-(L+s)f}, \tag{S6}$$

$$r_1(m) \le r_p(m) \le r_2(m).$$

A focused Bessel field $B(r,z,m)$ generated in response to r_p is expressed as:

$$B(r,z,m) = j\frac{2\pi md}{\lambda(L+s)^2 zA(z)} e^{jk(L+s+z)} e^{j\frac{\pi}{\lambda z}r^2}$$

$$e^{j\frac{\pi(md)^2}{\lambda(L+s)}} e^{-j\frac{\pi(md)^2}{\lambda(L+s)^2 A(z)}} J_0\left(2\pi r r_p(m)/\lambda z\right). \tag{S7}$$

According to Eqs. S6 and S7, the radius of the first zero of Bessel field $r_B(m,z)$ and the axial focusing region $S(m)$ are written as Eq. S8:

$$r_B(m,z) = \frac{2.4048\lambda\left[(L+s-f)z-(L+s)f\right]}{2\pi fmd},$$

$$\frac{f(2ms+L+s)}{2ms+L+s-2mf-f} \le S(m) \le \frac{f(2ms-L-s)}{2ms-L-s-2mf+f}. \tag{S8}$$

A similar analysis can be applied to input sources of a different field profile such as Gaussian.