Supporting information

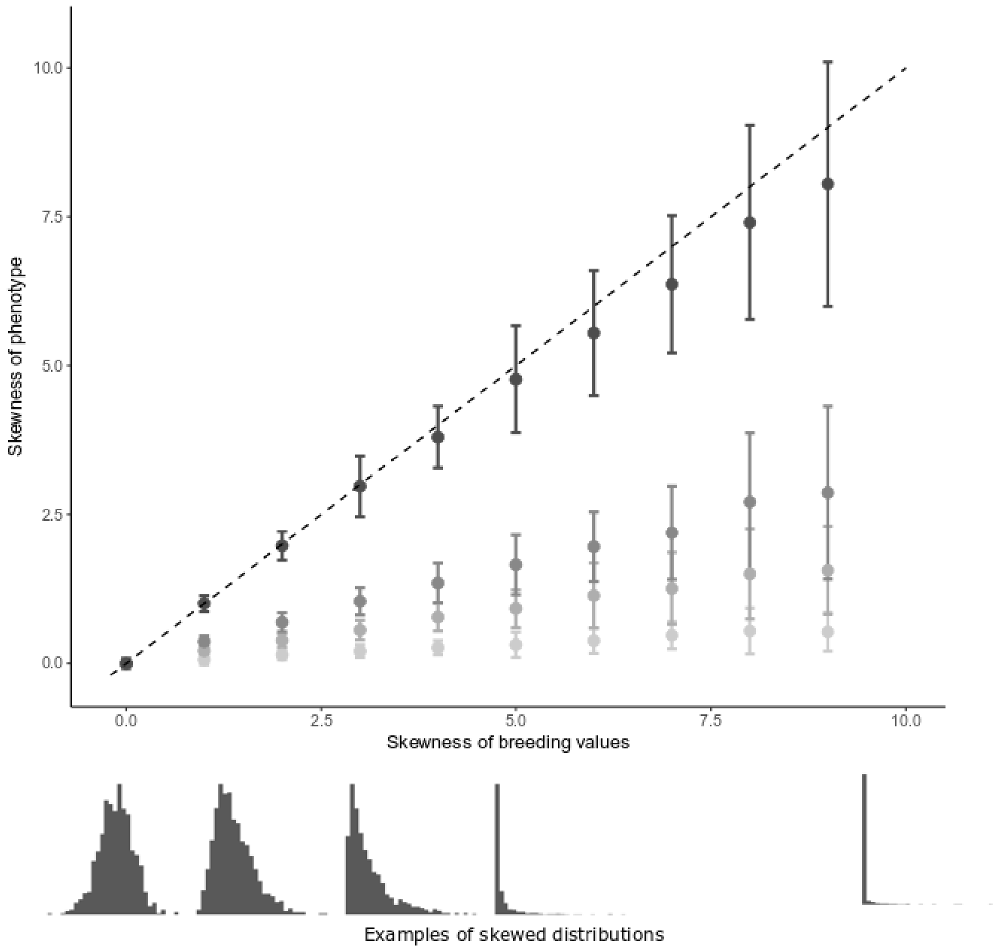

Figure S1 – Skewness in phenotypic distribution against skewness in breeding values. Mean estimations of skewness and their standard errors in simulations (1000 individuals) are shown for variable heritabilities ($\boldsymbol{h^2 = 1}; \frac{\mathbf{1}}{\mathbf{2}}; \frac{\mathbf{1}}{\mathbf{3}}$ or $\frac{\mathbf{1}}{\mathbf{6}}$ from black to light grey respectively). The dotted lines plots $\boldsymbol{y = x}$. Typical distributions with skewness $\boldsymbol{\alpha = 0, 1, 2, 5}$ and $\mathbf{10}$, are illustrated below the graph. Note that confidence intervals increase with expected skewness (parameter of the Gamma distribution). This is because realized skewness becomes more variable when samples are drawn from distributions with higher expected skewness, as a result of the increased probability to not pick values at the right tail of the distribution.

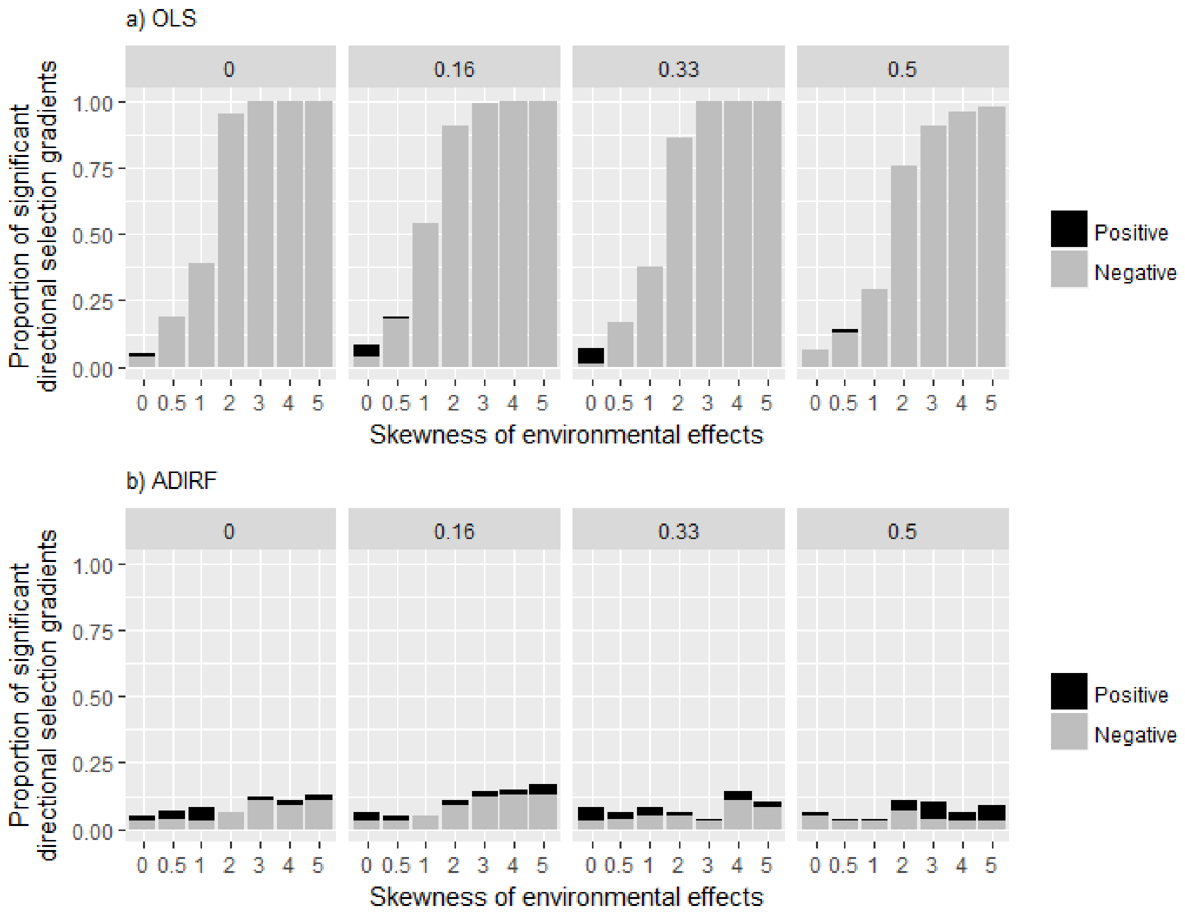

Figure S2 – Power to detect directional selection gradients under skewed environmental effects. The proportions of simulations where a significant selection gradient was estimated by the OLS method (a) or the ADIRF method (b) are shown for several combinations of skewness of environmental effects and heritability. In black, positive selection gradients; in grey, negative selection gradient. At the top of each frame, values of heritability: $h^2 = 0, \frac{1}{6}, \frac{1}{3}$ or $\frac{1}{2}$. Significance of selection gradients from zero is determined by a Student test at 5% (a) or a permutation test 1000 permutations) (b).

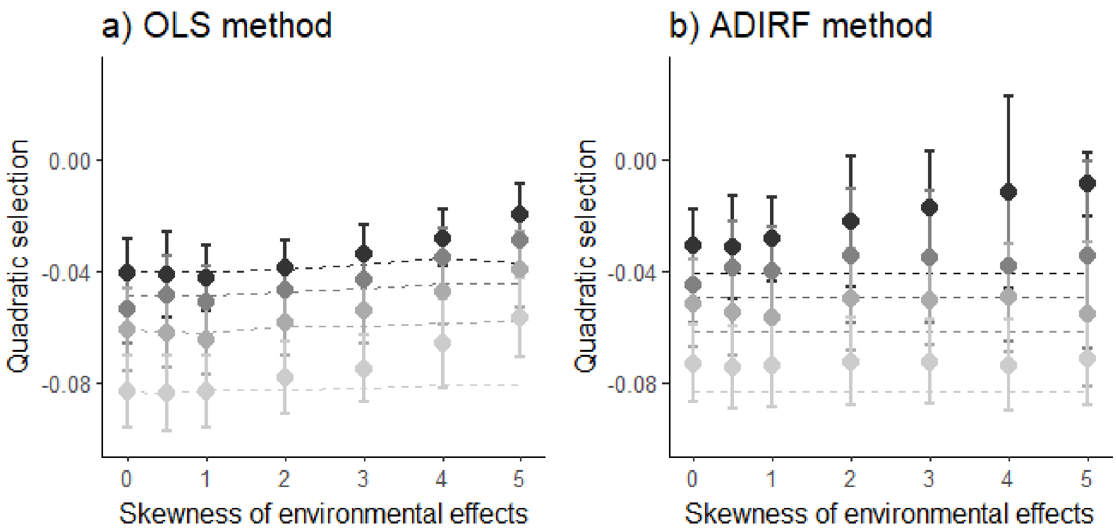

Figure S3 – Estimated quadratic selection gradients with skewed environmental effects. Standardized quadratic selection gradient (mean ± se) estimated by the OLS method (a) or the ADIRF method (b), are represented against the skewness of environmental effects, for variable heritabilities ($\boldsymbol{h^2} = \frac{1}{2};\frac{1}{3};\frac{1}{6}$ or $\mathbf{0}$ from black to light grey). Dotted lines show predictions from the model, using $\widehat{\boldsymbol{\gamma}} = \boldsymbol{\partial^2 ln}\,(\overline{\boldsymbol{W}})/\boldsymbol{\partial}\,\bar{\mathbf{z}}^2 + \boldsymbol{\beta}^2$ (Lande and Arnold (1983)) and the eq. (13) or eq. 6-7 for (a) and (b) respectively (see Table 2).

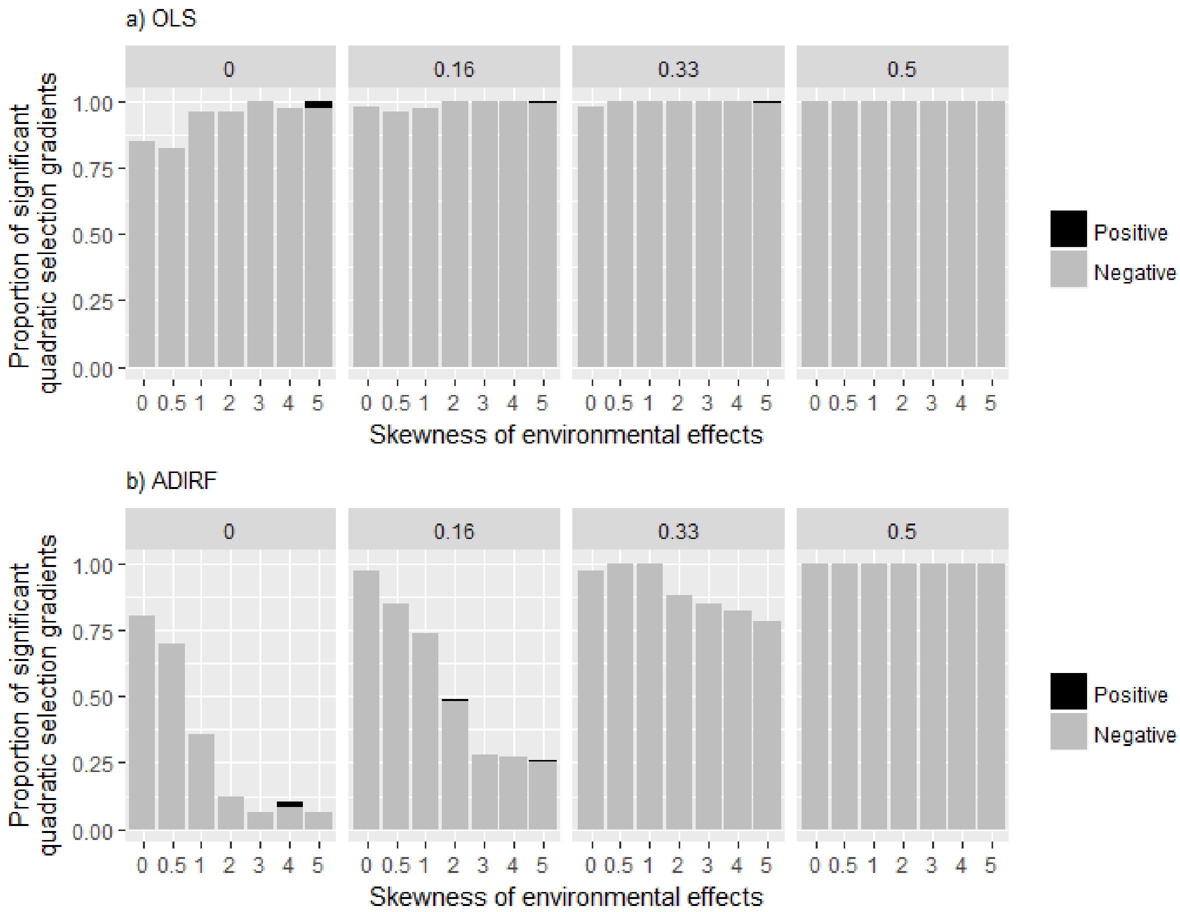

Figure S4 – Power to detect quadratic selection gradient under skewed environmental effects. The proportions of simulations where a significant selection gradients was estimated by the OLS method (a) or the ADIRF method (b) are shown for several combinations of skewness of environmental effects and heritability. In black, positive selection gradients; in grey, negative selection gradient. At the top of each frame, values of heritability: $h^2 = 0, \frac{1}{6}, \frac{1}{3}$ or $\frac{1}{2}$. Significance of selection gradients from zero is determined by a Student test at 5% (a) or a permutation test (1000 permutations) (b).

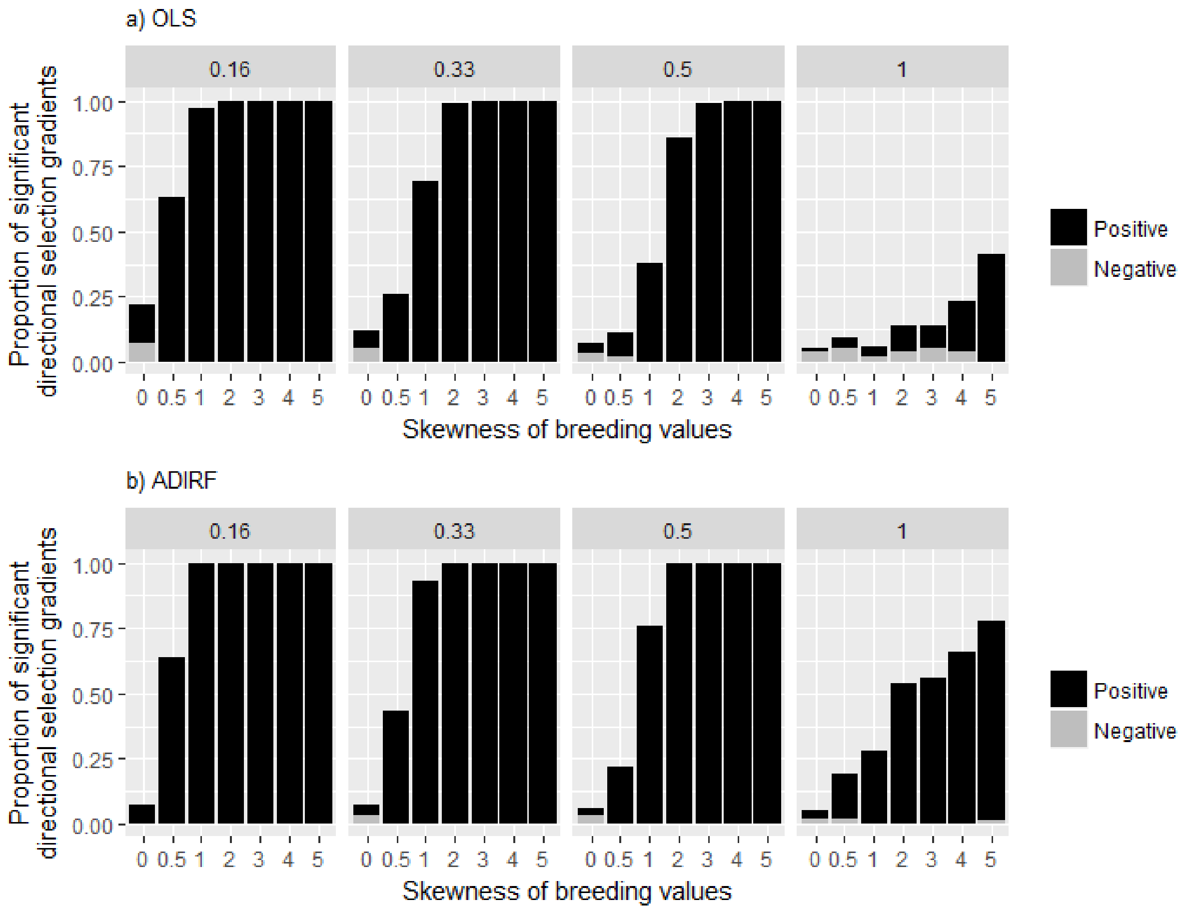

Figure S5 – Power to detect directional selection gradient under skewed breeding values. The proportions of simulations where a significant selection gradient was estimated by the OLS method (a) or the ADIRF method (b) are shown for several combinations of skewness of breeding values and heritability. In black, positive selection gradients; in grey, negative selection gradient. At the top of each frame, values of heritability: $h^2 = \frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ or 1. Significance of selection gradients from zero is determined by a student test at 5% (a) or a permutation test (1000 permutations) (b).

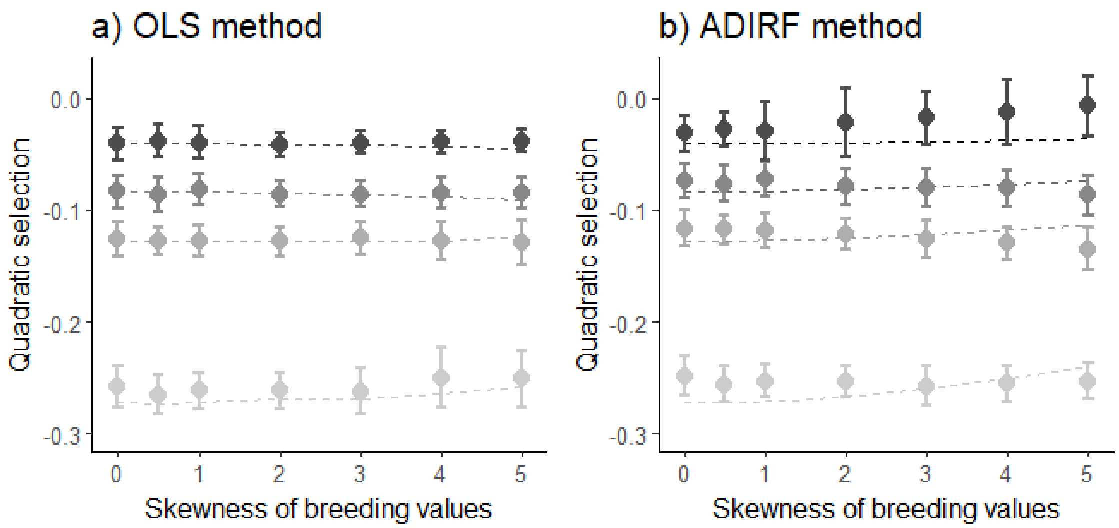

Figure S6 – Estimated quadratic selection gradients with skewed genetic effects. Standardized quadratic selection gradient (mean ± se) estimated by the OLS method (a) or the ADIRF method (b), are represented against the skewness of breeding values, for variable heritabilities ($h^2 = 1; \frac{1}{2}; \frac{1}{3}$ or $\frac{1}{6}$ from black to light grey). Dotted lines show predictions from the model, using $\hat{\gamma} = \partial^2 \ln(\bar{W}) / \partial \bar{z}^2 + \beta^2$ (Lande and Arnold (1983)) and the eq. (13) or eq. 6-7 for (a) and (b) respectively (see Table 2).

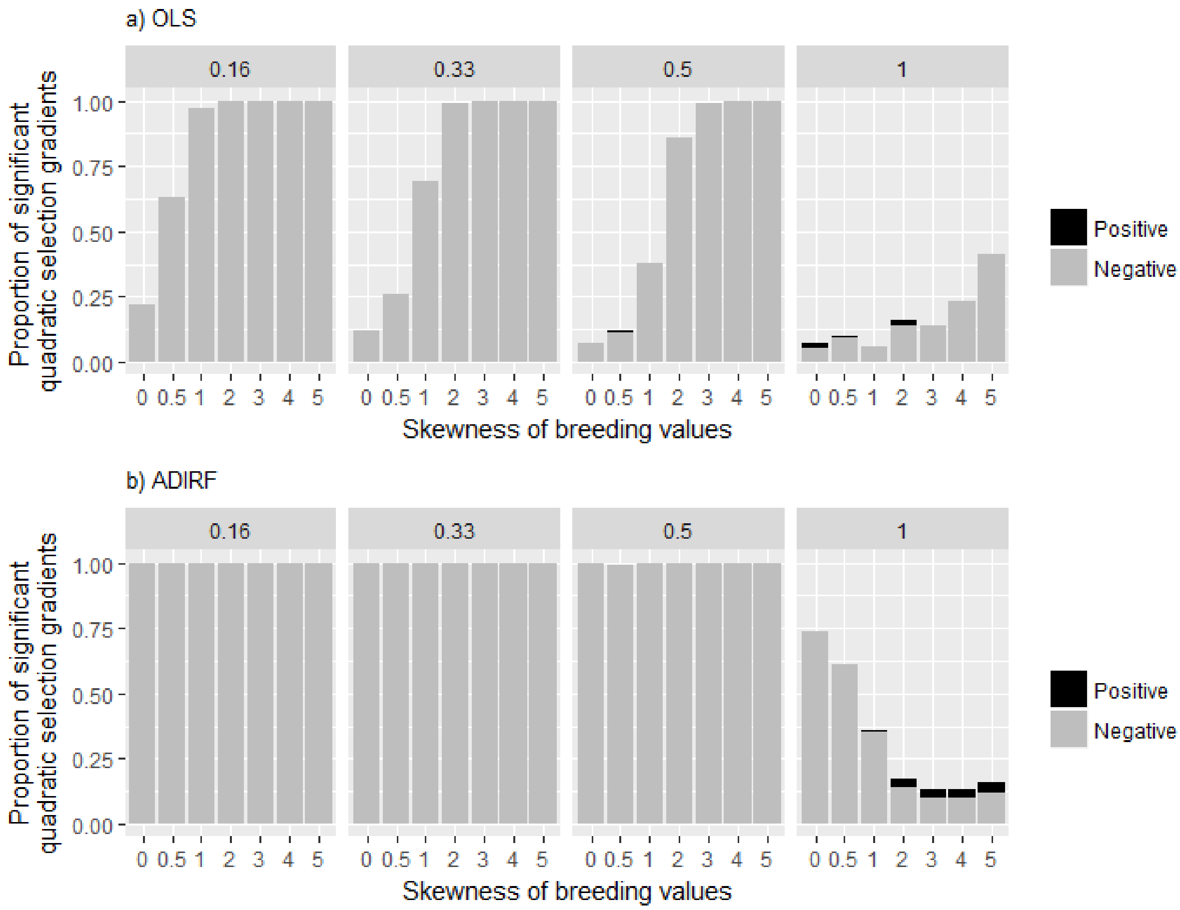

Figure S7 – Power to detect quadratic selection gradients under skewed breeding values. The proportions of simulations where a significant selection gradient was estimated by the OLS method (a) or the ADIRF method (b) are shown for several combinations of skewness of breeding values and heritability. In black, positive selection gradients; in grey, negative selection gradient. At the top of each frame, values of heritability: $h^2 = \frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ or 1. Significance of selection gradients from zero is determined by a Student test at 5% (a) or a permutation test (1000 permutations) (b).