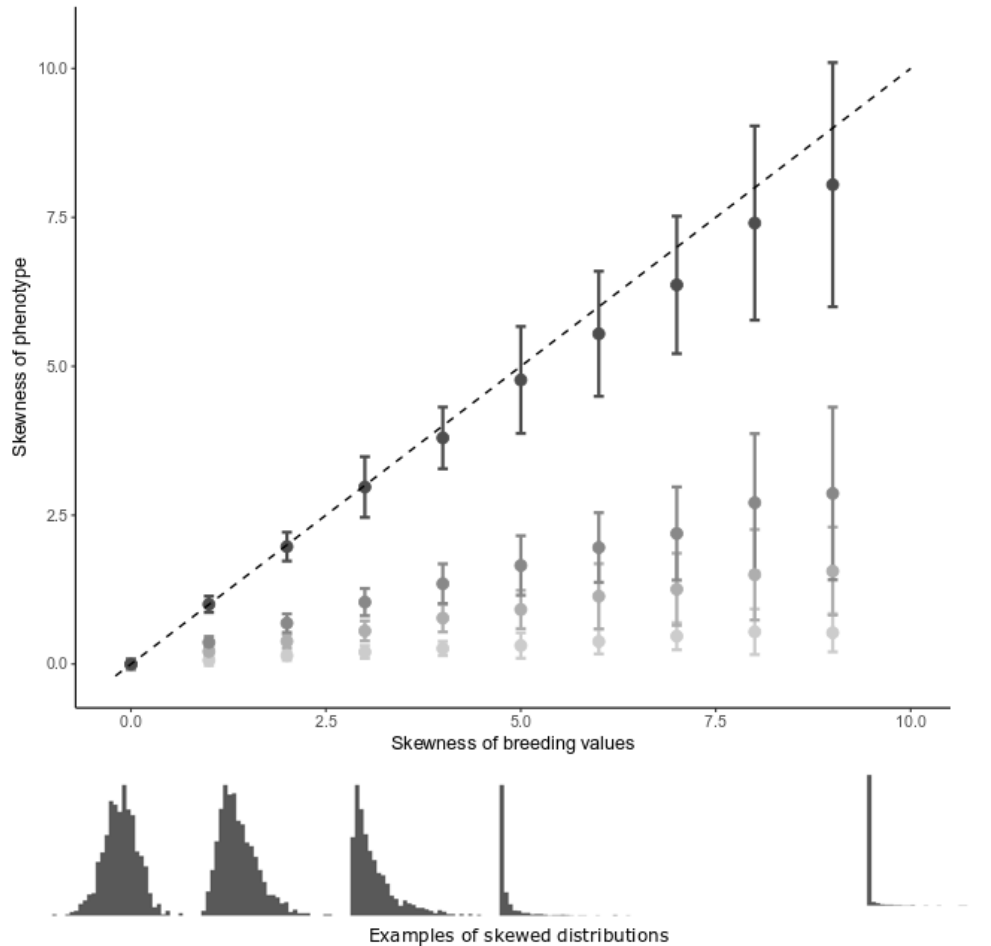


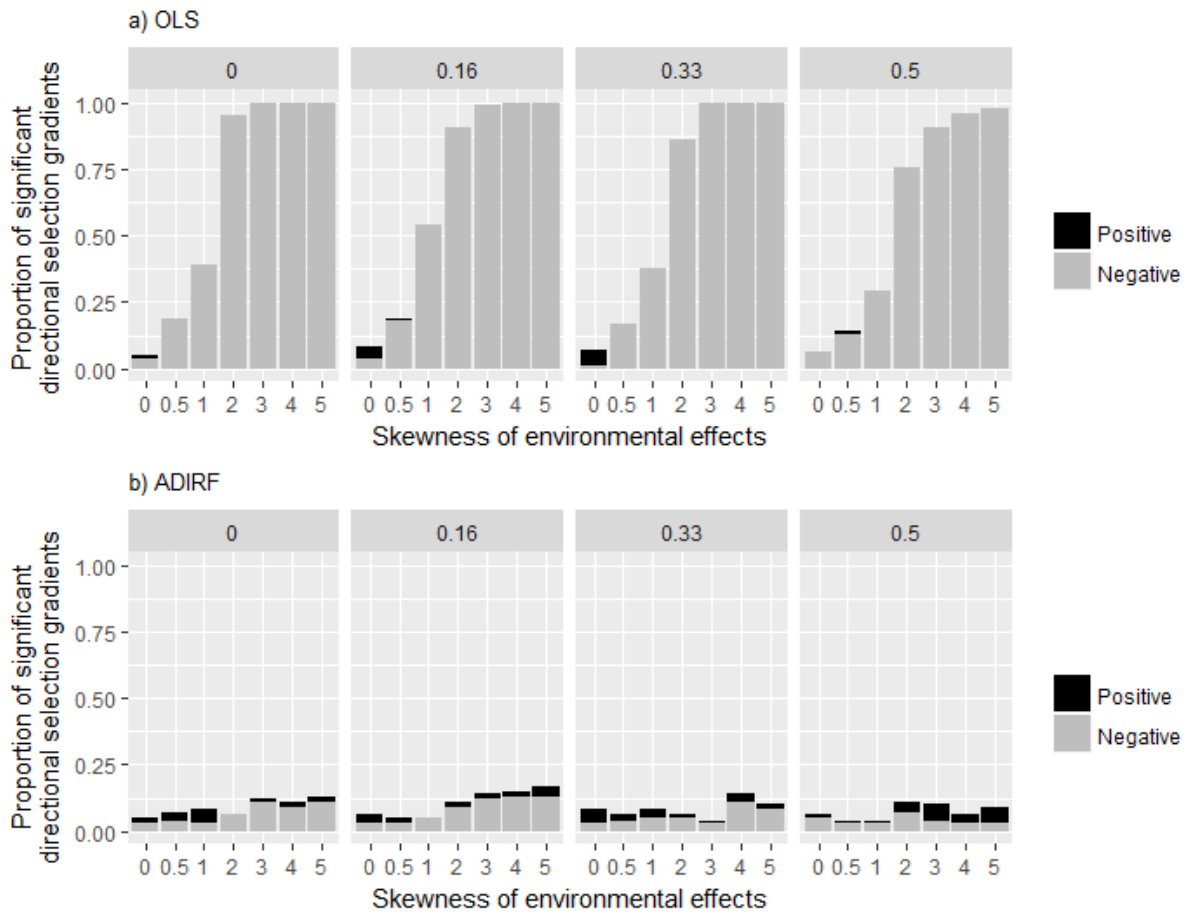
Supporting information



3

4 Figure S1 – Skewness in phenotypic distribution against skewness in breeding values. Mean
5 estimations of skewness and their standard errors in simulations (1000 individuals) are shown
6 for variable heritabilities ($h^2 = 1; \frac{1}{2}; \frac{1}{3}$ or $\frac{1}{6}$ from black to light grey respectively). The dotted
7 lines plots $y = x$. Typical distributions with skewness $\alpha = 0, 1, 2, 5$ and 10 , are illustrated
8 below the graph. Note that confidence intervals increase with expected skewness (parameter
9 of the Gamma distribution). This is because realized skewness becomes more variable when

- 10 samples are drawn from distributions with higher expected skewness, as a result of the
- 11 increased probability to not pick values at the right tail of the distribution.



12

13 Figure S2 – Power to detect directional selection gradients under skewed environmental effects.

14 The proportions of simulations where a significant selection gradient was estimated by the OLS

15 method (a) or the ADIRF method (b) are shown for several combinations of skewness of

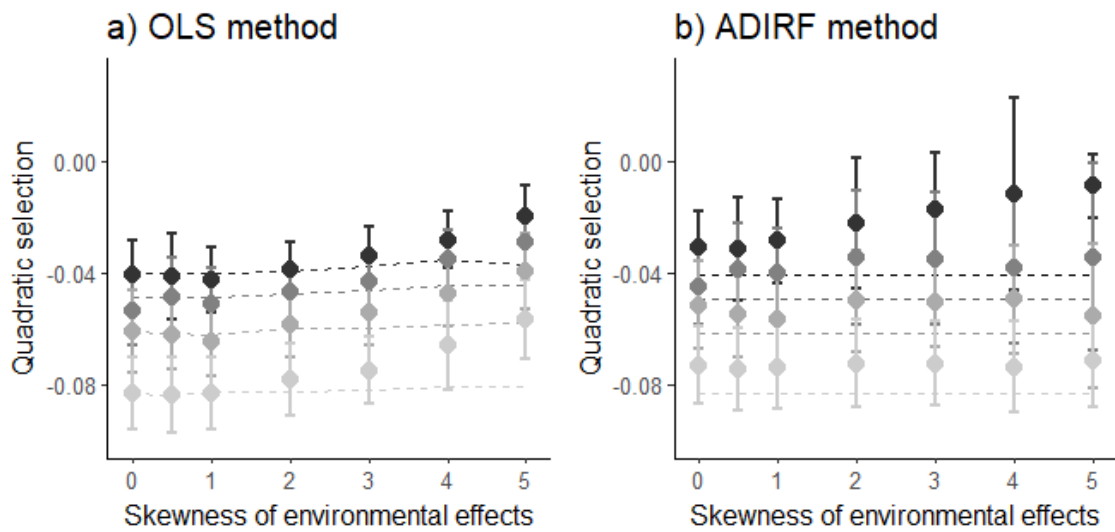
16 environmental effects and heritability. In black, positive selection gradients; in grey, negative

17 selection gradient. At the top of each frame, values of heritability: $h^2 = 0, \frac{1}{6}, \frac{1}{3}$ or $\frac{1}{2}$. Significance

18 of selection gradients from zero is determined by a Student test at 5% (a) or a permutation test

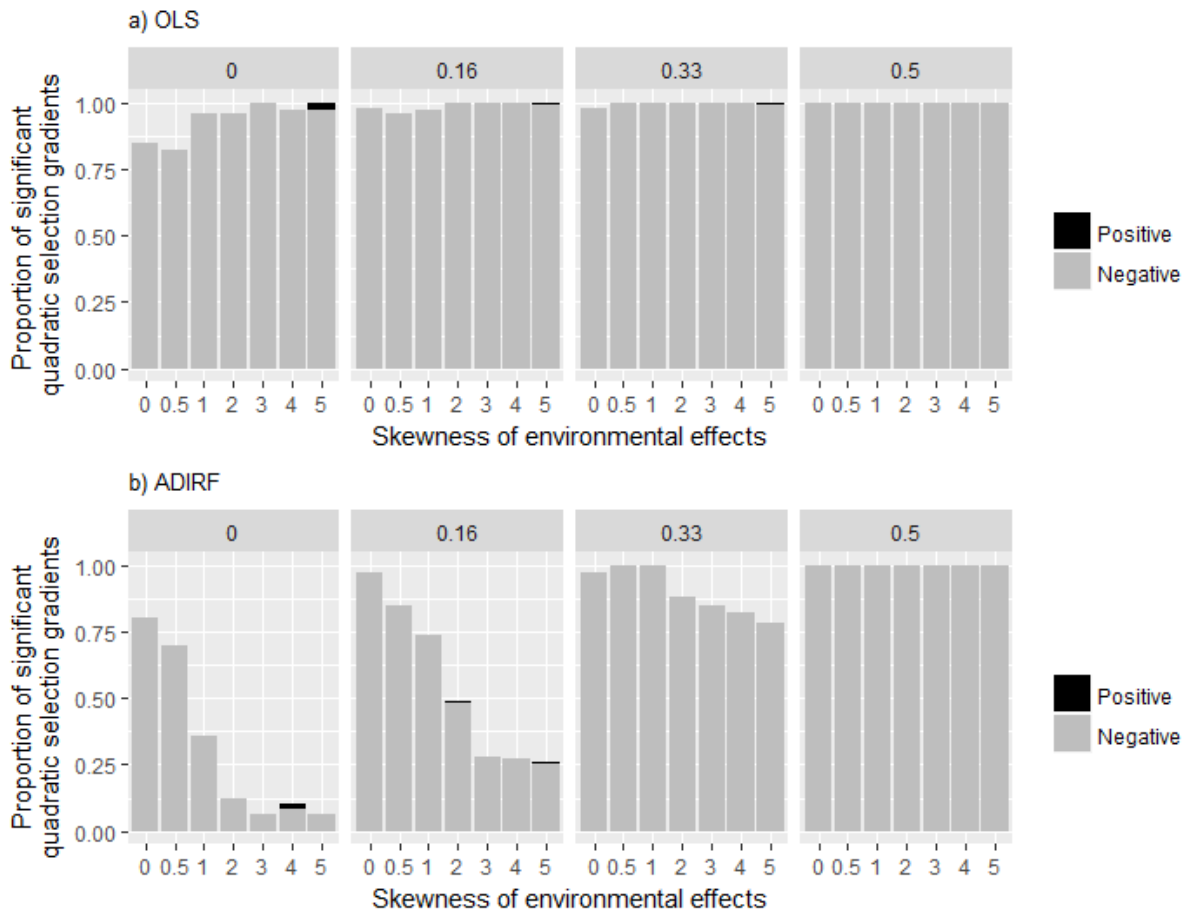
19 1000 permutations) (b).

20



22

23 Figure S3 – Estimated quadratic selection gradients with skewed environmental effects.
 24 Standardized quadratic selection gradient (mean \pm se) estimated by the OLS method (a) or the
 25 ADIRF method (b), are represented against the skewness of environmental effects, for variable
 26 heritabilities ($h^2 = \frac{1}{2}; \frac{1}{3}; \frac{1}{6}$ or 0 from black to light grey). Dotted lines show predictions from
 27 the model, using $\hat{\gamma} = \partial^2 \ln(\bar{W}) / \partial \bar{z}^2 + \beta^2$ (Lande and Arnold (1983)) and the eq. (13) or eq.
 28 6-7 for (a) and (b) respectively (see Table 2).



29

30 Figure S4 – Power to detect quadratic selection gradient under skewed environmental effects.

31 The proportions of simulations where a significant selection gradients was estimated by the

32 OLS method (a) or the ADIRF method (b) are shown for several combinations of skewness of

33 environmental effects and heritability. In black, positive selection gradients; in grey, negative

34 selection gradient. At the top of each frame, values of heritability: $h^2 = 0, \frac{1}{6}, \frac{1}{3}$ or $\frac{1}{2}$. Significance

35 of selection gradients from zero is determined by a Student test at 5% (a) or a permutation test

36 (1000 permutations) (b).

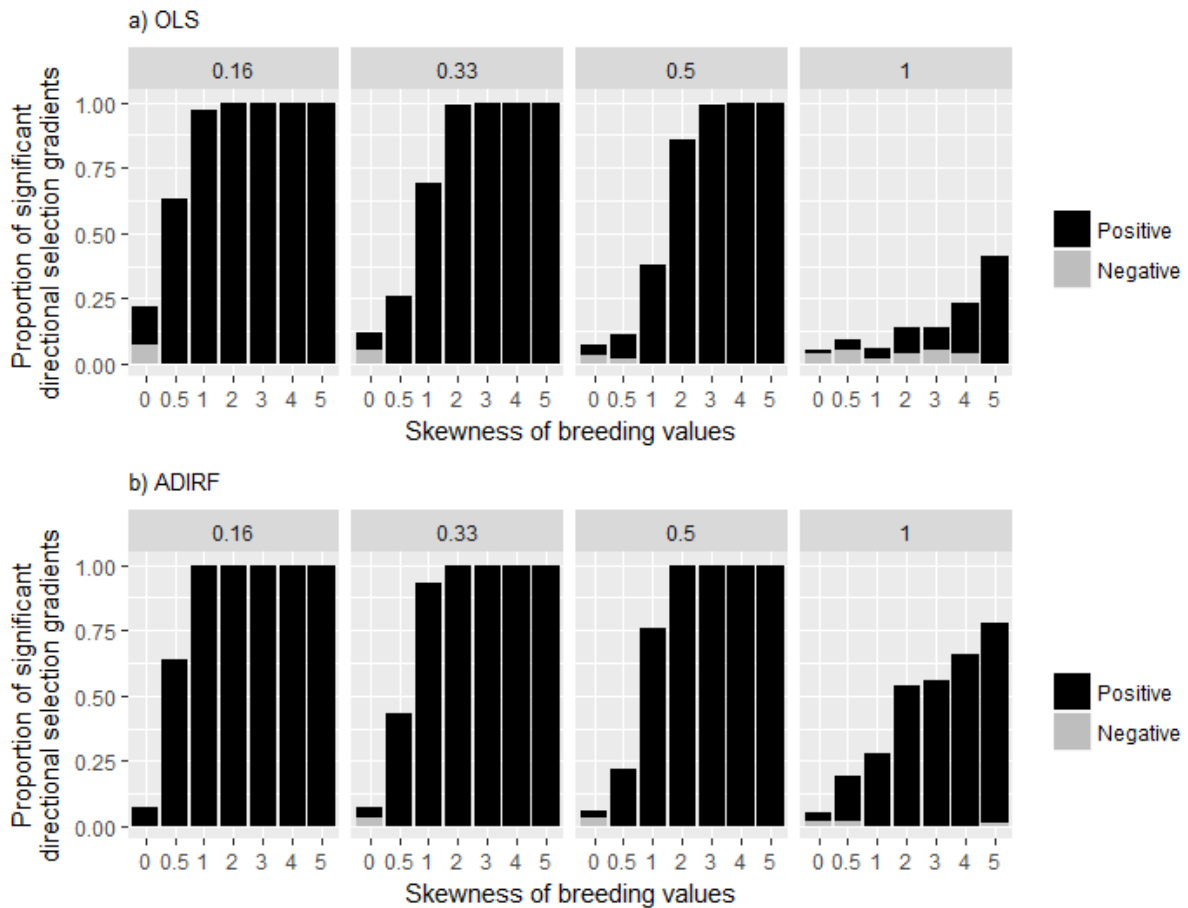
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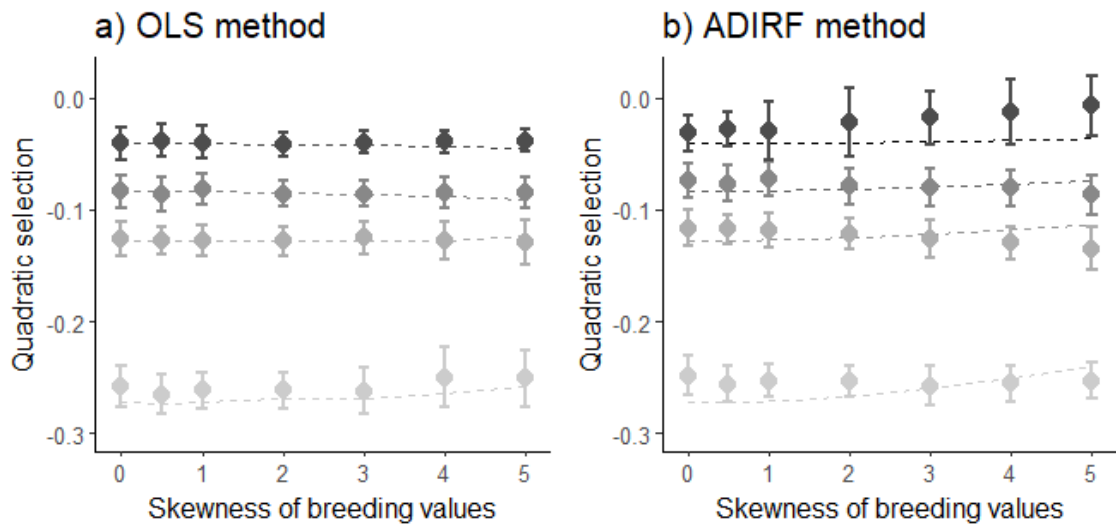


42

43 Figure S5 – Power to detect directional selection gradient under skewed breeding values. The
 44 proportions of simulations where a significant selection gradient was estimated by the OLS
 45 method (a) or the ADIRF method (b) are shown for several combinations of skewness of
 46 breeding values and heritability. In black, positive selection gradients; in grey, negative
 47 selection gradient. At the top of each frame, values of heritability: $h^2 = \frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ or 1. Significance
 48 of selection gradients from zero is determined by a student test at 5% (a) or a permutation test
 49 (1000 permutations) (b).

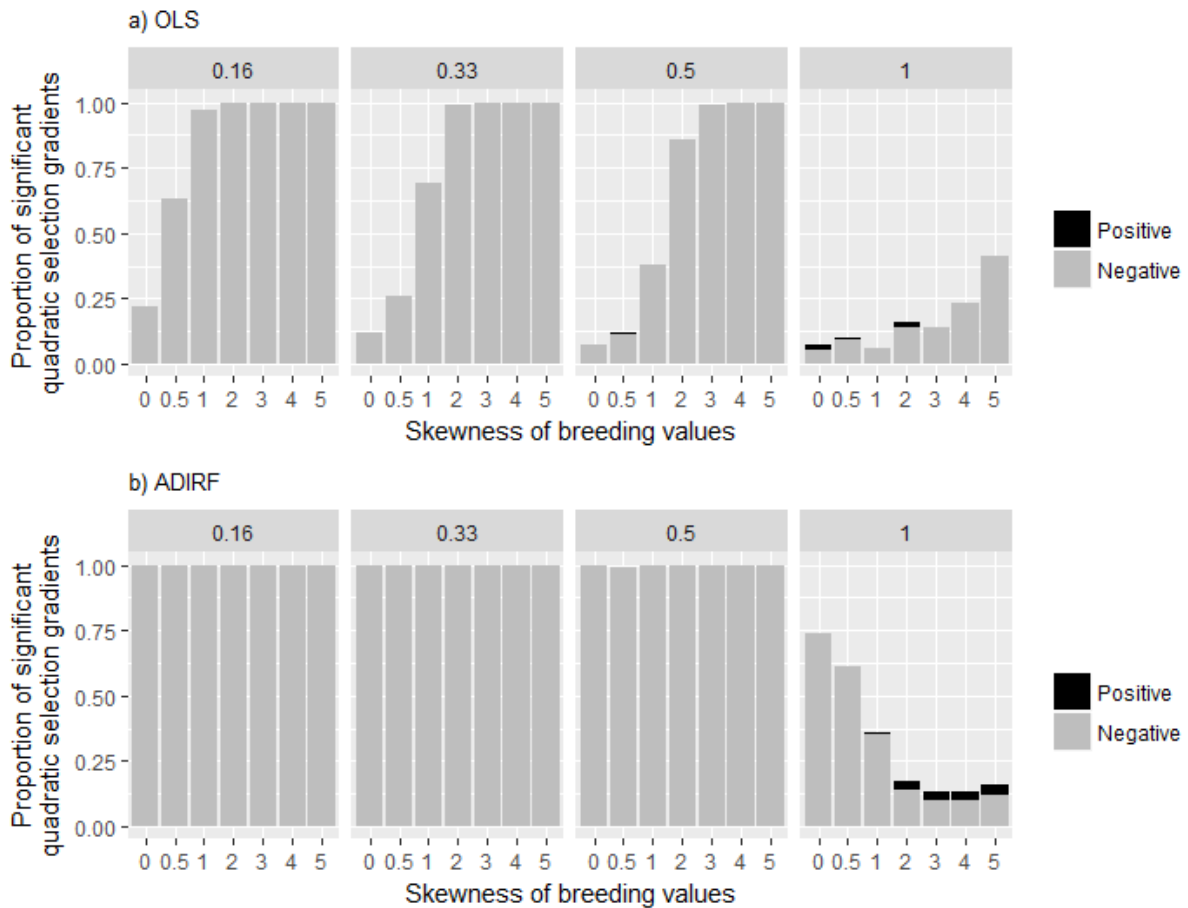
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51



52

53 Figure S6 – Estimated quadratic selection gradients with skewed genetic effects. Standardized
 54 quadratic selection gradient (mean \pm se) estimated by the OLS method (a) or the ADIRF method
 55 (b), are represented against the skewness of breeding values, for variable heritabilities ($h^2 =$
 56 $1; \frac{1}{2}; \frac{1}{3}$ or $\frac{1}{6}$ from black to light grey). Dotted lines show predictions from the model, using $\hat{\gamma} =$
 57 $\partial^2 \ln(\bar{W}) / \partial \bar{z}^2 + \beta^2$ (Lande and Arnold (1983)) and the eq. (13) or eq. 6-7 for (a) and (b)
 58 respectively (see Table 2).



59

60 Figure S7 – Power to detect quadratic selection gradients under skewed breeding values. The
 61 proportions of simulations where a significant selection gradient was estimated by the OLS
 62 method (a) or the ADIRF method (b) are shown for several combinations of skewness of
 63 breeding values and heritability. In black, positive selection gradients; in grey, negative
 64 selection gradient. At the top of each frame, values of heritability: $h^2 = \frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ or 1.
 65 Significance of selection gradients from zero is determined by a Student test at 5% (a) or a
 66 permutation test (1000 permutations) (b).