Understanding Causal Distributional and Subgroup Effects with the Instrumental Propensity Score

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Web Appendix 1

Proof of Result 2

Result 2 holds because

$$\begin{aligned}
P(Z|Y^{z,d}, D^z, s(X) = s) &= \int_x P(Z|X = x, Y^{z,d}, D^z, s(X) = s) P(X = x|Y^{z,d}, D^z, s(X) = s)dx \\
&= \int_{x:S(X)=s} P(Z|X = x, Y^{z,d}, D^z, s(X) = s) P(X = x|Y^{z,d}, D^z, s(X) = s)dx \\
&= \int_{x:S(X)=s} P(Z|X = x, Y^{z,d}, D^z) P(X = x|Y^{z,d}, D^z, S(X) = x)dx \\
&= \int_{x:S(X)=s} P(Z|X = x) P(X = x|Y^{z,d}, D^z, S(X) = s)dx \\
&= \int_{x:S(X)=s} P(Z|S(X) = s) P(X = x|Y^{z,d}, D^z, S(X) = s)dx \\
&= P(Z|s(X) = s),
\end{aligned}$$

where the second equality follows from that $P(X = x|Y^{z,d}, D^z, S(X) = s) = 0$ if $s(x) \neq s$. The fourth equality holds because Z is strongly ignorable given X. The condition $0 < P(Z = 1|S(X)) < 1$ follows from $0 < P(Z = 1|X) < 1$.

EL Estimation With the density ratio model, we will use the empirical likelihood methodology of Cheng, Qin and Zhang (2009) to make inferences about the potential outcome densities h_a^k, h_n^k, $h_{c^1}^k$ and $h_{c^0}^k$ in stratum k of the instrumental propensity score. Note that the treatment effect can be different across strata, and when $\alpha_{c^1}^k$ and $\beta_{c^1}^k$ are zero, there is no treatment effect for the compliers in stratum k. By (2) and (3), the log-likelihood for stratum k is

$$
\begin{aligned}
\ell^k &= n_{01}^k \log \phi_a^k + n_{00}^k \log(1-\phi_a^k) + n_{10}^k \log \phi_n^k + n_{11}^k \log(1-\phi_n^k) \\
&+ \sum_{i=1}^{n^k} [I(Z_i^k = 0, D_i^k = 1)(\alpha_3^k + \beta_3^k y_i) + I(Z_i^k = D_i^k = 0) \log\{\lambda^k + (1-\lambda^k)\exp(\alpha_1^k + \beta_1^k y_i)\}] \\
&+ \sum_{i=1}^{n^k} [I(Z_i^k = D_i^k = 1) \log\{\tau^k \exp(\alpha_2^k + \beta_2^k y_i) + (1-\tau^k)\exp(\alpha_3^k + \beta_3^k y_i)\}] \\
&+ \sum_{i=1}^{n^k} [I(Z_i^k = 1, D_i^k = 0)(\alpha_1^k + \beta_1^k y_i)] + \sum_{i=1}^{n^k} \log h_{c^0}^k(y_i)
\end{aligned}
$$

where n_{zd}^k is the number of observations under z and d, $h_{c^0}^k(\cdot)$ is unspecified and $h_{c^0}^k \in C = \{h_{c^0}^k | h_{c^0}^k(y_i) \geq 0, \sum_{i=1}^{n^k} h_{c^0}^k(y_i) = 1, \sum_{i=1}^{n^k} h_{c^0}^k(y_i)\exp(\alpha_j^k + \beta_j^k y_i) = 1, j = 1, 2, 3\}$. The constraint on $h_{c^0}^k$ ensures that the estimators for outcome distributions H_j are cumulative distribution functions. Maximize the log-likelihood with constraint through Lagrange multipliers ξ_j^k, we have the profiled log-likelihood in stratum k:

$$
\begin{aligned}
\ell^k &\propto n_{01}^k \log \phi_a^k + n_{00}^k \log(1-\phi_a^k) + n_{10}^k \log \phi_n^k + n_{11}^k \log(1-\phi_n^k) \\
&+ \sum_{i=1}^{n^k} I(Z_i^k = 1, D_i^k = 0)(\alpha_n^k + \beta_n^k y_i) \\
&+ \sum_{i=1}^{n^k} I(Z_i^k = D_i^k = 1) \log\{\tau^k \exp(\alpha_{c^1}^k + \beta_{c^1}^k y_i) + (1-\tau^k)\exp(\alpha_a^k + \beta_a^k y_i)\} \\
&+ \sum_{i=1}^{n^k} [I(Z_i^k = 0, D_i^k = 1)(\alpha_a^k + \beta_a^k y_i) + \\
&\quad I(Z_i^k = D_i^k = 0) \log\{\lambda^k + (1-\lambda^k)\exp(\alpha_n^k + \beta_n^k y_i)\}] \\
&- \sum_{i=1}^{n^k} \log[1 + \xi_n^k\{\exp(\alpha_n^k + \beta_n^k y_i) - 1\} + \xi_{c^1}^k\{\exp(\alpha_{c^1}^k + \beta_{c^1}^k y_i) - 1\} + \\
&\quad \xi_a^k\{\exp(\alpha_a^k + \beta_a^k y_i) - 1\}],
\end{aligned}
$$

Maximizing the log likelihood in stratum k we have

$$
\begin{aligned}
\hat{h}_{c^0}^k(y_i) &= \frac{1}{n^k} \frac{1}{1 + \sum_j \hat{\xi}_j^k\{\exp(\hat{\alpha}_j^k + \hat{\beta}_j^k y_i) - 1\}}, \\
\hat{h}_{c^1}^k(y_i) &= \hat{h}_{c^0}^k(y_i)\exp(\hat{\alpha}_{c^1}^k + \hat{\beta}_{c^1}^k y_i), \\
\hat{H}_{c^0}^k(y) &= \sum_i \hat{h}_{c^0}^k(y_i) I(y_i \le y) \\
\hat{H}_{c^1}^k(y) &= \sum_i \hat{h}_{c^0}^k(y_i)\exp(\hat{\alpha}_{c^1}^k + \hat{\beta}_{c^1}^k y_i) I(y_i \le y)
\end{aligned}
$$

Simulation

In the simulation, we simulated data on the covariates, IV, latent compliance classes and corresponding potential outcomes, and we knew the true causal effects. Observed treatment and outcomes were also simulated based on the IV and latent compliance classes. In real data analyses, we did not know the latent compliance classes and corresponding potential outcomes and could only use the observed treatment and outcomes for analyses, where latent compliance class was an unmeasured factor. The IV methods allow us to consistently estimate the causal effects of compliers of our interest.

I. Simulate covariates

$$X_{1i}^{Z} \sim N(0,1), \quad X_{2i}^{Z} \sim Bernoulli(0.5), \quad X_{3i}^{Z} \sim gamma(2,1)$$

II. Simulate the instrumental variable Z

$$P(Z=1|X^{Z}=x) = logit^{-1}(\eta^{T}X) = \frac{\exp(\eta_0 + \eta_1 X_{1i}^{Z} + \eta_2 X_{2i}^{Z} + \eta_3 X_{3i}^{Z})}{1+\exp(\eta_0 + \eta_1 X_{1i}^{Z} + \eta_2 X_{2i}^{Z} + \eta_3 X_{3i}^{Z})}$$

III. Simulate latent compliance class C with probabilities varying over IPS strata.

$$P(C_i = a) : 0.05 - 0.25; \quad P(C_i = n) : 0.05 - 0.25; \quad P(C_i = c) : 0.50 - 0.90$$

IV. Simulate observed treatment received D based on Z and C:

$$Z_i = 1, C_i = c : D_i = 1; \quad Z_i = 1, C_i = a : D_i = 1; \quad Z_i = 1, C_i = n : D_i = 0;$$

$$Z_i = 0, C_i = c : D_i = 0; \quad Z_i = 0, C_i = a : D_i = 1; \quad Z_i = 0, C_i = n : D_i = 0;$$

V. Simulate potential outcomes of latent compliance class over IPS strata:

Web Table 1 shows the true distributions of potential outcomes over IPS strata. Note that the density ratio model in equation (3) holds under all the distributions considered and the parameters of the density ratio models are functions of the potential outcome distributions of latent compliance classes.

VI: Simulate observed outcome:

Within each IPS stratum, the observed outcome was generated as:

$$
\begin{aligned}
Z_i = 1, D_i = 1: &\quad Y_i = \frac{\pi_c(x)}{\pi_c(x)+\pi_a(x)}(Y_i|C_i = c, Z_i = 1) + \frac{\pi_a(x)}{\pi_c(x)+\pi_a(x)}(Y_i|C_i = a);\\
Z_i = 1, D_i = 0: &\quad Y_i = (Y_i|C_i = n);\\
Z_i = 0, D_i = 0: &\quad Y_i = \frac{\pi_c(x)}{\pi_c(x)+\pi_n(x)}(Y_i|C_i = c, Z_i = 1) + \frac{\pi_n(x)}{\pi_c(x)+\pi_n(x)}(Y_i|C_i = n);\\
Z_i = 0, D_i = 1: &\quad Y_i = (Y_i|C_i = a);
\end{aligned}
$$

VII: True values of estimands

- Within each IPS stratum: true values of causal effects were computed with true outcome distributions and latent compliance classes;
- Overall causal effects over strata: true values of overall causal effects over strata were computed as a weighted average of the true stratum-specific causal effects with the true values of stratum-specific causal effects and compliance class probabilities .

Web Table 2 shows that the bias and root mean squared errors of our semiparametric (SEM) estimates on parameters of interest are small under all settings. The semiparametric empirical likelihood ratio statistic R was used to test $H_0 : h^k_{c^0}(y) = h^k_{c^1}(y)$ or equivalently $H_0 : \alpha^k_{c^1} = \beta^k_{c^1} = 0$, and it has a null distribution of χ^2_1 (Cheng et al. 2009). The empirical rejection rates show that the test rejects the null hypothesis around the nominal level (0.05) under the null and has good power (> 0.80) under the alternative hypothesis. Web Table 3 shows estimates of overall and stratum-specific CACE and CQCE, where the semiparametric (SEM) estimates were more efficient than the standard IV (SIV) estimates for CQCE.

Web Table 1: True distributions of potential outcomes over IPS strata.

Five equal-sized strata						
Dist'n		IPS stratum				
		1st	2nd	3rd	4th	5th
Normal	$Y_i^1 \vert C_i = c$*	N(1,1)	N(2,1)	N(3,1)	N(4,1)	N(5,1)
	$Y_i^0 \vert C_i = c$	N(0,1)	N(0,1)	N(0,1)	N(0,1)	N(0,1)
	$Y_i^d \vert C_i = a$	N(3,1)	N(4,1)	N(5,1)	N(6,1)	N(7,1)
	$Y_i^d \vert C_i = n$	N(1,1)	N(2,1)	N(3,1)	N(4,1)	N(5,1)
Log Normal	$Y_i^1 \vert C_i = c$*	LN(1,1)	LN(2,1)	LN(3,1)	LN(4,1)	LN(5,1)
	$Y_i^0 \vert C_i = c$	LN(0,1)	LN(0,1)	LN(0,1)	LN(0,1)	LN(0,1)
	$Y_i^d \vert C_i = a$	LN(3,1)	LN(4,1)	LN(5,1)	LN(6,1)	LN(7,1)
	$Y_i^d \vert C_i = n$	LN(1,1)	LN(2,1)	LN(3,1)	LN(4,1)	LN(5,1)
Exponential	$Y_i^1 \vert C_i = c$*	exp(2)	exp(3)	exp(4)	exp(5)	exp(6)
	$Y_i^0 \vert C_i = c$	exp(1)	exp(1)	exp(1)	exp(1)	exp(1)
	$Y_i^d \vert C_i = a$	exp(4)	exp(5)	exp(6)	exp(7)	exp(8)
	$Y_i^d \vert C_i = n$	exp(2)	exp(3)	exp(4)	exp(5)	exp(6)
Poisson	$Y_i^1 \vert C_i = c$*	Poi(2)	Poi(3)	Poi(4)	Poi(5)	Poi(6)
	$Y_i^0 \vert C_i = c$	Poi(1)	Poi(1)	Poi(1)	Poi(1)	Poi(1)
	$Y_i^d \vert C_i = a$	Poi(4)	Poi(5)	Poi(6)	Poi(7)	Poi(8)
	$Y_i^d \vert C_i = n$	Poi(2)	Poi(3)	Poi(4)	Poi(5)	Poi(6)

Four fixed range strata					
		IPS stratum			
		(0, 0.25]	(0.25, 0.5]	(0.5, 0.75]	(0.75, 1.0)
Normal	$Y_i^1 \vert C_i = c^a$	N(1,1)	N(2,1)	N(3,1)	N(4,1)
	$Y_i^0 \vert C_i = c$	N(0,1)	N(0,1)	N(0,1)	N(0,1)
	$Y_i^d \vert C_i = a$	N(3,1)	N(4,1)	N(5,1)	N(6,1)
	$Y_i^d \vert C_i = n$	N(1,1)	N(2,1)	N(3,1)	N(4,1)
Log Normal	$Y_i^1 \vert C_i = c^a$	LN(1,1)	LN(2,1)	LN(3,1)	LN(4,1)
	$Y_i^0 \vert C_i = c$	LN(0,1)	LN(0,1)	LN(0,1)	LN(0,1)
	$Y_i^d \vert C_i = a$	LN(3,1)	LN(4,1)	LN(5,1)	LN(6,1)
	$Y_i^d \vert C_i = n$	LN(1,1)	LN(2,1)	LN(3,1)	LN(4,1)
Exponential	$Y_i^1 \vert C_i = c^a$	exp(2)	exp(3)	exp(4)	exp(5)
	$Y_i^0 \vert C_i = c$	exp(1)	exp(1)	exp(1)	exp(1)
	$Y_i^d \vert C_i = a$	exp(4)	exp(5)	exp(6)	exp(7)
	$Y_i^d \vert C_i = n$	exp(2)	exp(3)	exp(4)	exp(5)
Poisson	$Y_i^1 \vert C_i = c^a$	Poi(2)	Poi(3)	Poi(4)	Poi(5)
	$Y_i^0 \vert C_i = c$	Poi(1)	Poi(1)	Poi(1)	Poi(1)
	$Y_i^d \vert C_i = a$	Poi(4)	Poi(5)	Poi(6)	Poi(7)
	$Y_i^d \vert C_i = n$	Poi(2)	Poi(3)	Poi(4)	Poi(5)

[a]In the setting of no treatment effect, $(Y_i^1 | C_i = c)$ has the same true distribution as $(Y_i^0 | C_i = c)$ over all IPS strata.

Web Table 2: Estimates of α_{c^1} and β_{c^1} and estimated rejection rate with R under different settings

				$\hat{\alpha}_{c^1}$			$\hat{\beta}_{c^1}$			R
k	$\begin{Bmatrix} h_{c^0} \\ h_n \\ h_{c^1} \\ h_a \end{Bmatrix}$	α_{c^1}	β_{c^1}	ave	SE	RMSE	ave	SE	RMSE	$\hat{rr}$
1	$\begin{Bmatrix} N(0,1) \\ N(1,1) \\ N(2,1) \\ N(3,1) \end{Bmatrix}$	-2	2	-2.119	0.354	0.374	2.072	0.285	0.294	1.000
2	$\begin{Bmatrix} N(0.5,1) \\ N(2,1) \\ N(3,1) \\ N(4,1) \end{Bmatrix}$	-4.375	2.5	-4.342	0.551	0.552	2.480	0.312	0.313	1.000
3	$\begin{Bmatrix} N(1,1) \\ N(3,1) \\ N(4,1) \\ N(5,1) \end{Bmatrix}$	-7.5	3	-7.348	0.967	0.979	2.957	0.426	0.428	1.000
4	$\begin{Bmatrix} N(2,1) \\ N(4,1) \\ N(5,1) \\ N(6,1) \end{Bmatrix}$	-10.5	3	-10.543	1.591	1.592	3.063	0.535	0.539	1.000
1	$\begin{Bmatrix} N(2,1) \\ N(1,1) \\ N(2,1) \\ N(3,1) \end{Bmatrix}$	0	0	-0.025	0.287	0.288	0.010	0.141	0.142	0.048
2	$\begin{Bmatrix} N(3,1) \\ N(2,1) \\ N(3,1) \\ N(4,1) \end{Bmatrix}$	0	0	-0.055	0.327	0.332	0.018	0.109	0.110	0.052
3	$\begin{Bmatrix} N(4,1) \\ N(3,1) \\ N(4,1) \\ N(5,1) \end{Bmatrix}$	0	0	-0.066	0.469	0.474	0.017	0.118	0.119	0.053
4	$\begin{Bmatrix} N(5,1) \\ N(4,1) \\ N(5,1) \\ N(6,1) \end{Bmatrix}$	0	0	-0.072	0.822	0.825	0.016	0.166	0.166	0.059

k	$\begin{Bmatrix} h_{c^0} \\ h_n \\ h_{c^1} \\ h_a \end{Bmatrix}$	α_{c^1}	β_{c^1}	$\hat{\alpha}_{c^1}$ ave	SE	RMSE	$\hat{\beta}_{c^1}$ ave	SE	RMSE	R $\hat{rr}$
1	$Exp(1)$ $Exp(2)$ $Exp(3)$ $Exp(4)$	1.099	-2	1.117	0.153	0.154	-2.081	0.389	0.398	1.000
2	$Exp(2)$ $Exp(3)$ $Exp(4)$ $Exp(5)$	0.693	-2	0.713	0.127	0.129	-2.056	0.415	0.419	1.000
3	$Exp(3)$ $Exp(4)$ $Exp(5)$ $Exp(6)$	0.511	-2	0.528	0.120	0.121	-2.059	0.473	0.476	0.996
4	$Exp(4)$ $Exp(5)$ $Exp(6)$ $Exp(7)$	0.405	-2	0.426	0.155	0.157	-2.073	0.714	0.717	0.831
1	$Exp(3)$ $Exp(2)$ $Exp(3)$ $Exp(4)$	0	0	0.002	0.139	0.139	-0.018	0.430	0.430	0.043
2	$Exp(4)$ $Exp(3)$ $Exp(4)$ $Exp(5)$	0	0	0.004	0.117	0.117	-0.019	0.473	0.474	0.058
3	$Exp(5)$ $Exp(4)$ $Exp(5)$ $Exp(6)$	0	0	0.000	0.115	0.115	0.010	0.582	0.582	0.049
4	$Exp(6)$ $Exp(5)$ $Exp(6)$ $Exp(7)$	0	0	-0.003	0.155	0.155	0.053	0.939	0.940	0.050

k	$\begin{Bmatrix} h_{c^0} \\ h_n \\ h_{c^1} \\ h_a \end{Bmatrix}$	α_{c^1}	β_{c^1}	$\hat{\alpha}_{c^1}$ ave	SE	RMSE	$\hat{\beta}_{c^1}$ ave	SE	RMSE	R $\hat{rr}$
1	$\begin{Bmatrix} Poi(3) \\ Poi(2) \\ Poi(5) \\ Poi(4) \end{Bmatrix}$	−2	0.511	−2.076	0.330	0.338	0.525	0.075	0.075	1.000
2	$\begin{Bmatrix} Poi(3.5) \\ Poi(3) \\ Poi(6) \\ Poi(5) \end{Bmatrix}$	−2.5	0.539	−2.560	0.309	0.315	0.552	0.065	0.066	1.000
3	$\begin{Bmatrix} Poi(4) \\ Poi(4) \\ Poi(7) \\ Poi(6) \end{Bmatrix}$	−3	0.560	−3.038	0.329	0.332	0.568	0.063	0.064	1.000
4	$\begin{Bmatrix} Poi(5) \\ Poi(5) \\ Poi(8) \\ Poi(7) \end{Bmatrix}$	−3	0.470	−3.093	0.447	0.456	0.489	0.077	0.079	1.000
1	$\begin{Bmatrix} Poi(5) \\ Poi(2) \\ Poi(5) \\ Poi(4) \end{Bmatrix}$	0	0	−0.013	0.305	0.305	0.002	0.060	0.060	0.049
2	$\begin{Bmatrix} Poi(6) \\ Poi(3) \\ Poi(6) \\ Poi(5) \end{Bmatrix}$	0	0	−0.039	0.269	0.271	0.006	0.045	0.045	0.052
3	$\begin{Bmatrix} Poi(7) \\ Poi(4) \\ Poi(7) \\ Poi(6) \end{Bmatrix}$	0	0	−0.025	0.298	0.299	0.004	0.043	0.043	0.052
4	$\begin{Bmatrix} Poi(8) \\ Poi(5) \\ Poi(8) \\ Poi(7) \end{Bmatrix}$	0	0	−0.021	0.434	0.434	0.003	0.055	0.055	0.054

ave: averaged estimate over the Monte Carlo replications; SE: standard error of the estimate; RMSE: root mean squared error; $\hat{rr}$: estimated rejection rate; k: IPS stratum.

Web Table 3: Standard IV and semiparametric estimates for CACE based on instrumental propensity score

k	$\begin{Bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{Bmatrix}$	CACE	$\widehat{CACE}^{SIV}$ ave	$\widehat{CACE}^{SIV}$ SD	$\widehat{CACE}^{SIV}$ RMSE	$\widehat{CACE}^{SEM}$ ave	$\widehat{CACE}^{SEM}$ SD	$\widehat{CACE}^{SEM}$ RMSE
Overall		2.654	2.635	0.073	0.075	2.629	0.068	0.072
1	$\begin{Bmatrix} N(0,1) \\ N(1,1) \\ N(2,1) \\ N(3,1) \end{Bmatrix}$	2.0	2.031	0.141	0.145	2.024	0.139	0.141
2	$\begin{Bmatrix} N(0.5,1) \\ N(2,1) \\ N(3,1) \\ N(4,1) \end{Bmatrix}$	2.5	2.520	0.119	0.121	2.517	0.114	0.115
3	$\begin{Bmatrix} N(1,1) \\ N(3,1) \\ N(4,1) \\ N(5,1) \end{Bmatrix}$	3.0	3.003	0.132	0.132	2.999	0.121	0.121
4	$\begin{Bmatrix} N(2,1) \\ N(4,1) \\ N(5,1) \\ N(6,1) \end{Bmatrix}$	3.0	3.029	0.186	0.188	3.020	0.167	0.168
Overall		0	0.022	0.069	0.072	0.020	0.067	0.070
1	$\begin{Bmatrix} N(2,1) \\ N(1,1) \\ N(2,1) \\ N(3,1) \end{Bmatrix}$	0	0.015	0.144	0.145	0.013	0.142	0.143
2	$\begin{Bmatrix} N(3,1) \\ N(2,1) \\ N(3,1) \\ N(4,1) \end{Bmatrix}$	0	0.023	0.116	0.118	0.021	0.114	0.116
3	$\begin{Bmatrix} N(4,1) \\ N(3,1) \\ N(4,1) \\ N(5,1) \end{Bmatrix}$	0	0.025	0.123	0.126	0.021	0.122	0.124
4	$\begin{Bmatrix} N(5,1) \\ N(4,1) \\ N(5,1) \\ N(6,1) \end{Bmatrix}$	0	0.023	0.168	0.169	0.021	0.165	0.166

k	$\begin{Bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{Bmatrix}$	CACE	$\widehat{CACE}^{SIV}$ ave	SD	RMSE	$\widehat{CACE}^{SEM}$ ave	SD	RMSE
Overall		-0.26	-0.291	0.024	0.039	-0.291	0.024	0.040
1	$\begin{Bmatrix} Exp(1) \\ Exp(2) \\ Exp(3) \\ Exp(4) \end{Bmatrix}$	-0.667	-0.658	0.067	0.067	-0.658	0.067	0.068
2	$\begin{Bmatrix} Exp(2) \\ Exp(3) \\ Exp(4) \\ Exp(5) \end{Bmatrix}$	-0.250	-0.261	0.042	0.043	-0.261	0.042	0.043
3	$\begin{Bmatrix} Exp(3) \\ Exp(4) \\ Exp(5) \\ Exp(6) \end{Bmatrix}$	-0.133	-0.137	0.031	0.031	-0.137	0.031	0.031
4	$\begin{Bmatrix} Exp(4) \\ Exp(5) \\ Exp(6) \\ Exp(7) \end{Bmatrix}$	-0.083	-0.086	0.034	0.034	-0.086	0.034	0.034
Overall		0	0.000	0.016	0.016	-0.001	0.016	0.016
1	$\begin{Bmatrix} Exp(3) \\ Exp(2) \\ Exp(3) \\ Exp(4) \end{Bmatrix}$	0	0.000	0.045	0.045	0.000	0.044	0.044
2	$\begin{Bmatrix} Exp(4) \\ Exp(3) \\ Exp(4) \\ Exp(5) \end{Bmatrix}$	0	-0.001	0.028	0.028	-0.001	0.028	0.028
3	$\begin{Bmatrix} Exp(5) \\ Exp(4) \\ Exp(5) \\ Exp(6) \end{Bmatrix}$	0	0.000	0.023	0.023	-0.001	0.025	0.025
4	$\begin{Bmatrix} Exp(6) \\ Exp(5) \\ Exp(6) \\ Exp(7) \end{Bmatrix}$	0	0.000	0.025	0.025	-0.001	0.025	0.025

k	$\begin{Bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{Bmatrix}$	CACE	$\widehat{CACE}^{SIV}$ ave	$\widehat{CACE}^{SIV}$ SD	$\widehat{CACE}^{SIV}$ RMSE	$\widehat{CACE}^{SEM}$ ave	$\widehat{CACE}^{SEM}$ SD	$\widehat{CACE}^{SEM}$ RMSE
Overall		2.654	2.641	0.142	0.143	2.641	0.143	0.143
1	$\begin{Bmatrix} Poi(3) \\ Poi(2) \\ Poi(5) \\ Poi(4) \end{Bmatrix}$	2.0	2.036	0.277	0.279	2.035	0.278	0.280
2	$\begin{Bmatrix} Poi(3.5) \\ Poi(3) \\ Poi(6) \\ Poi(5) \end{Bmatrix}$	2.5	2.539	0.242	0.245	2.538	0.241	0.244
3	$\begin{Bmatrix} Poi(4) \\ Poi(4) \\ Poi(7) \\ Poi(6) \end{Bmatrix}$	3.0	3.006	0.249	0.249	3.007	0.250	0.250
4	$\begin{Bmatrix} Poi(5) \\ Poi(5) \\ Poi(8) \\ Poi(7) \end{Bmatrix}$	3.0	3.025	0.356	0.357	3.025	0.356	0.357
Overall		0	0.028	0.166	0.168	0.026	0.164	0.166
1	$\begin{Bmatrix} Poi(5) \\ Poi(2) \\ Poi(5) \\ Poi(4) \end{Bmatrix}$	0	0.019	0.302	0.303	0.016	0.297	0.297
2	$\begin{Bmatrix} Poi(6) \\ Poi(3) \\ Poi(6) \\ Poi(5) \end{Bmatrix}$	0	0.043	0.274	0.277	0.040	0.268	0.271
3	$\begin{Bmatrix} Poi(7) \\ Poi(4) \\ Poi(7) \\ Poi(6) \end{Bmatrix}$	0	0.024	0.300	0.301	0.026	0.293	0.294
4	$\begin{Bmatrix} Poi(8) \\ Poi(5) \\ Poi(8) \\ Poi(7) \end{Bmatrix}$	0	0.021	0.431	0.431	0.019	0.425	0.426

Web Table 4: Standard IV and semiparametric estimates for CQCE based on instrumental propensity score

k	$\left\{ \begin{array}{c} h_0 \\ h_1 \\ h_2 \\ h_3 \end{array} \right\}$	CQCE $\left\{ \begin{array}{c} 0.1 \\ 0.25 \\ 0.5 \\ 0.75 \\ 0.9 \end{array} \right\}$	$\widehat{CQCE}^{SIV}$ ave	SD	RMSE	$\widehat{CQCE}^{SEM}$ ave	SD	RMSE
Overall		2.335	2.244	0.127	0.156	2.250	0.114	0.142
		2.485	2.406	0.100	0.128	2.402	0.093	0.124
		2.676	2.658	0.087	0.089	2.653	0.079	0.083
		2.841	2.879	0.107	0.114	2.873	0.099	0.104
		2.938	2.977	0.161	0.166	2.967	0.141	0.144
1	$\left\{ \begin{array}{c} N(0,1) \\ N(1,1) \\ N(2,1) \\ N(3,1) \end{array} \right\}$	2.0	2.010	0.210	0.210	2.025	0.180	0.182
		2.0	2.007	0.176	0.176	2.015	0.153	0.154
		2.0	2.023	0.168	0.169	2.020	0.154	0.156
		2.0	2.038	0.187	0.190	2.026	0.168	0.170
		2.0	2.056	0.262	0.268	2.034	0.206	0.209
2	$\left\{ \begin{array}{c} N(0.5,1) \\ N(2,1) \\ N(3,1) \\ N(4,1) \end{array} \right\}$	2.5	2.488	0.170	0.170	2.495	0.156	0.156
		2.5	2.509	0.137	0.137	2.507	0.128	0.129
		2.5	2.519	0.136	0.137	2.518	0.129	0.130
		2.5	2.534	0.166	0.169	2.529	0.151	0.153
		2.5	2.537	0.224	0.227	2.533	0.186	0.189
3	$\left\{ \begin{array}{c} N(1,1) \\ N(3,1) \\ N(4,1) \\ N(5,1) \end{array} \right\}$	3.0	2.975	0.182	0.184	2.981	0.177	0.178
		3.0	2.990	0.142	0.143	2.992	0.138	0.139
		3.0	3.002	0.139	0.139	3.003	0.134	0.134
		3.0	3.013	0.169	0.170	3.008	0.161	0.161
		3.0	3.017	0.251	0.252	3.018	0.203	0.204
4	$\left\{ \begin{array}{c} N(2,1) \\ N(4,1) \\ N(5,1) \\ N(6,1) \end{array} \right\}$	3.0	3.031	0.241	0.243	3.032	0.234	0.237
		3.0	3.027	0.194	0.196	3.026	0.190	0.192
		3.0	3.019	0.190	0.191	3.019	0.183	0.184
		3.0	3.017	0.231	0.231	3.016	0.210	0.211
		3.0	3.011	0.351	0.351	3.019	0.269	0.270

k	$\left\{\begin{array}{c} h_0 \\ h_1 \\ h_2 \\ h_3 \end{array}\right\}$	CQCE $\left\{\begin{array}{c} 0.1 \\ 0.25 \\ 0.5 \\ 0.75 \\ 0.9 \end{array}\right\}$	$\widehat{CQCE}^{SIV}$ ave	SD	RMSE	$\widehat{CQCE}^{SEM}$ ave	SD	RMSE
Overall		0	0.017	0.138	0.139	0.016	0.104	0.106
		0	0.022	0.105	0.108	0.017	0.081	0.083
		0	0.022	0.094	0.097	0.022	0.068	0.071
		0	0.024	0.111	0.114	0.020	0.088	0.090
		0	0.021	0.149	0.150	0.019	0.116	0.118
1	$\left\{\begin{array}{c} N(2,1) \\ N(1,1) \\ N(2,1) \\ N(3,1) \end{array}\right\}$	0	0.018	0.235	0.235	0.015	0.144	0.145
		0	0.008	0.185	0.185	0.012	0.144	0.144
		0	0.014	0.168	0.169	0.013	0.145	0.146
		0	0.018	0.179	0.180	0.013	0.147	0.147
		0	0.020	0.242	0.243	0.010	0.148	0.148
2	$\left\{\begin{array}{c} N(3,1) \\ N(2,1) \\ N(3,1) \\ N(4,1) \end{array}\right\}$	0	0.027	0.200	0.202	0.022	0.119	0.121
		0	0.029	0.149	0.152	0.022	0.159	0.166
		0	0.023	0.138	0.140	0.021	0.115	0.117
		0	0.022	0.156	0.158	0.021	0.119	0.121
		0	0.012	0.201	0.202	0.021	0.117	0.119
3	$\left\{\begin{array}{c} N(4,1) \\ N(3,1) \\ N(4,1) \\ N(5,1) \end{array}\right\}$	0	0.035	0.216	0.154	0.020	0.130	0.132
		0	0.022	0.216	0.219	0.020	0.125	0.127
		0	0.020	0.143	0.145	0.020	0.122	0.124
		0	0.024	0.158	0.160	0.022	0.124	0.126
		0	0.030	0.210	0.212	0.021	0.126	0.127
4	$\left\{\begin{array}{c} N(5,1) \\ N(4,1) \\ N(5,1) \\ N(6,1) \end{array}\right\}$	0	0.059	0.282	0.288	0.019	0.174	0.175
		0	0.037	0.213	0.216	0.019	0.169	0.170
		0	0.026	0.190	0.192	0.021	0.168	0.169
		0	0.020	0.208	0.208	0.022	0.162	0.163
		0	0.009	0.275	0.275	0.022	0.165	0.167

k	$\begin{Bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{Bmatrix}$	CQCE $\begin{Bmatrix} 0.1 \\ 0.25 \\ 0.5 \\ 0.75 \\ 0.9 \end{Bmatrix}$	$\widehat{CQCE}^{SIV}$ ave	SD	RMSE	$\widehat{CQCE}^{SEM}$ ave	SD	RMSE
Overall		−0.019	−0.020	0.008	0.008	−0.020	0.004	0.004
		−0.054	−0.055	0.013	0.013	−0.057	0.009	0.009
		−0.138	−0.147	0.023	0.025	−0.149	0.020	0.022
		−0.312	−0.344	0.040	0.051	−0.345	0.036	0.049
		−0.620	−0.714	0.075	0.121	−0.714	0.069	0.116
1	$\begin{Bmatrix} Exp(1) \\ Exp(2) \\ Exp(3) \\ Exp(4) \end{Bmatrix}$	−0.070	−0.068	0.026	0.026	−0.072	0.016	0.016
		−0.192	−0.187	0.043	0.043	−0.193	0.033	0.033
		−0.462	−0.451	0.072	0.073	−0.457	0.063	0.063
		−0.924	−0.908	0.116	0.118	−0.907	0.108	0.110
		−1.535	−1.523	0.202	0.202	−1.518	0.189	0.189
2	$\begin{Bmatrix} Exp(2) \\ Exp(3) \\ Exp(4) \\ Exp(5) \end{Bmatrix}$	−0.026	−0.027	0.014	0.014	−0.028	0.007	0.007
		−0.072	−0.073	0.025	0.025	−0.075	0.017	0.017
		−0.173	−0.176	0.041	0.041	−0.178	0.035	0.035
		−0.347	−0.357	0.070	0.071	−0.357	0.065	0.066
		−0.576	−0.604	0.122	0.125	−0.601	0.113	0.115
3	$\begin{Bmatrix} Exp(3) \\ Exp(4) \\ Exp(5) \\ Exp(6) \end{Bmatrix}$	−0.014	−0.015	0.011	0.011	−0.015	0.005	0.005
		−0.038	−0.040	0.019	0.019	−0.040	0.011	0.011
		−0.092	−0.095	0.032	0.032	−0.097	0.026	0.026
		−0.185	−0.189	0.055	0.055	−0.190	0.049	0.050
		−0.307	−0.314	0.092	0.092	−0.316	0.083	0.084
4	$\begin{Bmatrix} Exp(4) \\ Exp(5) \\ Exp(6) \\ Exp(7) \end{Bmatrix}$	−0.009	−0.011	0.012	0.012	−0.010	0.005	0.005
		−0.024	−0.025	0.020	0.020	−0.026	0.011	0.012
		−0.058	−0.060	0.035	0.035	−0.062	0.026	0.027
		−0.116	−0.120	0.058	0.058	−0.122	0.050	0.051
		−0.192	−0.197	0.101	0.101	−0.197	0.083	0.083

k	$\begin{Bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{Bmatrix}$	CQCE $\begin{Bmatrix} 0.1 \\ 0.25 \\ 0.5 \\ 0.75 \\ 0.9 \end{Bmatrix}$	$\widehat{CQCE}^{SIV}$ ave	$\widehat{CQCE}^{SIV}$ SD	$\widehat{CQCE}^{SIV}$ RMSE	$\widehat{CQCE}^{SEM}$ ave	$\widehat{CQCE}^{SEM}$ SD	$\widehat{CQCE}^{SEM}$ RMSE
Overall		0	0.000	0.005	0.005	0.000	0.002	0.002
		0	0.000	0.008	0.008	0.000	0.005	0.005
		0	0.000	0.015	0.015	0.000	0.011	0.011
		0	0.000	0.026	0.025	−0.001	0.021	0.021
		0	0.000	0.047	0.047	−0.002	0.038	0.039
1	$\begin{Bmatrix} Exp(3) \\ Exp(2) \\ Exp(3) \\ Exp(4) \end{Bmatrix}$	0	0.001	0.015	0.015	0.000	0.005	0.005
		0	0.001	0.024	0.024	0.000	0.014	0.014
		0	0.001	0.044	0.044	0.000	0.032	0.032
		0	0.000	0.072	0.072	0.000	0.062	0.062
		0	-0.004	0.133	0.133	-0.001	0.104	0.104
2	$\begin{Bmatrix} Exp(4) \\ Exp(3) \\ Exp(4) \\ Exp(5) \end{Bmatrix}$	0	0.000	0.009	0.009	0.000	0.003	0.003
		0	0.000	0.016	0.016	0.000	0.009	0.009
		0	0.000	0.028	0.028	-0.001	0.021	0.021
		0	-0.002	0.047	0.047	-0.001	0.040	0.040
		0	-0.003	0.086	0.086	-0.003	0.067	0.067
3	$\begin{Bmatrix} Exp(5) \\ Exp(4) \\ Exp(5) \\ Exp(6) \end{Bmatrix}$	0	0.000	0.008	0.008	0.000	0.003	0.003
		0	-0.001	0.013	0.013	0.000	0.007	0.007
		0	0.000	0.022	0.022	0.000	0.016	0.016
		0	0.000	0.039	0.039	-0.001	0.032	0.032
		0	0.001	0.067	0.067	0.000	0.054	0.054
4	$\begin{Bmatrix} Exp(6) \\ Exp(5) \\ Exp(6) \\ Exp(7) \end{Bmatrix}$	0	−0.001	0.008	0.008	0.000	0.003	0.003
		0	0.000	0.014	0.014	0.000	0.008	0.008
		0	-0.001	0.025	0.025	0.000	0.018	0.018
		0	-0.001	0.043	0.043	-0.001	0.035	0.035
		0	-0.001	0.075	0.075	-0.002	0.059	0.059

k	$\begin{Bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{Bmatrix}$	CQCE $\begin{Bmatrix} 0.1 \\ 0.25 \\ 0.5 \\ 0.75 \\ 0.9 \end{Bmatrix}$	$\widehat{CQCE}^{SIV}$ ave	SD	RMSE	$\widehat{CQCE}^{SEM}$ ave	SD	RMSE
Overall		2	1.941	0.303	0.308	1.958	0.201	0.205
		3	2.530	0.523	0.773	2.676	0.515	0.735
		2	2.160	0.378	0.410	2.111	0.321	0.339
		3	3.038	0.216	0.219	3.025	0.162	0.164
		3	3.358	0.525	0.636	3.314	0.477	0.571
1	$\begin{Bmatrix} Poi(3) \\ Poi(2) \\ Poi(5) \\ Poi(4) \end{Bmatrix}$	1	1.296	0.470	0.555	1.227	0.419	0.477
		1	1.272	0.534	0.713	1.245	0.509	0.676
		2	1.914	0.370	0.380	1.947	0.290	0.295
		2	2.424	0.526	0.675	2.418	0.502	0.653
		3	2.833	0.690	0.710	2.856	0.637	0.653
2	$\begin{Bmatrix} Poi(3.5) \\ Poi(3) \\ Poi(6) \\ Poi(5) \end{Bmatrix}$	2	1.957	0.411	0.414	1.968	0.212	0.215
		2	2.185	0.418	0.457	2.166	0.383	0.417
		3	2.775	0.457	0.509	2.823	0.424	0.460
		3	2.835	0.644	0.664	2.828	0.650	0.673
		3	3.291	0.509	0.586	3.265	0.474	0.543
3	$\begin{Bmatrix} Poi(4) \\ Poi(4) \\ Poi(7) \\ Poi(6) \end{Bmatrix}$	2	2.167	0.638	0.660	2.212	0.558	0.597
		2	2.427	0.547	0.694	2.408	0.521	0.662
		3	2.991	0.365	0.365	2.983	0.311	0.312
		4	3.608	0.552	0.677	3.654	0.520	0.625
		3	3.799	0.708	1.067	3.790	0.699	1.055
4	$\begin{Bmatrix} Poi(5) \\ Poi(5) \\ Poi(8) \\ Poi(7) \end{Bmatrix}$	3	2.223	0.703	1.048	2.300	0.636	0.946
		3	2.646	0.561	0.664	2.681	0.501	0.594
		3	3.063	0.572	0.576	3.084	0.497	0.504
		4	3.515	0.640	0.803	3.570	0.601	0.739
		4	3.821	0.792	0.812	3.847	0.691	0.708

k	$\begin{Bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{Bmatrix}$	CQCE $\begin{Bmatrix} 0.1 \\ 0.25 \\ 0.5 \\ 0.75 \\ 0.9 \end{Bmatrix}$	$\widehat{CQCE}^{SIV}$ ave	$\widehat{CQCE}^{SIV}$ SD	$\widehat{CQCE}^{SIV}$ RMSE	$\widehat{CQCE}^{SEM}$ ave	$\widehat{CQCE}^{SEM}$ SD	$\widehat{CQCE}^{SEM}$ RMSE
Overall		0	0.014	0.161	0.161	0.002	0.045	0.045
		0	0.059	0.699	0.701	0.075	0.643	0.648
		0	-0.006	0.253	0.253	0.002	0.184	0.184
		0	-0.010	0.303	0.303	-0.015	0.216	0.217
		0	-0.047	0.391	0.394	-0.043	0.267	0.270
1	$\begin{Bmatrix} Poi(5) \\ Poi(2) \\ Poi(5) \\ Poi(4) \end{Bmatrix}$	0	0.078	0.626	0.631	0.032	0.462	0.463
		0	0.086	0.685	0.690	0.055	0.615	0.618
		0	-0.071	0.385	0.391	-0.049	0.301	0.305
		0	0.072	0.714	0.717	0.068	0.666	0.670
		0	0.015	0.627	0.627	0.011	0.439	0.440
2	$\begin{Bmatrix} Poi(6) \\ Poi(3) \\ Poi(6) \\ Poi(5) \end{Bmatrix}$	0	0.063	0.453	0.457	-0.001	0.192	0.192
		0	0.061	0.529	0.532	0.037	0.431	0.433
		0	0.002	0.405	0.405	0.011	0.324	0.324
		0	0.047	0.711	0.712	0.076	0.655	0.659
		0	0.083	0.599	0.604	0.077	0.501	0.507
3	$\begin{Bmatrix} Poi(7) \\ Poi(4) \\ Poi(7) \\ Poi(6) \end{Bmatrix}$	0	0.069	0.633	0.637	0.060	0.425	0.429
		0	-0.013	0.439	0.440	0.002	0.286	0.286
		0	0.046	0.436	0.438	0.055	0.363	0.368
		0	0.087	0.643	0.649	0.077	0.591	0.596
		0	0.037	0.758	0.759	0.034	0.633	0.634
4	$\begin{Bmatrix} Poi(8) \\ Poi(5) \\ Poi(8) \\ Poi(7) \end{Bmatrix}$	0	-0.018	0.844	0.844	-0.012	0.620	0.620
		0	0.021	0.614	0.614	0.026	0.419	0.420
		0	0.090	0.645	0.651	0.110	0.555	0.566
		0	0.111	0.715	0.724	0.121	0.615	0.627
		0	0.058	0.884	0.886	0.084	0.702	0.707