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Supplemental Information

Active Prestress Leads to an Apparent Stiffening of Cells through Geometrical Effects

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I. GEOMETRICAL COUPLING WITH PHASE-SHIFTED, FREQUENCY-DEPENDENT ACTIVE FORCE CONTRIBUTIONS

In the first and second example in the main text, geometrical coupling of active prestress has contributed only a frequency-independent addition to the measured storage modulus of the system. In the following, we will present an example where geometrical coupling gives rise to a complex-valued, frequency-dependent addition to the effective elastic modulus of the system for the case of a deflected prestressed fibre with a viscoelastic connector between the bead and the fiber (see Fig. 1B, main text). The bead is deflected in a vertical manner. The combined system has a (complex) spring constant

$$k_{comb} = 1/(1/k_{conn} + 1/k_{fibre}),$$

where k_{conn} and k_{fibre} are the effective (complex) spring constants of the connector and the fibre with respect to vertical deflection. One finds

$$k_{fibre}(\sigma_{\rm act}, \omega) = 16h_0^4/(4h_0^2 + L^2)^{3/2} G^*_{\rm meas}(\sigma_{\rm act}, \omega),$$

where $G^*_{\rm meas}$ equals the r.h.s of formula (6), main text. Thus, in the stiffness of the combined system k_{comb}, the active term in $G^*_{\rm meas}$ contributes in general also to the imaginary part of the system and is frequency-dependent.

II. DETERMINING THE GEOMETRICAL FACTOR FOR THE CASE OF A DEFLECTED PRESTRESSED CYTOSKELETAL FIBRE

In the main text, we where discussing the example of geometrical coupling of active stress in a prestressed cytoskeletal fibre tethered at its end points at a distance L (see Fig. 1A, main text). Height oscillations are imposed on the fibre center with $h(t) = h_0 + \tilde{h}\exp(i\omega t)$. Here, we derive the factor of geometrical coupling $1/g_0 \cdot \tilde{g}/\tilde{\epsilon}$ for this specific example.

We make a perturbation calculation determining the fibre oscillation dynamics up to first order in height amplitude $\tilde{h}$. We make the following expansions of the dynamic fibre length $l(t)$ and the dynamic angle $\alpha(t)$ between the substrate and the fibre

$$\begin{aligned} l(t) &= l_0 + \tilde{l}\exp(i\omega t) + \mathcal{O}(\tilde{h}^2), \\ \alpha(t) &= \alpha_0 + \tilde{\alpha}\exp(i\omega t) + \mathcal{O}(\tilde{h}^2). \end{aligned}$$

Using the geometrical relations $\sin(\alpha(t)) = h(t)/(l(t)/2)$ and $h(t)/(L/2) = \tan(\alpha(t))$, we obtain

$$\begin{aligned} l_0 &= \sqrt{4h_0^2 + L^2}, \\ \alpha_0 &= \arctan(2h_0/L), \\ \tilde{\alpha} &= (2\tilde{h}L)/(4h_0^2 + L^2), \\ \tilde{l} &= (4h_0\tilde{h})/\sqrt{4h_0^2 + L^2}. \end{aligned} \tag{1}$$

We can thus calculate the time variation of the geometrical factor $g(t) = 2\sin(\alpha(t))$ to first order

$$\begin{aligned} g(t) &\equiv g_0 + \tilde{g}\exp(i\omega t) + \mathcal{O}(\tilde{h}^2) \\ &= 2\sin(\alpha_0) + 2\cos(\alpha_0)\tilde{\alpha}\exp(i\omega t) + \mathcal{O}(\tilde{h}^2) \\ &= \frac{2\tan(\alpha_0)}{\sqrt{1+\tan(\alpha_0^2)}} + \frac{2\tilde{\alpha}}{\sqrt{1+\tan(\alpha_0^2)}}\exp(i\omega t) + \mathcal{O}(\tilde{h}^2) \\ &= \frac{4h_0}{\sqrt{4h_0^2+L^2}} + \frac{4\tilde{h}L^2}{(4h_0^2+L^2)^{3/2}}\exp(i\omega t) + \mathcal{O}(\tilde{h}^2), \end{aligned}$$

where we have used Eqn. (1) in the last transformation step. We conclude that $g_0 = \frac{4h_0}{\sqrt{4h_0^2+L^2}}$ and $\tilde{g} = \frac{4\tilde{h}L^2}{(4h_0^2+L^2)^{3/2}}$. The strain amplitude of the fibre is $\tilde{\epsilon} = \tilde{l}/l_0$. We thus obtain the coupling geometrical factor as

$$\begin{aligned} \frac{1}{g_0}\frac{\tilde{g}}{\tilde{\epsilon}} &= \frac{1}{\frac{4h_0}{\sqrt{4h_0^2+L^2}}} \frac{\frac{4\tilde{h}L^2}{(4h_0^2+L^2)^{3/2}}}{\frac{\tilde{l}}{l_0}} \\ &= \frac{L^2}{4h_0^2}. \end{aligned}$$