Appendix: Matrix Computation of KL Divergence

Consider 61 points spanning from -3 to 3 with 0.1 increments along each ability dimension, and let $\boldsymbol{\pi}_1$ denote a 61-by-D matrix, with the (*l*, *d*)th element being $\prod_{j=1}^{n_d} P(\boldsymbol{\theta}_i^d)^{y_{ij}}(1-P(\boldsymbol{\theta}_i^d))^{1-y_{ij}}$. $\boldsymbol{\pi}_2$ is computed similarly with item parameters and the response vector from the second occasion. Then the likelihood at every possible ability point (there are 61^D points in total) forms a 61^{D-1}-by-61 matrix Υ_t computed recursively as $\boldsymbol{\Upsilon}_t^d = \boldsymbol{\Upsilon}_t^{d-1} \otimes \boldsymbol{\pi}_t^d$, for t = 1, or 2, and D > 2, d = 2, … ,D. $\boldsymbol{\pi}_t^d$ denotes the *d*th column of $\boldsymbol{\pi}_t$, $\otimes$ denotes Kronecker products, and $\boldsymbol{\Upsilon}_t^2 = \boldsymbol{\pi}_t^1(\boldsymbol{\pi}_t^2)^T$ when D = 2. After expressing the posterior likelihood as a matrix operation in matrix form, the computation of Equation 9 becomes much simpler. In all computations, only one loop with 61 iterations is needed, regardless of the size of D.