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Supplementary Materials for

Parity-time–symmetric optoelectronic oscillator

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The PDF file includes:

- section S1. Balanced photodetection
- section S2. PT symmetry
- section S3. Loop length–dependent phase noise
- fig. S1. Block diagram of the BPD used in the experiment.
- fig. S2. Optical to electrical conversion at the BPD.
- fig. S3. Open-loop response of the OEO.
- fig. S4. Phase noise enhancement with a long OEO loop.

Other Supplementary Material for this manuscript includes the following:
(available at advances.sciencemag.org/cgi/content/full/4/6/eaar6782/DC1)

- movie S1 (.mp4 format). Phase noise of a 5.26 GHz signal from a PT-symmetric OEO.
- movie S2 (.mp4 format). Electrical spectrum of the 5.26 GHz signal.
- movie S3 (.mp4 format). A 10-GHz microwave signal generated by the OEO with PT symmetry.
- movie S4 (.mp4 format). Output of the OEO without PT symmetry.

Supplementary Materials

section S1. Balanced photodetection

Balanced photodetection is widely used in coherent optical communications. Figure S1 shows a block diagram of a balanced photodetector (BPD) used in the experiment. The BPD consists of two InGaAs photodiodes (PD1 and PD2) with each capable of handling up to 10-dBm optical power and with a bandwidth greater than 40 GHz. The photocurrents generated by the two PDs are combined electrically, generating a signal that is the difference between the signals from the two PDs. A 50-Ω resistor is connected between the common port of the two PDs and the ground, converting the photocurrent signal into a voltage signal.

The BPD has two optical input ports that direct two input optical signals to the two PDs. A tunable delay line (TDL) that provides a tunable time delay with 600-ps tuning range is incorporated in one of the optical paths. The other optical path is a length of optical fiber with a fixed length.

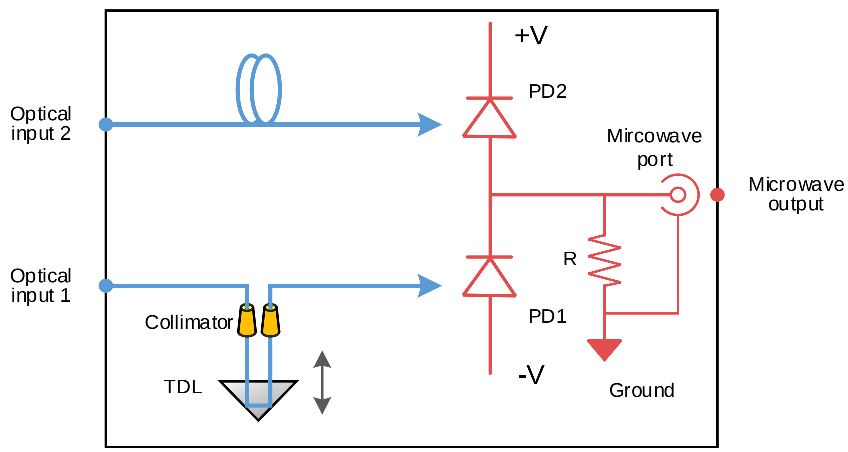

fig. S1. Block diagram of the BPD used in the experiment.

In the PT symmetric opto-electronic oscillator (OEO), the optical carrier modulated by the oscillating microwave signal is split by a polarization beam splitter (PBS) into two signals, which are directed to the two optical input ports of the BPD. The TDL is tuned to a position where the optical path difference for the optical signals reaching PD1 and PD2 equals to half the wavelength of the oscillating microwave signal. Consequently, the signals at the inputs of PD1 and PD2 have a π-phase difference. The π-phase difference is then cancelled after differential detection at the BPD. The phase relationship between the two optical signals and the two microwave signals is illustrated in fig. S2.

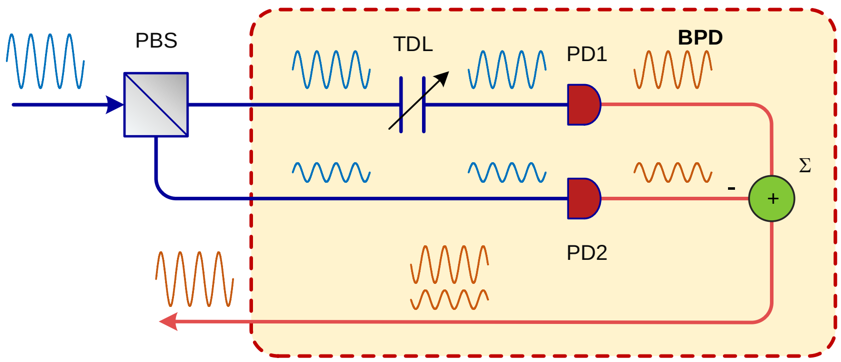

fig. S2. Optical to electrical conversion at the BPD. The phase relationship between the two optical signals (blue color) and the two microwave signals (red color) is shown.

To achieve single mode oscillation, a gain loop and a loss loop are constructed to form a PT-symmetric OEO. Although sharing most of the electrical and optical components, the two loops use their own PDs in the BPD module to perform optical to electrical conversion. The signals in the two loops cross-couple when the detected signals at the outputs of PD1 and PD2 are combined. The tuning of the gain and loss in the two loops is realized by tuning PC2, to change the polarization state of the light delivered to the PBS and thus determines the splitting ratio of the optical power directed to the two PDs. The more optical power a PD receives, the higher the gain the corresponding loop will have. Hence, tuning PC2 can break the PT symmetry to achieve single-mode oscillation without the use of a high-Q optical or microwave filter.

section S2. PT symmetry

Since the gain and loss loops have identical optical lengths, the coupling between the two OEO loops breaks the degeneracy of the modes in each cavity, resulting in a frequency splitting of the original eigenmodes in each loop. The coupling equations of the modes in the two cavities are given by

$$\frac{da_n}{dt} = -i\omega_n a_n + i\kappa b_n + \gamma_{a_n} a_n \qquad \text{(S1)}$$

$$\frac{db_n}{dt} = -i\omega_n b_n + i\kappa a_n + \gamma_{b_n} b_n \qquad \text{(S2)}$$

where a_n and b_n are the amplitudes of the n-th modes in the gain and loss loops, respectively, ω_n is the eigenfrequency of the longitudinal modes of the two loops without PT symmetry coupling, κ is the coupling coefficient between the two loops, and γ_{a_n} and γ_{b_n} represent the gain coefficients of the gain and loss loops for the n-th mode. Solving (S1) and (S2), we can get the eigenfrequencies of the PT symmetric system

$$\omega_n^{(1,2)} = \omega_n + i\frac{\gamma_{a_n} + \gamma_{b_n}}{2} \pm \sqrt{\kappa_n^2 - \left(\frac{\gamma_{a_n} - \gamma_{b_n}}{2}\right)^2} \qquad \text{(S3)}$$

Assuming that the exact PT condition can be satisfied by tuning PC2 to achieve single-mode oscillation for a mode denoted by $n = 0$, i.e., $\gamma_{a_0} = -\gamma_{b_0} = \gamma_0$, (S3) can be written as

$$\omega_0^{(1,2)} = \omega_0 \pm \sqrt{\kappa_0^2 - \gamma_0^2} \quad \text{(S4)}$$

To find the capability for single-mode oscillation, it is important to analyze the gain difference between the main oscillating mode, which would experience the highest gain, and a secondary mode would experience the second highest gain. The main and secondary modes are denoted as $n = 0$ and $n = 1$. The gain difference in the gain loop is

$$\Delta g = \gamma_{a_0} - \gamma_{a_1} \quad \text{(S5)}$$

Assuming the secondary mode is in critical condition in a PT-symmetric OEO, (S3) is rewritten for $n = 1$ as

$$\omega_1^{(1,2)} = \omega_1 + i\frac{\gamma_{a_1} + \gamma_{b_1}}{2} \pm \sqrt{\kappa_1^2 - \left(\frac{\gamma_{a_1} - \gamma_{b_1}}{2}\right)^2} \quad \text{(S6)}$$

The critical condition for the emergence of the conjugate amplifying/decaying secondary mode is

$$\sqrt{\kappa_1^2 - \left(\frac{\gamma_{a_1} - \gamma_{b_1}}{2}\right)^2} = 0 \quad \text{(S7)}$$

Or

$$\kappa_1 = \frac{\gamma_{a_1} - \gamma_{b_1}}{2} \quad \text{(S8)}$$

Due to the small mode spacing for $n = 0$ and $n = 1$, we may assume that the coupling ratio of the two modes are equal, i.e., $\kappa_0 = \kappa_1$. The third term in (S3) for the primary mode $n = 0$ can be written as

$$g_0 = \sqrt{\kappa_0^2 - \left(\frac{\gamma_{a_0} - \gamma_{b_0}}{2}\right)^2} = \sqrt{\kappa_1^2 - \gamma_0^2} = \sqrt{\left(\frac{\gamma_{a_1} - \gamma_{b_1}}{2}\right)^2 - \gamma_0^2} \quad \text{(S9)}$$

Note that PT-symmetry is broken for the primary mode when the secondary mode is in critical condition, and g_0 is an imaginary number. The gain difference between $n = 0$ and $n = 1$ modes with PT-symmetry can be given by

$$\Delta g_{PT} = |g_0| - |g_1| = j \cdot g_0 - 0 = \sqrt{\gamma_0^2 - \left(\frac{\gamma_{a_1} - \gamma_{b_1}}{2}\right)^2} \quad \text{(S10)}$$

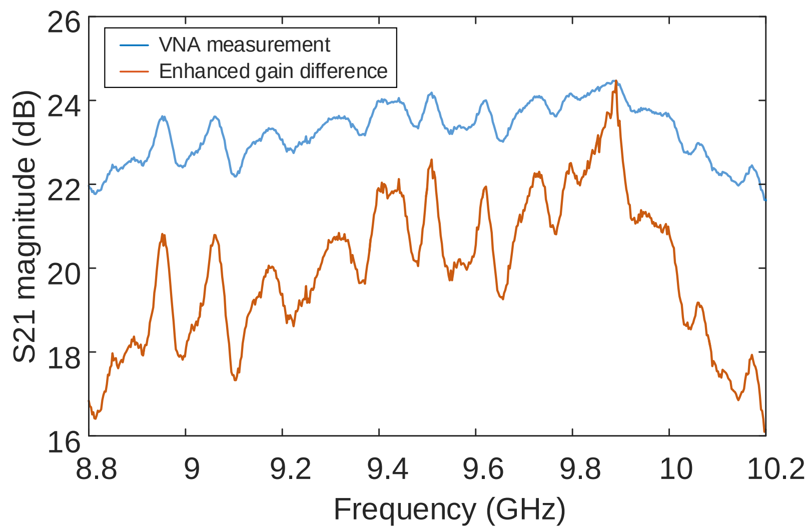

fig. S3. Open-loop response of the OEO. The VNA measurement is a zoom-in view of that shown in Fig. 3. With gain difference enhancement by PT-symmetry, the small ripples in the gain spectrum can be magnified to sharp the high peaks, which indicate significantly different gain coefficients for two adjacent longitudinal modes near the frequency with a maximum loop gain.

The equivalent gain spectrums of a PT-symmetric OEO and a single-loop OEO are shown in fig. S3. The VNA measurement corresponds to the gain spectrum for a single-loop OEO. For the PT-symmetric OEO, the gain spectrum is achieved by modifying the gain spectrum of the VNA measurement with the gain difference calculated by (S5) and (S9), in relative to the peak gain at 9.867 GHz. As can be observed, PT symmetry magnifies the small ripples in the gain spectrum into sharp and high peaks, thus it increases the loop gain difference between the oscillating mode and the other sidemodes. Single mode oscillation is then achieved for the mode experiencing the highest loop gain even without using a narrow band filter. The magnification of the ripples is particularly strong for the modes experiencing a gain that is almost as strong as the main oscillating mode. In a practical implementation, the gain ripples always exist due to the uneven frequency response of EA, the PD and the MZM in the OEO.

section S3. Loop length–dependent phase noise

The phase noise of an OEO is given by[15]

$$S(f') = \frac{\delta}{(\delta/2\tau)^2 + (2\pi\tau f')^2} \tag{S11}$$

where f' is the offset frequency from the carrier, and δ is a noise factor. The round trip time of an OEO is given by $\tau = n_g l/c$, where n_g and l are the group index and physical length of the optical fiber, respectively, and c is the light velocity in vacuum. The time delay in the electrical path is negligible compared to that in the optical paths, and thus is ignored. The noise factor δ contributed by the noise from the LD, the BPD and the EA, can be expressed as $\delta = \rho_N G_A^2 / P_{OSC}$, where ρ_N and P_{OSC} are the noise power density and microwave power at the output of the PD, respectively, and G_A is the voltage gain of the EA. fig. S4 shows that the phase noise of an OEO as a function of loop length. As can be seen the phase noise is reduced almost linearly when the loop length is increased. The noise factor in an OEO ranges from 10^{-13} Hz^{-1} for a regular OEO to 10^{-15} Hz^{-1} for a perfectly optimized OEO using state-of-the-art low noise components. In the PT-symmetric OEO, we have achieved an estimated noise factor of 5.1×10^{-14} Hz^{-1}.

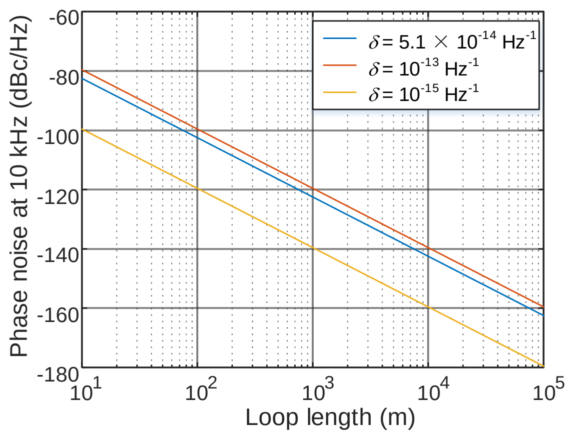

fig. S4. Phase noise enhancement with a long OEO loop. The phase noise of an OEO at 10 kHz offset frequency when the OEO loop length is increased. Different values of noise factors are used, which correspond to the OEO loop in our demonstration, a typical OEO loop and an OEO loop with state-of-the-art low noise components.