SUPPLEMENTARY MATERIAL

Appendix A

Here we summarize each individual power calculation.

3DOF Ankle: Rotational Power

3DOF Ankle power (P_{rot_ank}, Eqn. A1) captures power due to rotation of the shank about the foot (Fig. 1B). Rotational power is obtained by multiplying the ankle joint moment ($\vec{M}_{ank}$) by the relative angular velocity of the shank with respect to the foot ($\vec{\omega}_{shank} - \vec{\omega}_{foot}$) (Elftman, 1939; Robertson et al., 2013; Whittle, 2014). Rotational power terms are summed about all 3 body planes (i.e., sagittal, frontal, transverse) to obtain net 3DOF Ankle power. The foot is typically modeled as a single rigid body, and tracked in 3D space via markers distributed along the hindfoot and forefoot (Buczek et al., 1994; Honert and Zelik, 2016; Robertson et al., 2013; Takahashi et al., 2014; Winter, 1991; Zelik et al., 2015). This estimate fails to fully capture power due to the interaction between the foot and ground.

[A1] $$P_{rot\_ank} = \vec{M}_{ank} \cdot (\vec{\omega}_{shank} - \vec{\omega}_{foot})$$

Ankle: Rotational + Translational Power

6DOF Ankle power (P_{ank}) captures power due to both rotation and translation of the shank relative to the foot (Eqn. A2, Fig. 1C). 6DOF Ankle power is computed by summing 3DOF rotational Ankle power (Eqn. A1) with 3DOF translational Ankle power. The translational power is defined as the dot product of net ankle force on the shank segment ($\vec{F}_{ank}$) and the relative translational velocity of the distal shank with respect to the proximal foot at the ankle (Buczek et al., 1994). Distal shank velocity and proximal foot velocities are defined here as the velocity of the ankle joint center based on rigid-body motion of the shank ($\vec{v}_{ank,shank}$) and foot ($\vec{v}_{ank,foot}$), respectively. Note that this formulation accounts for (i.e., captures) power due to relative segment endpoint displacement (Ebrahimi et al., 2017). The addition of the translational power term helps account for imperfect joint modeling as well as any physical compression or translation that occurs at the ankle (Buczek et al., 1994; Zelik et al., 2015). As with 3DOF Ankle power, the foot is typically modeled as a single rigid body. This estimate also fails to fully capture foot power, due to the interaction between the foot and ground.

[A2] $$P_{ank} = \vec{F}_{ank} \cdot (\vec{v}_{ank,shank} - \vec{v}_{ank,foot}) + P_{rot\_ank}$$

Anklefoot: Ankle + Distal Foot Power

This approach estimates power due to both the ankle and foot, by summing 6DOF Ankle power (summarized above) with Distal Foot power (P_{distal_foot}, Eqn. A3). This combined power estimate ($P_{ank+distal\_foot}$, Eqn. A4) represents power due to motion of the shank relative to the foot (Ankle power) plus power of the foot relative to the ground (Distal

Foot power, Fig. 1D). Both power terms are still based on calculations that assume the foot is a single rigid-body segment. Distal Foot power is sometimes interpreted to reflect deformation of structures within/around the foot, such as compression of the heel pad, foot arches or shoe, and rotation about the metatarsophalangeal joints (see Discussion for why this must be done cautiously and is often not recommended since it only models/captures a portion of foot power). Also note that Distal Foot power (and all other Distal Segment powers) captures power due to any ground deformation or foot slippage relative to the ground. Distal Foot power calculations also assume that mass and inertia distal to the foot segment is negligible (e.g., due to the toes). Distal Foot power is computed as:

[A3] $$P_{distal\_foot} = \vec{F}_{grf} \cdot \left(\vec{v}_{foot} + \vec{\omega}_{foot} \times \vec{r}_{cop/_{foot}}\right) + \vec{M}_{free} \cdot \vec{\omega}_{foot}$$

where $\vec{F}_{grf}$ is the ground reaction force, $\vec{v}_{foot}$ is the velocity of the foot's center-of-mass (COM), $\vec{\omega}_{foot}$ is the angular velocity of the foot, $\vec{r}_{cop/_{foot}}$ is the position of the COP relative to the foot's COM, and $\vec{M}_{free}$ is the free moment (Siegel et al., 1996; Takahashi and Stanhope, 2013; Zelik et al., 2015).

[A4] $$P_{ank+distal\_foot} = P_{ank} + P_{distal\_foot}$$

Anklefoot: AJC + Distal Calcaneus Power

This combined anklefoot power estimate ($P_{ajc+distal\_cal}$) is analogous to the preceding method (Ankle + Distal Foot power), except that it uses calcaneus instead of foot motion. Experimentally, a cluster of markers is placed on the calcaneus (Fig. 2a) and used to estimate motion of this portion of the foot, based on rigid-body assumptions. AJC power (P_{ajc}, Eqn. A5) is defined as the power due to motion (translation and rotation) of the shank relative to the calcaneus. Distal Calcaneus power (P_{distal_cal}, Eqn. A6) then captures power due to motion between the calcaneus and ground, reflecting translation and rotation of various foot structures (e.g., heel pad, foot arch, metatarsophalangeal joints, Takahashi et al., 2017). Again, mass and inertia due to structures distal to the calcaneus (e.g., forefoot, toes) is neglected.

AJC power is define as:

[A5] $$P_{ajc} = \vec{F}_{ank} \cdot \left(\vec{v}_{ank,shank} - \vec{v}_{ank,cal}\right) + \vec{M}_{ank} \cdot (\vec{\omega}_{shank} - \vec{\omega}_{cal})$$

where $\vec{v}_{ank,cal}$ is the ankle joint center velocity based on motion of the calcaneus, and $\vec{\omega}_{cal}$ is angular velocity of the calcaneus. Distal Calcaneus power is then defined as:

[A6] $$P_{distal\_cal} = \vec{F}_{grf} \cdot \left(\vec{v}_{cal} + \vec{\omega}_{cal} \times \vec{r}_{cop/_{cal}}\right) + \vec{M}_{free} \cdot \vec{\omega}_{cal}$$

where $\vec{v}_{cal}$ is the velocity of the calcaneus' COM and $\vec{r}_{cop/_{cal}}$ is the position of the COP relative to the calcaneus' COM.

[A7] $$P_{ajc+distal\_cal} = P_{ajc} + P_{distal\_cal}$$

Although AJC power (shank relative to calcaneus) is unconventional, it is appealing for several reasons. First, ankle power is often studied to understand the soleus, gastrocnemius and/or Achilles tendon; the main sources of plantarflexion power. These calf muscles (via the Achilles tendon) insert directly onto the calcaneus. Thus, this approximation of shank-relative-to-calcaneus power seems more consistent with human anatomy than shank-relative-to-foot power (Ankle power). Second, prior studies indicate that the mid-foot joint undergoes substantial articulation during locomotor tasks like walking (Kelly et al., 2015). If the foot is modeled as a single rigid body (as with Ankle power estimates), then this mid-foot articulation bleeds over into (i.e., appear as) ankle rotation causing ankle angle (and angular velocity, and thus power) to be overestimated (Leardini et al., 2007). Third, the calcaneus is a better approximation of a rigid-body than the entire foot. Again, this is due to motion of the mid-foot and other joints in the foot during walking and other locomotor tasks (Bruening et al., 2012; Kelly et al., 2015; MacWilliams et al., 2003). Fourth, Figure 4 indicates that Distal Calcaneus power results were qualitatively consistent with cadaver studies on the foot arch (e.g., Ker et al., 1987), exhibiting energy storage and return by the foot. In contrast, Distal Foot power estimates are known to be highly inconsistent with these same cadaver studies, exhibiting large energy dissipation in late stance with little or no energy return. In summary, AJC + Distal Calcaneus power provides an appealing, more physiologically-relevant alternative to partition power sources within the body, which is more consistent with *in vitro* evidence of foot function than Ankle + Distal Foot power partitioning. As such, choice of methods can have important implications for how we interpret ankle vs. foot contributions in gait (see Discussion for further details and specific examples).

Anklefoot: Distal Shank Power

Distal Shank power (P_{distal_shank}) provides a lumped estimate of anklefoot power, by directly computing the power due to 6DOF motion of the shank relative to the ground (see Appendix B for detailed derivation). This estimate, described by Takahashi et al. (2012) as originating from a Unified Deformable segment model, has been applied to quantify net power contributions from prosthetic feet (more aptly called anklefeet). It is generally preferable to compute combined anklefoot power for prostheses since they often lack a clearly identifiable ankle joint and a well-defined foot segment that is distinct from the ankle. For prosthetic feet (or other interventions, e.g., anklefoot orthoses) where the ankle and/or foot deviate substantially from the anatomical norm, this Distal Shank power provides a means of capturing net power due to all structures distal to the shank. This approach assumes a rigid-body shank. However, it makes no assumptions about the rigidity or dynamics of the structures distal to the shank, except that these structures have negligible mass and inertia.

[A8] $$P_{distal\_shank} = \vec{F}_{grf} \cdot \left(\vec{v}_{shank} + \vec{\omega}_{shank} \times \vec{r}_{cop/_{shank}}\right) + \vec{M}_{free} \cdot \vec{\omega}_{shank}$$

where $\vec{v}_{shank}$ is the velocity of the shank's COM, and $\vec{r}_{cop/_{shank}}$ is the position of the center-of-pressure (COP) relative to the shank's COM.

Anklefoot: Intersegmental Power

Intersegmental power ($P_{interseg}$) is similar to the Distal Shank power estimate in that it provides a lumped estimate of anklefoot power. In fact, these two power estimates are analytically equivalent when foot mass and inertia are assumed to be negligible (see derivation in Appendix C). The main difference is that Intersegmental power is nominally formulated to account for inertial effects of the foot (Prince et al., 1994), whereas Distal Shank power is not. Intersegmental power can be computed at any arbitrary point on a rigid-body segment (e.g., at the segmental center-of-mass, or at the segment's distal end), and it represents the net power flow into or out of that point. When computed for a point on the shank, Intersegmental power reflects an estimate of the net power flow to/from the combined anklefoot. Similar to Distal Shank power, this Intersegmental analysis is commonly used to analyze prosthetic power (Prince et al., 1994), when anatomically-inspired models of the ankle and foot may not be applicable or appropriate. Of note, the point at which Intersegmental power is computed must be proximal to all prosthetic foot components (e.g., located on the socket or rigid pylon), in order to fully capture prosthetic foot power. Below is an example of Intersegmental power, computed at the ankle joint center (i.e., distal end of the shank). Note, $\vec{F}_{ank}$ in [A9] represents the net force on the shank at the ankle.

[A9] $$P_{interseg} = \vec{F}_{ank} \cdot \vec{v}_{ank} + \vec{M}_{ank} \cdot \vec{\omega}_{shank}$$

where $\vec{v}_{ank}$ is the translational velocity of the ankle joint center.

Appendix B

The purpose of this Appendix is to demonstrate analytically that Distal Shank power provides an estimate of the 6DOF joint power between the shank and the ground.

In generalized form, the 6DOF power at/about a given joint (j), which is located between a proximal segment/body (p) and distal segment/body (d), is defined as:

[B1] $$P_{p/d} = \vec{F}_j \cdot (\vec{v}_{j,p} - \vec{v}_{j,d}) + \vec{M}_j \cdot (\vec{\omega}_p - \vec{\omega}_d)$$

where $\vec{F}_j$ is the net force at the joint on the proximal segment/body, $\vec{v}_{j,p}$ is the estimated velocity of the joint based on proximal segment/body motion, $\vec{v}_{j,d}$ is the estimated velocity of the joint based on distal segment/body motion, $\vec{M}_j$ is the net moment about the joint on the proximal segment/body, $\vec{\omega}_p$ is the angular velocity of the proximal segment/body, and $\vec{\omega}_d$ is the angular velocity of the distal segment/body, based on rigid-body assumptions.

The 6DOF power between the shank and the ground (gnd), both assumed to be rigid, using the ankle joint center to represent the modeled joint, is therefore:

[B2] $$P_{shank/gnd} = \vec{F}_{ank} \cdot (\vec{v}_{ank,shank} - \vec{v}_{ank,gnd}) + \vec{M}_{ank} \cdot (\vec{\omega}_{shank} - \vec{\omega}_{gnd})$$

If we assume negligible foot mass and inertia, then the force and moment balances about the ankle joint center on the shank result in:

[B3] $$\vec{F}_{ank} = \vec{F}_{grf}$$

[B4] $$\vec{M}_{ank} = \vec{M}_{free} + \vec{r}_{cop/ank} \times \vec{F}_{grf}$$

where $\vec{F}_{grf}$ is the ground reaction force measured under the foot, $\vec{M}_{free}$ is the free moment and $\vec{r}_{cop/ank}$ is the position vector from the ankle joint center to the COP under the foot.

Furthermore, if the ground is not translating or rotating in the inertial frame (i.e., the ground is not moving relative to the motion of the Earth), then the velocity and angular velocity of the ground ($\vec{v}_{ank,gnd}$ and $\vec{\omega}_{gnd}$) are zero:

[B5] $$\vec{v}_{ank,gnd} = \vec{\omega}_{gnd} = 0$$

Plugging Eqns. B3-B5 into Eqn. B2 yields:

[B6] $$P_{shank/gnd} = \vec{F}_{grf} \cdot \vec{v}_{ank,shank} + (\vec{M}_{free} + \vec{r}_{cop/ank} \times \vec{F}_{grf}) \cdot \vec{\omega}_{shank}$$

Terms can be rearranged to:

[B7] $$P_{shank/gnd} = \vec{F}_{grf} \cdot \left(\vec{v}_{ank,shank} + \vec{\omega}_{shank} \times \vec{r}_{cop/ank}\right) + \vec{M}_{Free} \cdot \vec{\omega}_{shank}$$

Next, $\vec{v}_{ank,shank}$ and $\vec{r}_{cop/ank}$ can be written using vector addition:

[B8] $$\vec{v}_{ank,shank} = \vec{v}_{shank} + \vec{v}_{ank,shank/shank}$$

[B9] $$\vec{r}_{cop/ank} = \vec{r}_{cop/shank} - \vec{r}_{ank/shank}$$

where $\vec{v}_{shank}$ is the velocity of the shank's COM in the lab reference frame, $\vec{v}_{ank,shank/shank}$ is the velocity of the ankle joint center relative to the shank's COM, $\vec{r}_{cop/shank}$ is the position of the COP relative to the shank's COM, and $\vec{r}_{ank/shank}$ is the position of the ankle joint center with respect to the shank's COM.

Assuming a rigid shank segment, then:

[B10] $$\vec{v}_{ank,shank/shank} = \vec{\omega}_{shank} \times \vec{r}_{ank/shank}$$

Plugging Eqns. B8-B10 into Eqn. B7, then simplifying, yields:

[B11] $$P_{shank/gnd} = \vec{F}_{grf} \cdot \left(\vec{v}_{shank} + \vec{\omega}_{shank} \times \vec{r}_{cop/shank}\right) + \vec{M}_{free} \cdot \vec{\omega}_{shank}$$

This is identical to Distal Shank power (Eqn. A8), demonstrating that Distal Shank power represents 6DOF joint power due to motion (translation and rotation) between the rigid-body shank segment and ground.

Analogous derivations can be performed for any other Distal Segment powers (e.g., Distal Calcaneus power), showing that they are estimates of 6DOF power between the Segment and ground. However, note that the negligible mass and inertia assumption becomes increasingly less valid as one moves up with leg (e.g., Distal Thigh power would be expected to include considerable errors due to neglected inertia). Also note that any power due to ground deformation, or to slippage of the foot relative to the ground, would also be captured by Distal Segment power calculations. Finally, care should be taken in treadmill studies because errors in treadmill belt speed (e.g., actual vs. assumed/programmed speed) can appear in the analysis as relative motion between the segment and ground, resulting in a misestimate of Distal Segment power.

Appendix C

The purpose of this Appendix is to demonstrate that Intersegmental power from Prince et al. (1994) computed at the ankle joint center is analytically equivalent to Distal Shank power, when foot mass and inertia are assumed to be negligible.

Intersegmental power ($P_{interseg}$) at the ankle joint center (ank), is defined as:

[C1] $$P_{interseg} = \vec{F}_{ank} \cdot \vec{v}_{ank} + \vec{M}_{ank} \cdot \vec{\omega}_{shank}$$

where $\vec{F}_{ank}$ is the net force at the ankle on the shank segment, $\vec{v}_{ank}$ is the translational velocity of the ankle, $\vec{M}_{ank}$ is the net moment about the ankle on the shank segment, and $\vec{\omega}_{shank}$ is the angular velocity of the shank based on rigid-body assumptions.

Assuming that foot mass and inertia are negligible, the force and moment balances about the ankle joint on the shank segment result in:

[C2] $$\vec{F}_{ank} = \vec{F}_{grf}$$

[C3] $$\vec{M}_{ank} = \vec{M}_{free} + \vec{r}_{cop/ank} \times \vec{F}_{grf}$$

where $\vec{F}_{grf}$ is the ground reaction force measured under the foot, $\vec{M}_{free}$ is the free moment and $\vec{r}_{cop/ank}$ is the position vector from the ankle joint center to the COP under the foot.

Plugging Eqns. C2-C3 into Eqn. C1, then simplifying yields:

[C4] $$P_{interseg} = \vec{F}_{grf} \cdot \left(\vec{v}_{ank} + \vec{\omega}_{shank} \times \vec{r}_{cop/ank}\right) + \vec{M}_{free} \cdot \vec{\omega}_{shank}$$

Next, the velocity of the ankle joint center ($\vec{v}_{ank}$) and the position of the COP relative to the ankle ($\vec{r}_{cop/ank}$), in the lab reference frame, can be written using vector addition:

[C5] $$\vec{v}_{ank} = \vec{v}_{shank} + \vec{v}_{ank/shank}$$

[C6] $$\vec{r}_{cop/ank} = \vec{r}_{cop/shank} - \vec{r}_{ank/shank}$$

where $\vec{v}_{shank}$ is the velocity of the shank's COM in the lab reference frame, $\vec{v}_{ank/shank}$ is the velocity of the ankle joint center relative to the shank's COM, $\vec{r}_{cop/shank}$ is the position of the COP relative to the shank's COM, and $\vec{r}_{ank/shank}$ is the position of the ankle joint center relative to the shank's COM.

Assuming a rigid shank segment, then:

[C7] $$\vec{v}_{ank/_{shank}} = \vec{\omega}_{shank} \times \vec{r}_{ank/_{shank}}$$

Plugging Eqns. C5-C7 into Eqn. C4, and simplifying, then yields:

[C8] $$P_{interseg} = \vec{F}_{grf} \cdot \left(\vec{v}_{shank} + \vec{\omega}_{shank} \times \vec{r}_{cop/_{shank}} \right) + \vec{M}_{free} \cdot \vec{\omega}_{shank}$$

Eqn. C8 is identical to the Distal Shank power calculation in Eqn. A8.

Note that Takahashi et al. (2012) previously demonstrated that Ankle + Distal Foot power ($P_{ank+distal\_foot}$, Eqn. A4) is also analytically equivalent to Distal Shank power when distal foot mass and inertia are negligible. Finally, see Figure 1 for conceptual visualizations of these anklefoot power equivalencies.

Supplementary Material References

Bruening, D.A., Cooney, K.M., Buczek, F.L., 2012. Analysis of a kinetic multi-segment foot model part II: Kinetics and clinical implications. Gait Posture 35, 535–540. https://doi.org/10.1016/j.gaitpost.2011.11.012

Buczek, F.L., Kepple, T.M., Siegel, K.L., Stanhope, S.J., 1994. Translational and rotational joint power terms in a six degree-of-freedom model of the normal ankle complex. J. Biomech. 27, 1447–1457. https://doi.org/10.1016/0021-9290(94)90194-5

Ebrahimi, A., Collins, J.D., Kepple, T.M., Takahashi, K.Z., Higginson, J.S., Stanhope, S.J., 2017. A mathematical analysis to address the 6 degree-of-freedom segmental power imbalance. J. Biomech. https://doi.org/10.1016/j.jbiomech.2017.10.034

Elftman, H., 1939. Forces and energy changes in the leg during walking. Am. J. Physiol. Content 125, 339–356.

Honert, E.C., Zelik, K.E., 2016. Inferring Muscle-Tendon Unit Power from Ankle Joint Power during the Push-Off Phase of Human Walking: Insights from a Multiarticular EMG-Driven Model. PLOS ONE 11, e0163169. https://doi.org/10.1371/journal.pone.0163169

Kelly, L.A., Lichtwark, G., Cresswell, A.G., 2015. Active regulation of longitudinal arch compression and recoil during walking and running. J. R. Soc. Interface 12, 20141076. https://doi.org/10.1098/rsif.2014.1076

Ker, R.F., Bennett, M.B., Bibby, S.R., Kester, R.C., Alexander, R.M., 1987. The spring in the arch of the human foot. Nature 325, 147–9.

Leardini, A., Benedetti, M.G., Berti, L., Bettinelli, D., Nativo, R., Giannini, S., 2007. Rear-foot, mid-foot and fore-foot motion during the stance phase of gait. Gait Posture 25, 453–462. https://doi.org/10.1016/j.gaitpost.2006.05.017

MacWilliams, B.A., Cowley, M., Nicholson, D.E., 2003. Foot kinematics and kinetics during adolescent gait. Gait Posture 17, 214–224. https://doi.org/10.1016/S0966-6362(02)00103-0

Prince, F., Winter, D.A., Sjonnesen, G., Wheeldon, R.K., 1994. A new technique for the calculation of the energy stored, dissipated, and recovered in different ankle-foot prostheses. Rehabil. Eng. IEEE Trans. On 2, 247–255.

Robertson, D.G.E., Caldwell, G.E., Hamill, J., Kamen, G., Whittlesey, 2013. Research Methods In Biomechanics 2nd Edition. Human Kinetics.

Siegel, K.L., Kepple, T.M., Caldwell, G.E., 1996. Improved agreement of foot segmental power and rate of energy change during gait: Inclusion of distal power terms and use of three-dimensional models. J. Biomech. 29, 823–827. https://doi.org/10.1016/0021-9290(96)83336-7

Takahashi, K.Z., Horne, J.R., Stanhope, S.J., 2014. Comparison of mechanical energy profiles of passive and active below-knee prostheses: A case study. Prosthet. Orthot. Int. 0309364613513298. https://doi.org/10.1177/0309364613513298

Takahashi, K.Z., Kepple, T.M., Stanhope, S.J., 2012. A unified deformable (UD) segment model for quantifying total power of anatomical and prosthetic below-knee structures during stance in gait. J. Biomech. 45, 2662–2667. https://doi.org/10.1016/j.jbiomech.2012.08.017

Takahashi, K.Z., Stanhope, S.J., 2013. Mechanical energy profiles of the combined ankle–foot system in normal gait: Insights for prosthetic designs. Gait Posture. https://doi.org/10.1016/j.gaitpost.2013.04.002

Takahashi, K.Z., Worster, K., Bruening, D.A., 2017. Energy neutral: the human foot and ankle subsections combine to produce near zero net mechanical work during walking. Sci. Rep. 7, 15404. https://doi.org/10.1038/s41598-017-15218-7

Whittle, M.W., 2014. Gait Analysis: An Introduction. Butterworth-Heinemann.

Winter, D.A., 1991. The biomechanics and motor control of human gait: normal, elderly and pathological. University of waterloo press Waterloo.

Zelik, K.E., Takahashi, K.Z., Sawicki, G.S., 2015. Six degree-of-freedom analysis of hip, knee, ankle and foot provides updated understanding of biomechanical work during human walking. J. Exp. Biol. 218, 876–886.