Supplementary Materials for

Species coexistence through simultaneous fluctuation-dependent mechanisms

Andrew D. Letten,[a,b*] Manpreet Dhami,[a,c] Po-Ju Ke,[a] & Tadashi Fukami[a]

[a]Department of Biology, Stanford University, Stanford, California, 94305-5020, USA
[b]University of Canterbury, Christchurch, New Zealand
[c]Biodiversity and Conservation, Landcare Research, Lincoln, New Zealand

[*]To whom correspondence should be addressed; E-mail: andrewletten@gmail.com

This PDF file includes:

Figs. S1 to S12
Tables S1 to S3
Supplementary text

www.pnas.org/cgi/doi/10.1073/pnas.1801846115

Supplementary figures

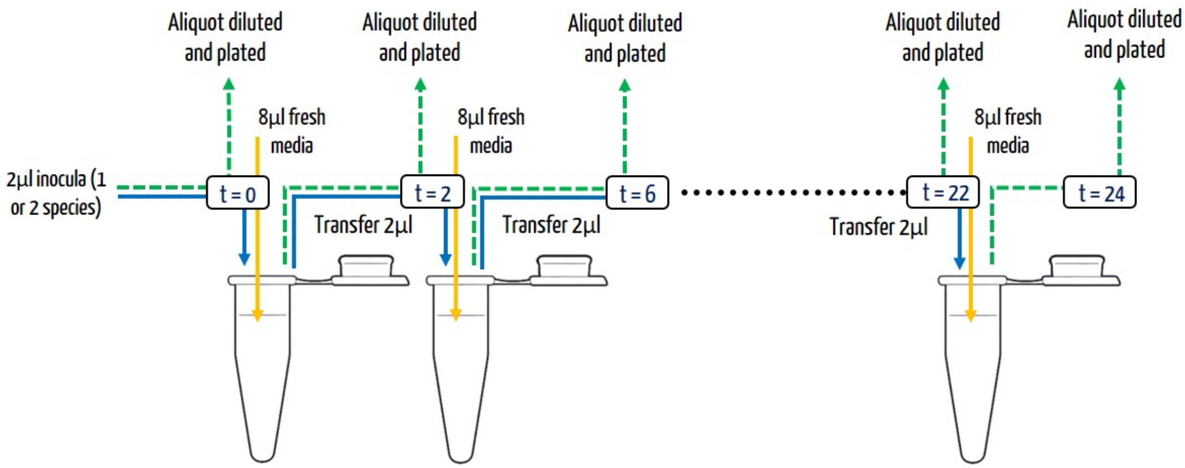

Figure S1: Illustration of the design of mixed- and mono-culture simulations and experiments.

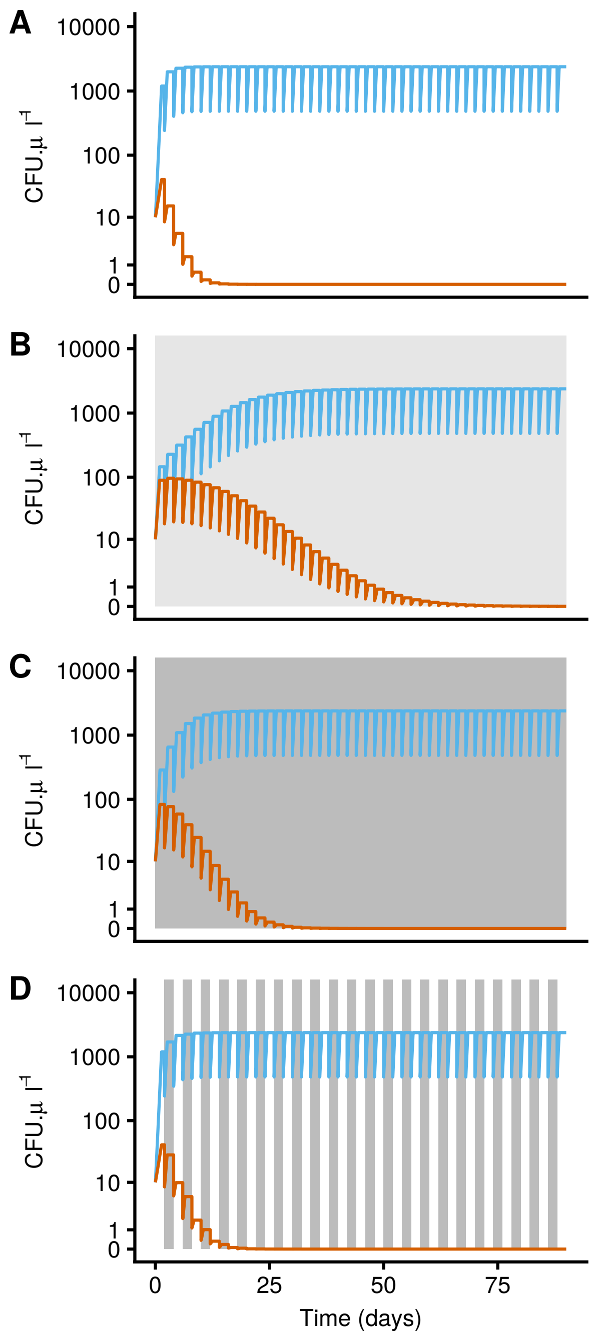

Figure S2: Simulated resource competition between *M. reukaufii* (blue) and *M. gruessii* (dark orange) at constant 10% (A), 30% (B), 50% (C) and fluctuating 10-50% (D) sucrose.

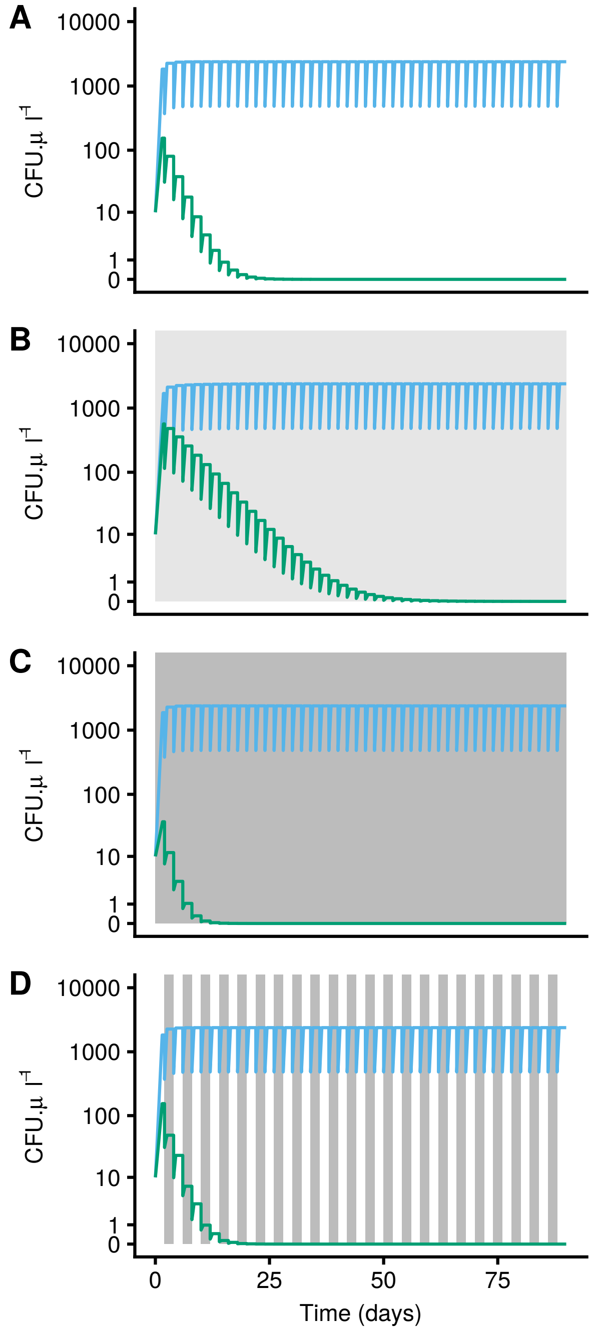

Figure S3: Simulated resource competition between *M. reukaufii* (blue) and *S. bombicola* (green) at constant 10% (A), 30% (B), 50% (C) and fluctuating 10-50% (D) sucrose.

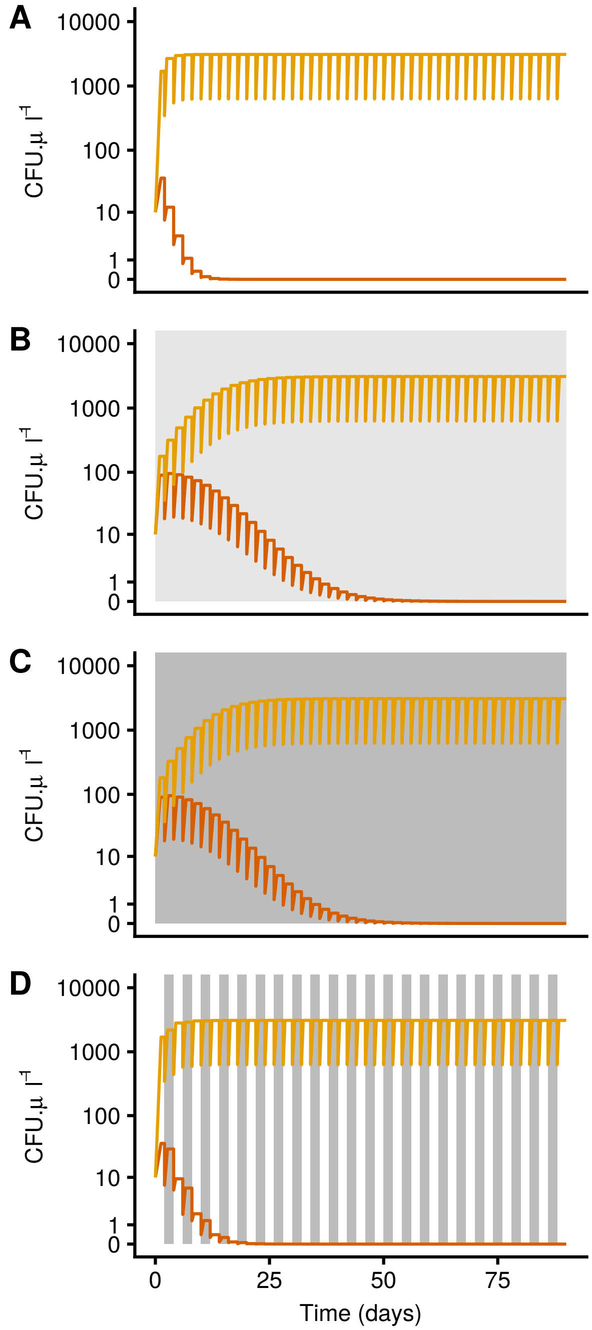

Figure S4: Simulated resource competition between *M. gruessii* (dark orange) and *M. koreensis* (light orange) at constant 10% (A), 30% (B), 50% (C) and fluctuating 10-50% (D) sucrose.

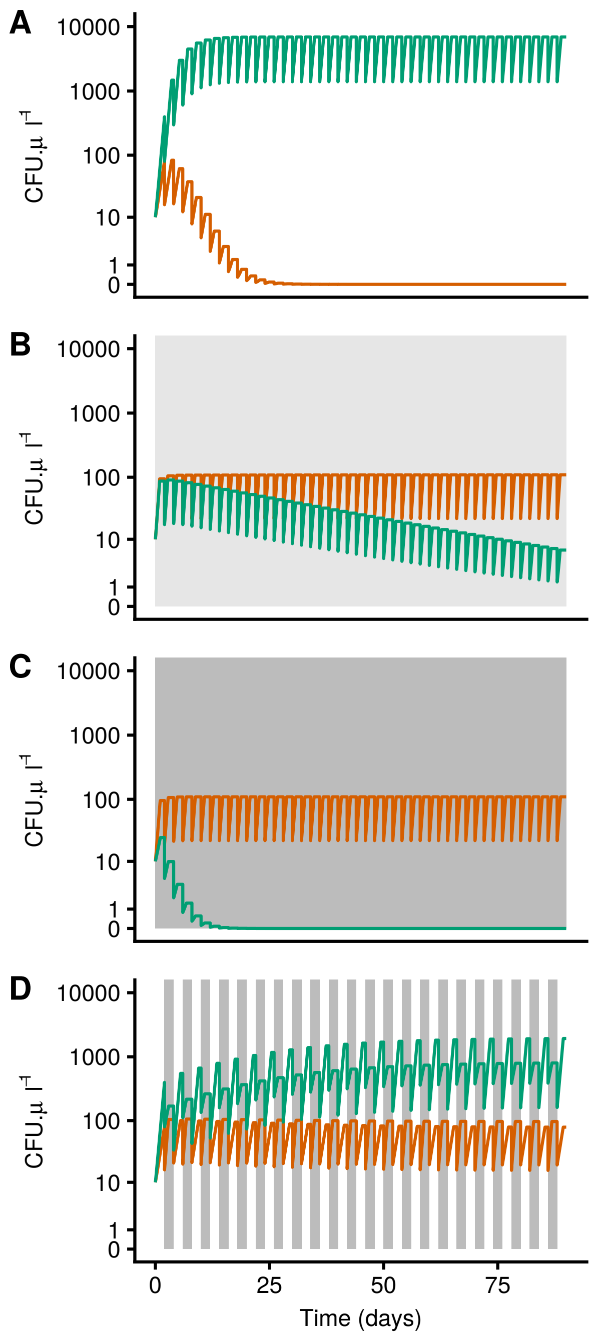

Figure S5: Simulated resource competition between *M. gruessii* (dark orange) and *S. bombicola* (green) at constant 10% (A), 30% (B), 50% (C) and fluctuating 10-50% (D) sucrose.

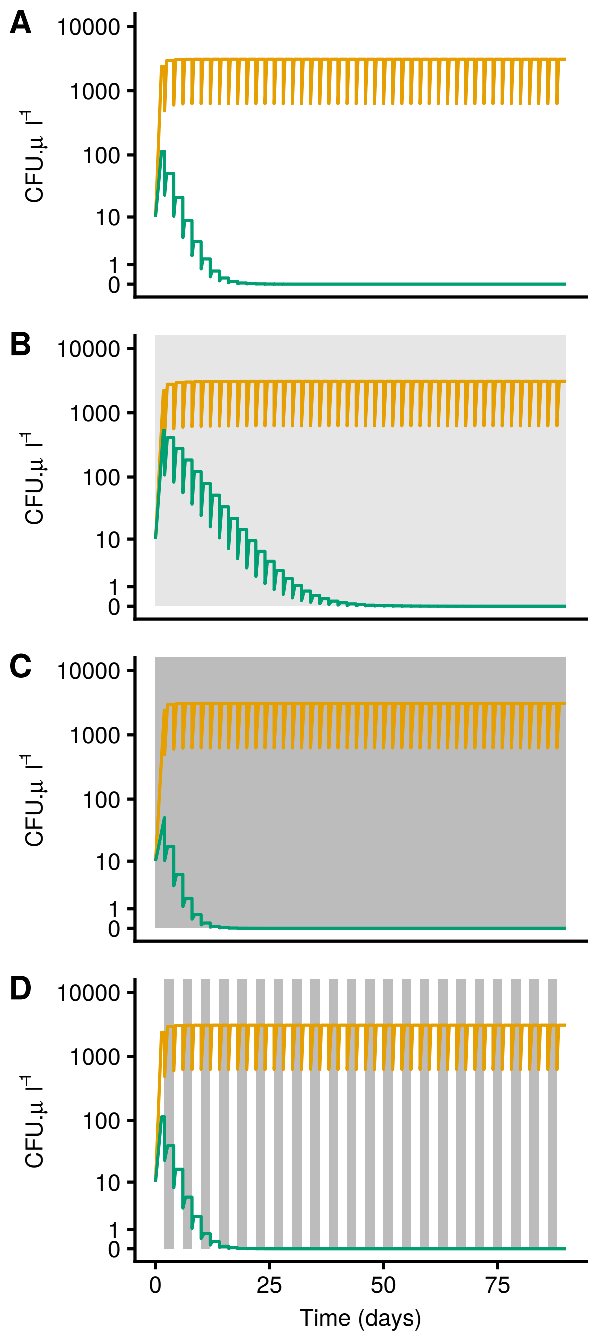

Figure S6: Simulated resource competition between *M. koreensis* (light orange) and *S. bombicola* (green) at constant 10% (A), 30% (B), 50% (C) and fluctuating 10-50% (D) sucrose.

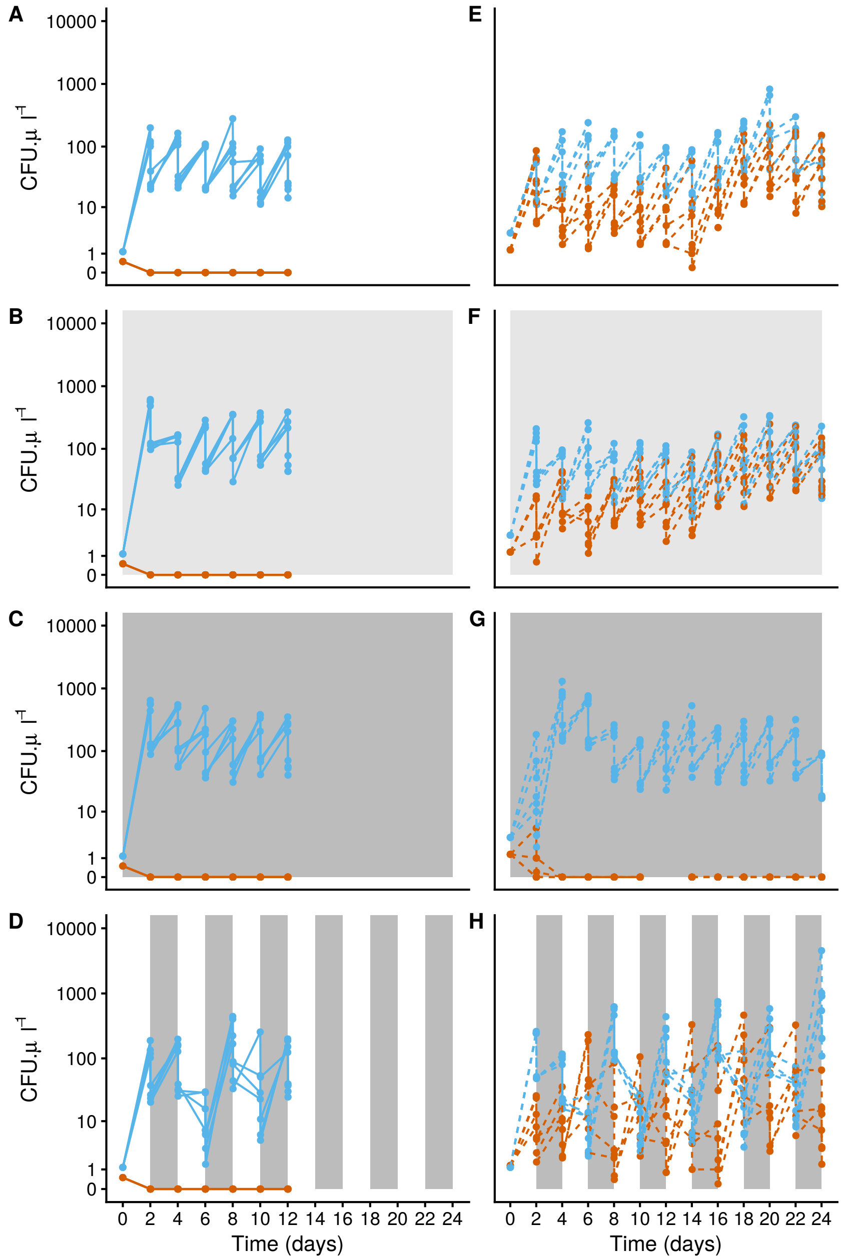

Figure S7: Mixed (solid lines) and monoculture (dashed lines) times-series for *M. gruessii* (dark orange) and *M. reukaufii* (blue) at constant 10% (A,E), 30% (B,F), 50% (C,G) and fluctuating 10-50% (D,H) sucrose. Due to rapid exclusion of *M. gruessii*, mixed culture assays were terminated after 12 days.

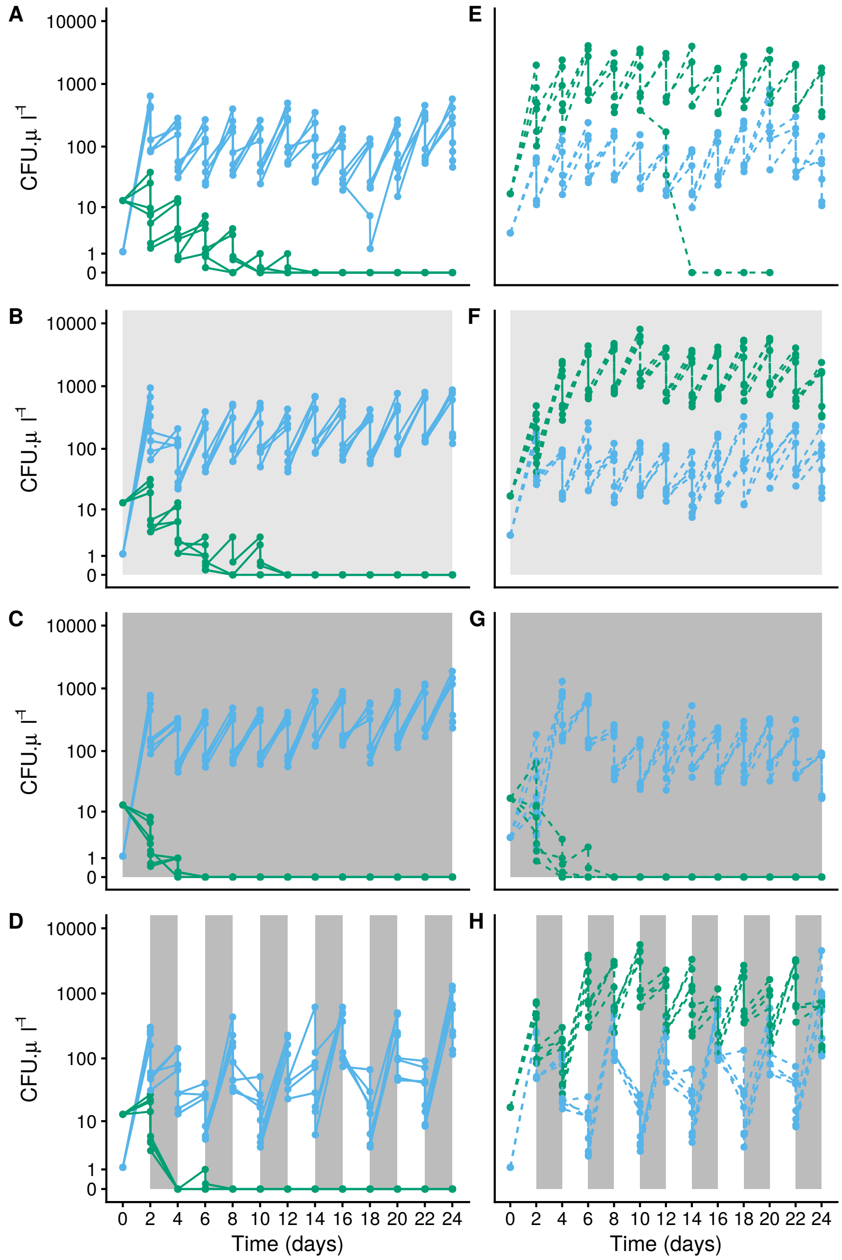

Figure S8: Mixed (solid lines) and monoculture (dashed lines) times-series for *S. bombicola* (green) and *M. reukaufii* (blue) at constant 10% (A,E), 30% (B,F), 50% (C,G) and fluctuating 10-50% (D,H) sucrose.

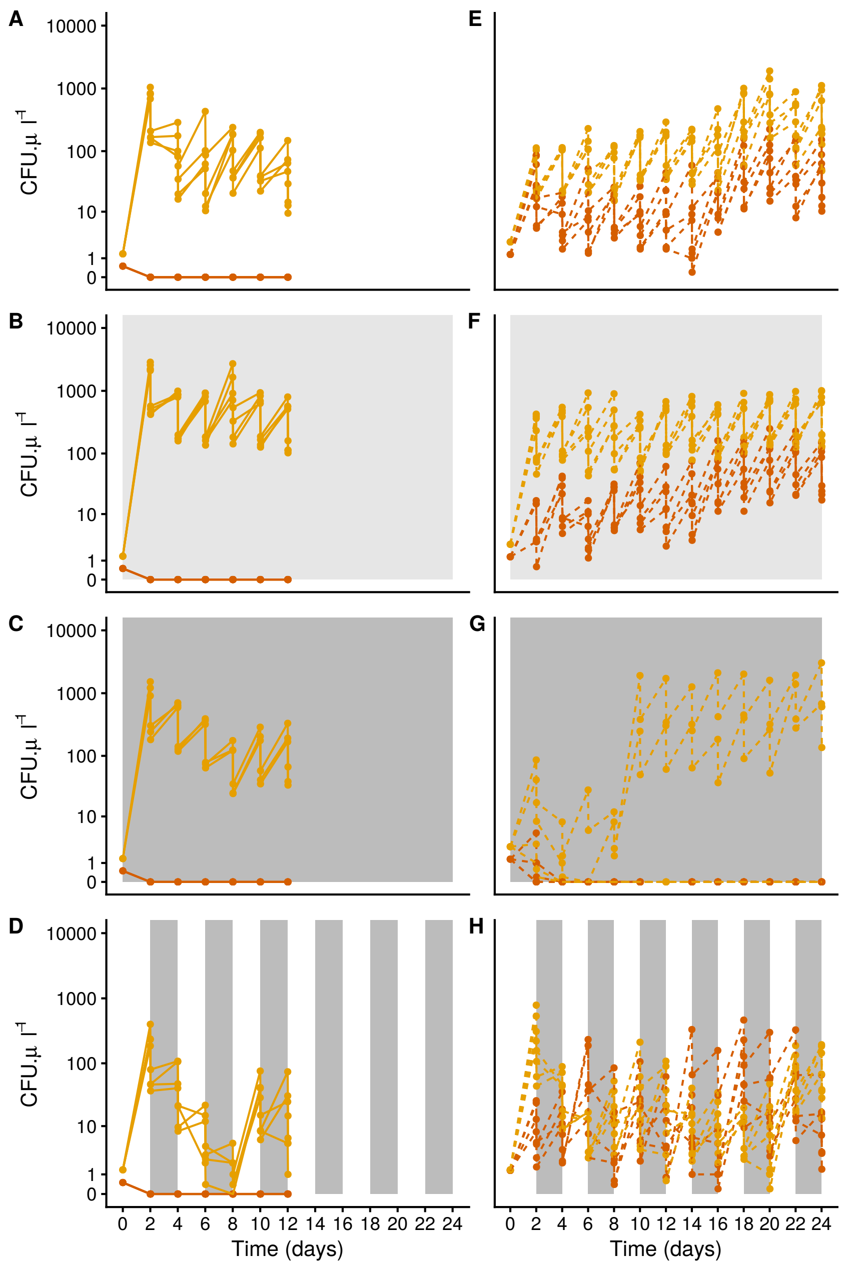

Figure S9: Mixed (solid lines) and monoculture (dashed lines) times-series for *M. gruessii* (dark orange) and *M. koreensis* (light orange) at constant 10% (A,E), 30% (B,F), 50% (C,G) and fluctuating 10-50% (D,H) sucrose. Due to rapid exclusion of *M. gruessii*, mixed culture assays were terminated after 12 days.

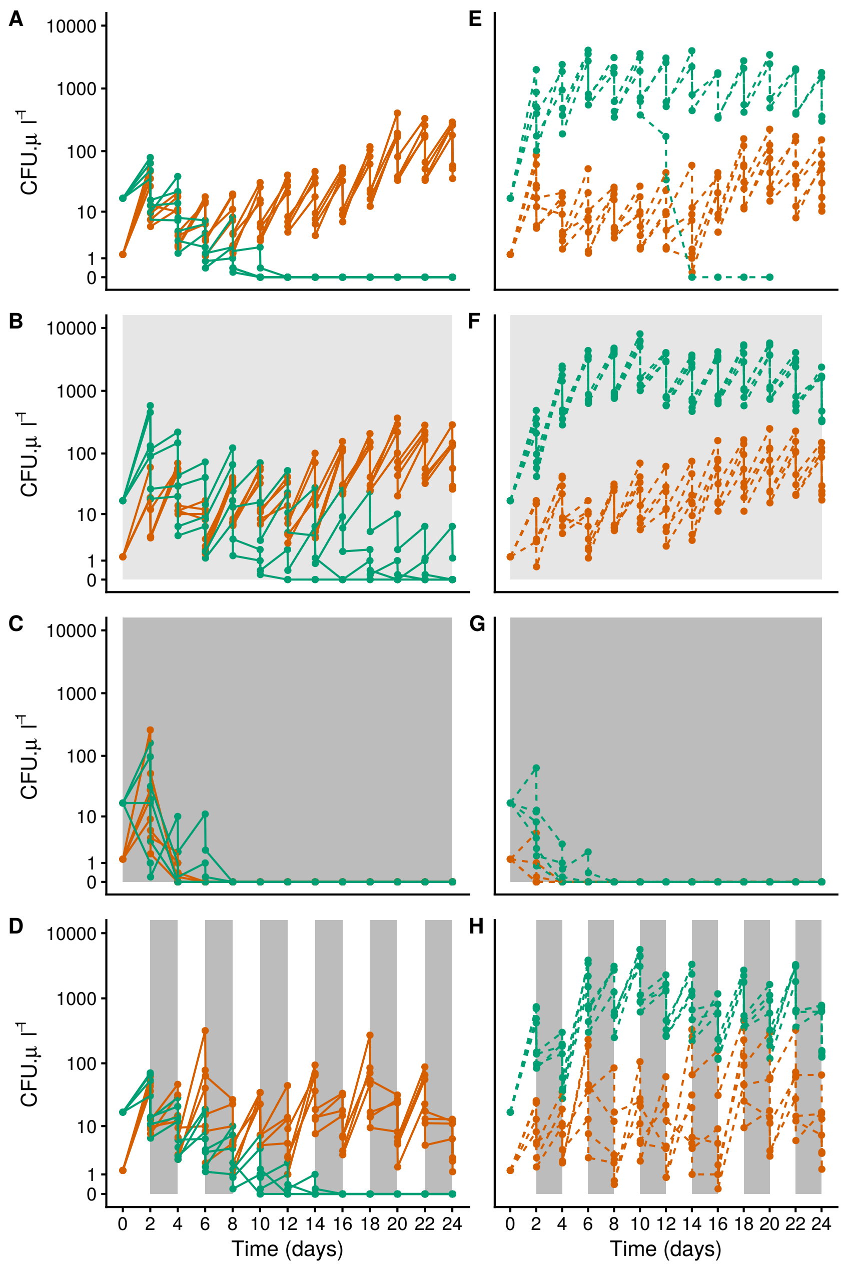

Figure S10: Mixed (solid lines) and monoculture (dashed lines) times-series for *S. bombicola* (green) and *M. gruessii* (dark orange) at constant 10% (A,E), 30% (B,F), 50% (C,G) and fluctuating 10-50% (D,H) sucrose.

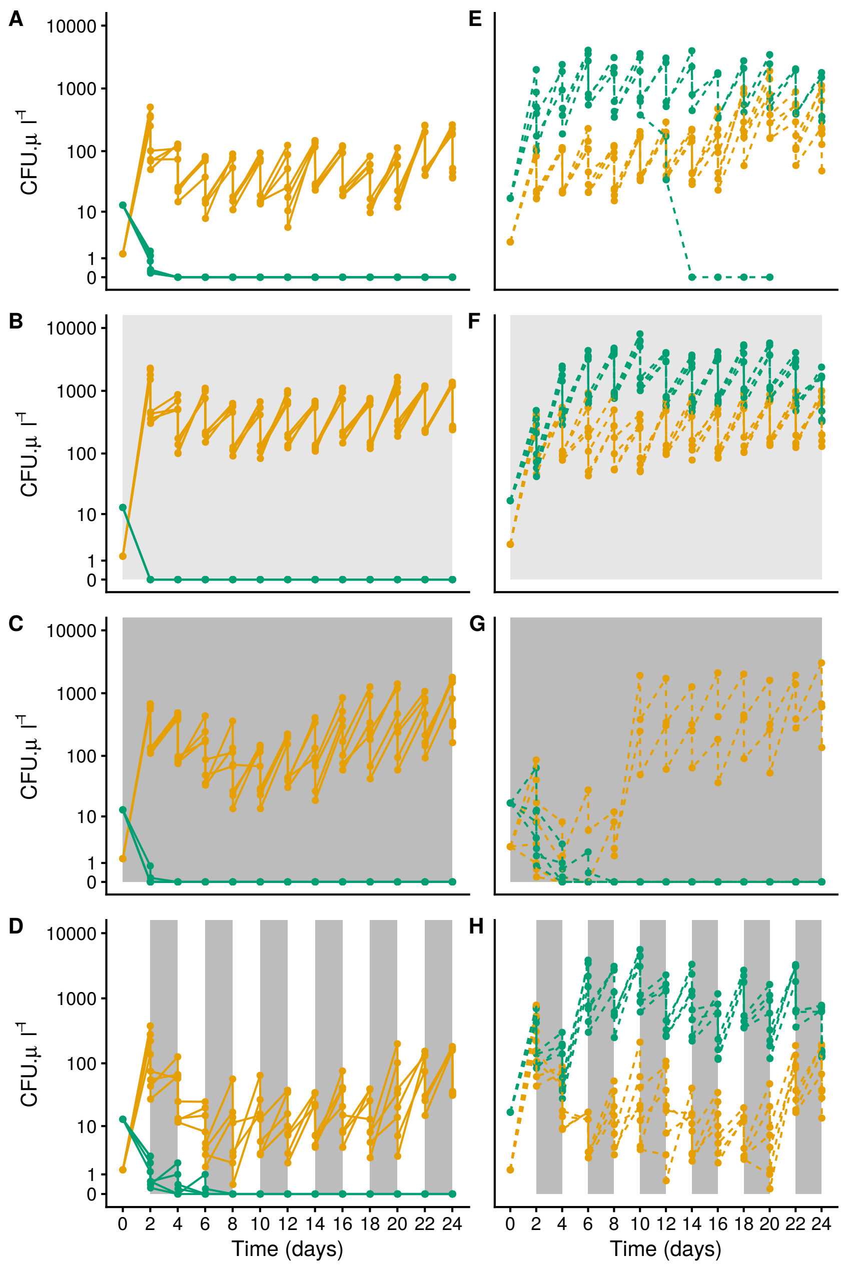

Figure S11: Mixed (solid lines) and monoculture (dashed lines) times-series for *S. bombicola* (green) and *M. koreensis* (light orange) at constant 10% (A,E), 30% (B,F), 50% (C,G) and fluctuating 10-50% (D,H) sucrose.

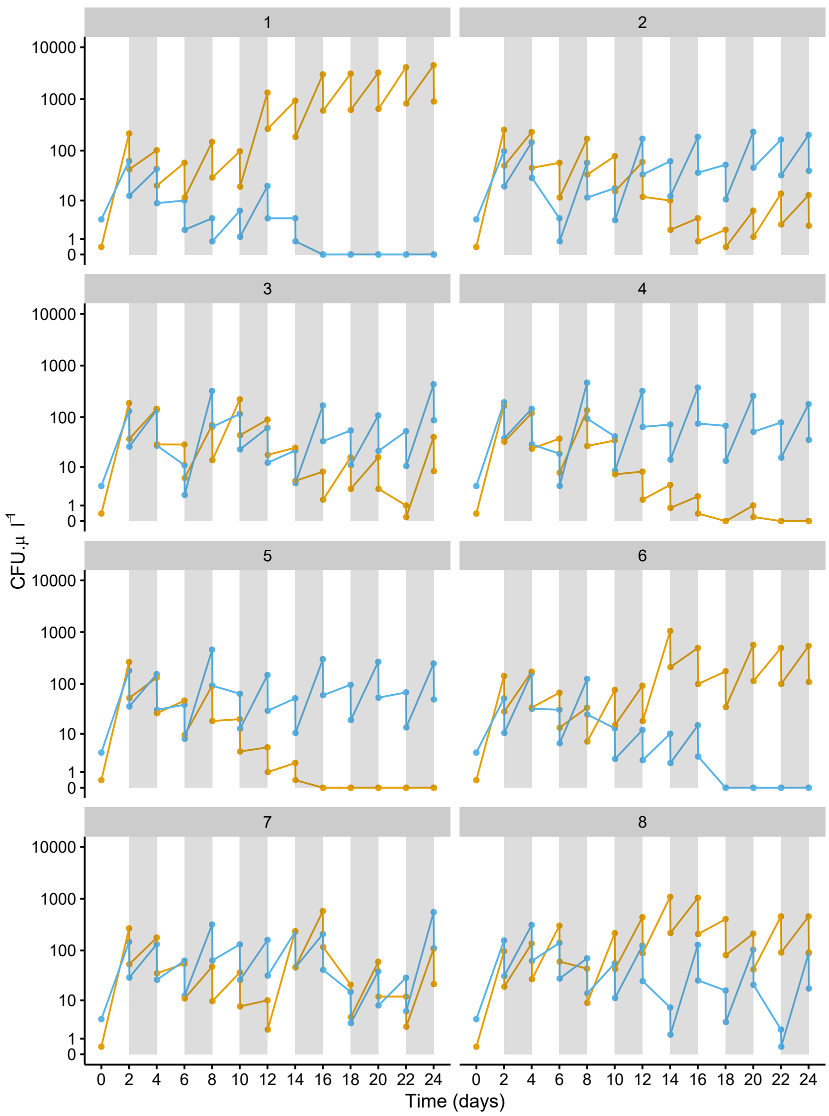

Figure S12: Replicated experiment. Mixed culture times-series for *M. koreensis* (light orange) and *M. reukaufii* (blue) under fluctuating 10-50% (D) sucrose.

Supplementary tables

Table S1: Model parameters and R^* for each species at each sucrose concentration ($R^* = \frac{DK}{\mu_{max}-D}$, where $D = 0.03352$ hr^{-1}).

Species	Sucrose (%)	K (mM)	μ_{max} (hr^{-1})	Q (mM)	R^*
M. koreensis	10	0.0058	0.1741	0.0009	0.0014
	30	0.0029	0.1305	0.0009	0.0010
	50	0.0365	0.1219	0.0009	0.0140
M. reukaufii	10	0.0047	0.1469	0.0011	0.0014
	30	0.0067	0.1225	0.0011	0.0025
	50	0.0084	0.1409	0.0011	0.0026
S. bombicola	10	0.0048	0.0772	0.0004	0.0037
	30	0.0083	0.0966	0.0004	0.0044
	50	0.0021	0.0349	0.0004	0.0526
M. gruessii	10	0.0169	0.0442	0.0245	0.0530
	30	0.0003	0.0992	0.0245	0.0002
	50	0.0426	0.0930	0.0245	0.0240

Table S2: Contribution of ΔI and ΔN to invader growth rate for all species pairs in all treatments, alongside simulation predictions and experimental outcomes. Experimental results inconsistent with model predictions are bolded. ΔI boxes greyed-out under constant treatments to indicate there is no storage effect in the absence of environmental fluctuations.

Treatment	Species pair	r_{inv}	ΔI	ΔN	Predicted outcome	Observed outcome
Fluctuating	Mr	0.00073	0.00092	-0.00092	coexist	coexist*
	Mk	0.000065	0	0.0013		
	Mr	0.05	0.0054	-0.032	Mg excluded	Mg excluded
	Mg	-0.017	0.0011	0.06		
	Mr	0.065	0.0048	-0.027	Sb excluded	Sb excluded
	Sb	-0.021	-0.00091	0.065		
	Mk	0.055	0.01	-0.033	Mg excluded	Mg excluded
	Mg	-0.017	0.0021	0.059		
	Mk	0.06	-0.00074	-0.029	Sb excluded	Sb excluded
	Sb	-0.021	0.0000074	0.064		
	Mg	0.018	0.011	-0.0037	coexist	**Sb excluded**
	Sb	0.0029	0.0088	0.0043		
Constant 10	Mr	-0.005		0.021	Mr excluded	Mr excluded
	Mk	0.006		-0.02		
	Mr	0.08		-0.023	Mg excluded	Mg excluded
	Mg	-0.02		0.08		
	Mr	0.03		-0.039	Sb excluded	Sb excluded
	Sb	-0.016		0.053		
	Mk	0.1		-0.029	Mg excluded	Mg excluded
	Mg	-0.025		0.1		
	Mk	0.042		-0.054	Sb excluded	Sb excluded
	Sb	-0.019		0.076		
	Mg	-0.015		0.019	Mg excluded	**Sb excluded**
	Sb	0.026		-0.007		
Constant 30	Mr	-0.0024		0.007	Mr excluded	Mr excluded
	Mk	0.0023		-0.0068		
	Mr	0.0073		-0.014	Mg excluded	Mg excluded
	Mg	-0.0061		0.015		
	Mr	0.0089		-0.017	Sb excluded	Sb excluded
	Sb	-0.0073		0.019		
	Mk	0.01		-0.02	Mg excluded	Mg excluded
	Mg	-0.008		0.022		
	Mk	0.012		-0.023	Sb excluded	Sb excluded

	Sb	-0.0091		0.026		
	Mg Sb	0.0014 -0.0014		-0.0026 0.0029	Sb excluded	Sb excluded
Constant 50	Mr Mk	0.0068 -0.0057		-0.02 0.022	Mk excluded	**coexist**†
	Mr Mg	0.02 -0.012		-0.033 0.043	Mg excluded	Mg excluded
	Mr Sb	0.1 -0.025		-0.0032 0.078	Sb excluded	Sb excluded
	Mk Mg	0.011 -0.0083		-0.017 0.02	Mg excluded	Mg excluded
	Mk Sb	0.081 -0.023		-0.0038 0.053	Sb excluded	Sb excluded
	Mg Sb	0.053 -0.02		-0.0028 0.031	Sb excluded	**Non-persistence**

* coexistence in 3 out of 4 replicates in original experiment and 4 out of 8 replicates in follow-up experiment; † coexistence in 3 out of 4 replicates.

Table S3: Scaling factor adjusted contribution of ΔI and ΔN to invader growth rate for all species pairs in all treatments. ΔI boxes greyed-out under constant treatments to indicate there is no storage effect in the absence of environmental fluctuations.

Treatment	Species pair	r_{inv}	q_{ir}	$adjusted\text{-}\Delta I$	$adjusted\text{-}\Delta N$
Fluctuating	Mr	0.00073	0.33	0.00021	-0.072
	Mk	0.000065	3	(0) 2.4e-05	0.22
	Mr	0.05	0.49	0.0029	-0.047
	Mg	-0.017	1.9	0.0013	0.15
	Mr	0.065	4.8	0.026	0.037
	Sb	-0.021	0.21	-0.001	-0.019
	Mk	0.055	1.6	0.013	-0.018
	Mg	-0.017	0.62	0.0017	0.019
	Mk	0.06	16	0.08	0.22
	Sb	-0.021	0.069	-0.00098	-0.034
	Mg	0.018	9.9	0.061	0.15
	Sb	0.0029	0.1	0.0045	-0.02
Constant 10	Mr	-0.005	0.69		-0.021
	Mk	0.006	2.9		0.19
	Mr	0.08	0.95		-0.02
	Mg	-0.02	2		0.19
	Mr	0.03	1.9		-0.0016
	Sb	-0.016	1.1		0.06
	Mk	0.1	1.4		-0.025
	Mg	-0.025	1.4		0.15
	Mk	0.042	2.71		0.02
	Sb	-0.019	0.74		0.019
	Mg	-0.015	1.9		0.059
	Sb	0.026	1		-0.007
Constant 30	Mr	-0.0024	2.1		0.12
	Mk	0.0023	0.94		-0.012
	Mr	0.0073	25		1.53
	Mg	-0.0061	0.082		-0.065
	Mr	0.0089	1		-0.016
	Sb	-0.0073	2		0.1
	Mk	0.01	11		0.67
	Mg	-0.008	0.18		-0.057
	Mk	0.012	0.48		-0.055
	Sb	-0.0091	4.2		0.33
	Mg	0.0014	0.042		-0.063

	Sb	-0.0014	48		3.2
Constant 50	Mr	0.0068	0.31		-0.074
	Mk	-0.0057	6.4		0.58
	Mr	0.02	0.34		-0.067
	Mg	-0.012	5.5		0.51
	Mr	0.1	16		0.0096
	Sb	-0.025	0.13		-0.013
	Mk	0.011	1.1		-0.0094
	Mg	-0.0083	1.7		0.076
	Mk	0.081	57		0.045
	Sb	-0.023	0.04		-0.021
	Mg	0.053	50		0.04
	Sb	-0.02	0.044		-0.019

Supplementary text

Derivation of scaling factors

In this section we describe the derivation of the scaling factors, q_{ir}. As in [1], we start by defining the competitive effects experienced by each species j when species i is invading, $\mathscr{C}_{j\setminus i}$, as:

$$\mathscr{C}_{j\setminus i} = -r_j(E_j^*, C_{j\setminus i}). \tag{S1}$$

Here, E_j^* is the baseline environment and is typically set as the central value of $E_j(t)$. Then, the scaling factors, q_{ir}, are defined as follows:

$$q_{ir} = \frac{\partial \mathscr{C}_{i\setminus i}}{\partial \mathscr{C}_{r\setminus i}}. \tag{S2}$$

For our model, if we set E_j^* as the mean value $\overline{\mu_{max_j}}$, we arrive at the following expression:

$$\mathscr{C}_{j\setminus i} = D - E_j^*/C_j = D - \frac{\overline{\mu_{max_j}} R}{K_j + R}. \tag{S3}$$

To derive q_{ir} for our model, we turn to equation (SI.16) in [2] since an explicit formula for $\mathscr{C}_{i\setminus i}$ as a function of $\mathscr{C}_{r\setminus i}$ cannot be easily found. With this approach, we derived q_{ir} as follows:

$$q_{ir} = \frac{\partial \mathscr{C}_{i\setminus i}}{\partial \mathscr{C}_{r\setminus i}} = \frac{\partial \mathscr{C}_{i\setminus i}/\partial R}{\partial \mathscr{C}_{r\setminus i}/\partial R} = \frac{\overline{\mu_{max_i}}\,\overline{K_i}}{\overline{\mu_{max_r}}\,\overline{K_r}} \times \frac{(\overline{K_r} + \overline{R_r})^2}{(\overline{K_i} + \overline{R_r})^2}. \tag{S4}$$

We used the mean value of K_j as the half-saturation constant varies with environmental variability in our model. More importantly, we used the mean resource level set by the resident, $\overline{R_r}$, instead of its respective R^*. This is because in our system the pulsing of resources set by the resident results in a mean resource level set by the resident that is slightly greater than expected based on each species R^*. According to the small-variance assumption of [1], this choice should be immaterial assuming $\overline{R_r}$ and R^* differ by $O(\sigma^2)$. Nevertheless, as we found in this work, we expect this assumption to break down in many empirical contexts. We refer to this derivation of the scaling factors as semi-analytical as the mean resource level set by the resident is obtained through numerical simulation.

References

[1] Chesson P (1994) Multispecies competition in variable environments. *Theoretical Population Biology* 45(3):227–276.

[2] Ellner SP, Snyder RE, Adler PB (2016) How to quantify the temporal storage effect using simulations instead of math. *Ecology Letters* 19(11):1333–1342.