Supplementary Information for

Excitation and propagation of surface plasmon polaritons on a non-structured surface with a permittivity gradient

Xi Wang[1], Yang Deng[1], Qitong Li[1], Yijing Huang[1], Zilun Gong[1,2], Kyle Tom[1,3] and Jie Yao[1,3]

[1]Department of Materials Science and Engineering, University of California, Berkeley, California 94720, USA

[2]Environmental Energy Technologies Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

[3]Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

yaojie@berkeley.edu

Supplemental 1:

We started from solving the curl equations of Maxwell's equations by considering harmonic time dependence $e^{-i\omega t}$ and only one set of self-consistent solutions for a TM polarized (H_x=H_z=0, E_y=0) wave.

$$\begin{cases} \dfrac{\partial E_x}{\partial z} - \dfrac{\partial E_z}{\partial x} = i\omega\mu_0 H_y \\ \dfrac{\partial H_z}{\partial y} - \dfrac{\partial H_y}{\partial z} = -i\omega\varepsilon_0\varepsilon E_x \\ \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = -i\omega\varepsilon_0\varepsilon E_z \end{cases} \tag{1}$$

where ω is the angular frequency of the wave, μ_0 and ε_0 are the vacuum permeability and permittivity respectively, and ε is the relative permittivity. Applying a condition of homogeneous properties along the y direction, we can get:

$$\begin{cases} \left[\dfrac{\partial^2}{\partial z^2} + \varepsilon(x)\dfrac{\partial}{\partial x}(\dfrac{1}{\varepsilon(x)}\dfrac{\partial}{\partial x}) + k_0^2\varepsilon(x)\right] H_y = 0 \\ E_x = -i\dfrac{1}{\omega\varepsilon_0\varepsilon}\dfrac{\partial H_y}{\partial z} \\ E_z = i\dfrac{1}{\omega\varepsilon_0\varepsilon}\dfrac{\partial H_y}{\partial x} \end{cases} \tag{2}$$

where k_0=ω/c is the free space wave vector.

Since materials are homogeneous along the z direction in each half space (z>0 and z<0 respectively), we can decouple H_y as:

$$\begin{cases} H_y = G(x)e^{-k_{\text{zd}}(x)z}, z > 0, \text{Re}(k_{\text{zd}}(x)) > 0 \\ H_y = G(x)e^{k_{\text{zm}}(x)z}, z < 0, \text{Re}(k_{\text{zm}}(x)) > 0 \end{cases} \tag{3}$$

where $G(x) = H_y(x, z = 0)$ describes the magnetic field at the interface; and k_{zd} and k_{zm} are the decay factors of the SPP in air and in the GNM, respectively.

Thus, E_x can be rewritten as:

$$E_x = -i\frac{1}{\omega\varepsilon_0\varepsilon}\frac{\partial H_y}{\partial z} = \begin{cases} i\dfrac{k_{\mathrm{zd}}(x)}{\omega\varepsilon_0\varepsilon_{\mathrm{d}}}G(x)e^{-k_{\mathrm{zd}}(x)z}, z>0 \\ -i\dfrac{k_{\mathrm{zm}}(x)}{\omega\varepsilon_0\varepsilon_{\mathrm{m}}(x)}G(x)e^{k_{\mathrm{zm}}(x)z}, z<0 \end{cases} \tag{4}$$

At the interface (z=0), by combining Eq. (1) and (2), we get:

$$\begin{cases} \dfrac{\partial^2 G(x)}{\partial x^2} - \dfrac{\varepsilon_{\mathrm{m}}'(x)}{\varepsilon_{\mathrm{m}}(x)}\dfrac{\partial G(x)}{\partial x} + (k_0^2\varepsilon_{\mathrm{m}}(x) + k_{\mathrm{zm}}^2(x))G(x) = 0 \\ \dfrac{\partial^2 G(x)}{\partial x^2} + (k_0^2\varepsilon_{\mathrm{d}} + k_{\mathrm{zd}}^2(x))G(x) = 0 \rightarrow k_{\mathrm{zd}}^2(x) = -(k_0^2\varepsilon_{\mathrm{d}} + \dfrac{G''(x)}{G(x)}) \end{cases} \tag{5}$$

Also, when z=0, continuity of tangential fields (H_y and E_x) yields that: (from (3) and (4))

$$\frac{k_{\mathrm{zd}}(x)}{k_{\mathrm{zm}}(x)} = \frac{-\varepsilon_{\mathrm{d}}}{\varepsilon_{\mathrm{m}}(x)} \rightarrow k_{\mathrm{zm}}(x) = -\frac{k_{\mathrm{zd}}(x)\varepsilon_{\mathrm{m}}(x)}{\varepsilon_{\mathrm{d}}} \tag{6}$$

Solving the second order differential equation group (5) with equation (6), we have:

$$G''(x) + \frac{\varepsilon_{\mathrm{m}}'(x)\varepsilon_{\mathrm{d}}^2}{\varepsilon_{\mathrm{m}}(x)[\varepsilon_{\mathrm{m}}^2(x) - \varepsilon_{\mathrm{d}}^2]}G'(x) + k_0^2\frac{\varepsilon_{\mathrm{m}}(x)\varepsilon_{\mathrm{d}}}{\varepsilon_{\mathrm{m}}(x) + \varepsilon_{\mathrm{d}}}G(x) = 0 \tag{7}$$

Supplemental 2:

The basic relationship in the gradient-index material system:

$$\varepsilon = 1 + \chi$$

$$P = \varepsilon_0 \chi E$$

The coupling between propagating waves and surface waves can be explained by single layer inhomogeneous dipole radiation at the interface. Consider a single dipole at the interface of the GNM with the coordinates $(x', 0, 0)$ illuminated by a normally incident *x*-polarized plane wave:

$$dp(x',t) = e^{-i\omega t}\varepsilon_0 \chi(x') E_0 dv' = E_0 e^{-i\omega t}\varepsilon_0 \chi(x') dx' dy' dz' = E_0 e^{-i\omega t}\varepsilon_0 \chi(x') \Delta z dx'$$

where *x*' represents the position vector of the dipole. We set the effective length along the *z* axis as Δz and neglect the *y* coordinate in the calculation since the system is insensitive along the *y* axis and k_y should always be kept as zero. In the GNM, the permittivity is not a constant and can be expressed as $\varepsilon(x')$. As a result, the polarizability also should be rewritten as $\chi(x')$, which indicates the inhomogeneity of the GNM.

Now we write the standard form of the radiation field of the single dipole first:

$$H(r,t,x') = \frac{-\omega^2}{4\pi c|r - x'|} dp(x',t)\sin(\theta) e^{ik|r-x'|} e^{-i\omega t} \varphi(r,x')$$

Where θ and φ represent the elevation angle of the *x*-axis and the azimuth angle in the *y*-*z* plane, respectively. Here we set (*x*', 0, 0) as the origin of the coordinate and φ as the unit vector along local φ-axis. Thus, the radiation field of a single dipole at the interface of the GNM has the form below:

$$H(r,t,x')=\frac{-\omega^2}{4\pi c|r-x'|}E_0e^{-i\omega t}\varepsilon_0\chi(x')\Delta zdx'\cdot\sin(\theta)e^{ik|r-x'|}e^{-i\omega t}\varphi(r,x')$$

In the 2D case, the total radiation field should be given by an infinite row of *x*-polarized dipoles along the *x*-axis:

$$H(r,t,x')=\int\frac{-\omega^2}{4\pi c|r-x'|}E_0e^{-i\omega t}\varepsilon_0\chi(x')\Delta z\cdot\sin(\theta)e^{ik|r-x'|}e^{-i\omega t}\varphi(r,x')dx'$$

Then, the spatial spectra of the radiation field equals to:

$$\begin{aligned}&H_{\text{total}}(k_x,k_y=0)=\int H_{\text{total}}(r,t)e^{-ik_xx}dxdy\\&=\int e^{-ik_xx}dxdy\int\frac{-\omega^2}{4\pi c|r-x'|}E_0e^{-i\omega t}\varepsilon_0\chi(x')\Delta z\cdot\sin(\theta)e^{ik|r-x'|}e^{-i\omega t}\varphi(r,x')dx'\end{aligned}$$

We change the integral order and use the variable transformation:

$$\begin{aligned}&H_{\text{total}}(k_x,k_y=0)=\int dx'\int\frac{-\omega^2}{4\pi c|r-x'|}E_0e^{-i\omega t}\varepsilon_0\chi(x')\Delta z\cdot\sin(\theta)e^{ik|r-x'|}e^{-i\omega t}\varphi(r,x')e^{-ik_xx}dxdy\\&=\int\chi(x')e^{-ik_xx'}dx'\int\frac{-\omega^2}{4\pi cr}E_0e^{-i\omega t}\varepsilon_0\Delta z\cdot\sin(\theta)e^{ikr}e^{-i\omega t}\varphi(r,x')e^{-ik_xx}dxdy\\&=A\int\chi(x')e^{-ik_xx'}dx'\end{aligned}$$

Where *A* represents the spatial frequency spectra of the single dipole.

According to the ref. [33], *A* should have the form below:

$$H_y(k_x,z)\propto[p_z\frac{k_x}{k_z}-p_x]e^{ik_zz},z>0$$

In our case, since $p_z = 0$, $|A|$ should equal a constant when k_x changes. Thus, we can focus on the formula:

$$\int\chi(x')e^{-ik_xx'}dx'$$

We calculate the integral as the function of k_x and can then get the relative coefficient of the spatial frequency spectra as shown in Figure 1(d).