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Supplementary materials for
"Model diagnostics for the proportional hazards model with length-biased data"

Web Appendix A: Figure S1 for Section 4.1

In Figure S1, we show how the exponential function changes with increasing βs.

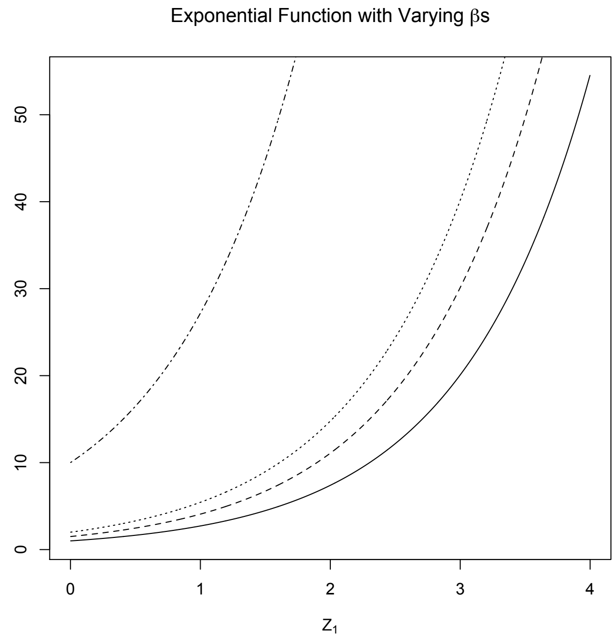

Fig. S1 Exponential functions in solid, dashed, dotted, and dashed-dotted lines with $\beta = 1, 1.5, 2$ and 10, respectively

Web Appendix B: Table S1 for Section 4.2

The additional simulation results are summarized in Table S1.

Table S1 Summary of type I error rate under the null hypothesis and the powers under the alternative hypotheses for testing the proportional hazards assumption using $f(Z_{ij}) = Z_{ij}^2$ with sample sizes (n) of 200 and 400 and censoring rates (cr) of 30% and 45% at significance levels of 5% and 10%.

		Null hypothesis		(a) Time-dependent I		(b) Time-dependent II	
n	cr	5%	10%	5%	10%	5%	10%
200	30%	0.044	0.086	0.319	0.486	0.611	0.710
	45%	0.040	0.097	0.264	0.410	0.557	0.671
400	30%	0.031	0.078	0.682	0.818	0.898	0.938
	45%	0.039	0.087	0.537	0.704	0.859	0.909

Web Appendix C: Figure S2 for Section 5

In Figure S2, we compare the survival curves based on Vardi's estimator and the estimated Cox model for each subgroup.

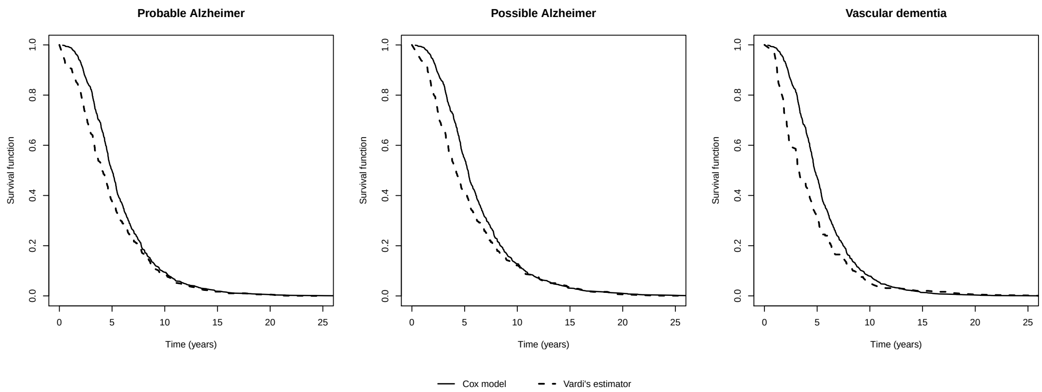

Fig. S2 Comparison between Vardi's estimator and the estimated Cox model

Web Appendix D: Table S2 for Section 6

We conducted a set of simulation studies to examine the performance of the proposed test procedure under covariate-dependent censoring. We generated 1000 datasets, following a setting similar to that described in Section 4.2 to test the proportional hazards assumption. To consider a censoring time that is dependent on the covariates, we sampled the residual censoring times from a gamma distribution with shape and rate parameters equal to 3 and $6/\tau_C$, respectively, when $Z_2 = 1$; and a uniform distribution $(0, \tau_C)$ when $Z_2 = 0$, where two different values of τ_C generate censoring rates of 30% and 45%. First, we applied the proposed test procedure to the simulated datasets without accounting for covariate-dependent censoring to examine how the proposed method behaves. The results are presented in the upper panel of Table S2. We found that the proposed test is quite robust to the misspecified censoring distribution under moderate levels of censoring rates, and maintained the type I error rates under the specified levels. Then, we generalized the proposed test procedure to accommodate censoring that is dependent on the covariates. The results can be found in the lower panel of Table S2. Under the null hypothesis, the estimated type I error rates are close to the nominal levels at significance levels of 5% and 10%. The generalized test provides adequate levels of power under the alternative hypotheses.

Table S2 Summary of type I error rate under the null hypothesis and the powers under the alternative hypotheses for testing the proportional hazards assumption using the proposed and the generalized test procedures under covariate-dependent censoring with sample sizes (n) of 200 and 400 and censoring rates (cr) of 30% and 45% at significance levels of 5% and 10%.

		Null hypothesis		(a) Time-dependent I		(b) Time-dependent II	
n	cr	5%	10%	5%	10%	5%	10%
The proposed test without considering covariate-dependent censoring							
200	30%	0.053	0.117	0.403	0.562	0.676	0.769
	45%	0.073	0.137	0.297	0.468	0.684	0.784
400	30%	0.040	0.083	0.697	0.846	0.926	0.961
	45%	0.053	0.119	0.580	0.750	0.904	0.948
The proposed test adjusted for covariate-dependent censoring							
200	30%	0.051	0.103	0.378	0.540	0.692	0.790
	45%	0.066	0.120	0.276	0.424	0.667	0.769
400	30%	0.042	0.085	0.672	0.836	0.931	0.966
	45%	0.041	0.099	0.566	0.737	0.900	0.938