

```

Get[NotebookDirectory[] <> "IncompatibilityMatrices.m"];
Get[NotebookDirectory[] <> "RelaxedAncestralMatrices.m"];

BranchLength["(S1,S2)S3 int"] = {t1, q[t2], t1, q[t2], t1, t2, 1, t2 - q[t2], 1};
BranchLength["(S1,S2)S3 anc"] = {t1, t2, 1/3, t1, t2, 1/3, t1, t2, 4/3, 1};
BranchLength["(S2,S3)S1"] = {t1, t2, 4/3, t1, t2, 1/3, t1, t2, 1/3, 1};
BranchLength["(S1,S3)S2"] = {t1, t2, 1/3, t1, t2, 4/3, t1, t2, 1/3, 1};

q[x_] := 1 - (x / (E^x - 1));

ParseMatrix[treematrix_, incompatibility_] :=
  Boole[Map[SameQ[#, incompatibility] &, M[treematrix], {2}]]

GetBranchProducts[tree1_, tree2_, incompatibility_] :=
  If[MatrixQ[M[StringJoin[tree1, ";", tree2]]],
    Simplify[BranchLength[tree1].ParseMatrix[
      StringJoin[tree1, ";", tree2], incompatibility].BranchLength[tree2]],
    Simplify[BranchLength[tree2].ParseMatrix[StringJoin[tree2, ";", tree1],
      incompatibility].BranchLength[tree1]]]

GetAncestralProducts[tree1_, tree2_, incompatibility_] :=
  If[MatrixQ[M[StringJoin[tree1, ";", tree2]]],
    Simplify[BranchLength[tree1].
      (ParseMatrix[StringJoin[tree1, ";", tree2], incompatibility] *
        RDA[StringJoin[tree1, ";", tree2]])].
      BranchLength[tree2]],
    Simplify[BranchLength[tree2].
      (ParseMatrix[StringJoin[tree2, ";", tree1], incompatibility] *
        RDA[StringJoin[tree2, ";", tree1]])].
      BranchLength[tree1]]]

trees = {"(S1,S2)S3 int", "(S1,S2)S3 anc", "(S2,S3)S1", "(S1,S3)S2"};
tree$probabilities = {1 - E^-t2, E^-t2/3, E^-t2/3, E^-t2/3};

```

Calculating the probability of concordance for a locus involved in an $S_1 \times S_2$ incompatibility.

`ParseMatrix` returns the indicator matrix for a specific incompatibility pattern from a general matrix of incompatibilities for a gene tree pair.

`GetBranchProducts` returns the inner product of all branch lengths from a particular pair of trees, filtered by the indicator matrix for that pair from `ParseMatrix`. This corresponds to the probability of two mutations that could give rise to a DMI on a specific gene tree pair.

To get these probabilities for all gene tree pairs, `GetBranchProducts` is applied to each pairwise combination. Here, we use the outer product of the tree vector with itself and apply `GetBranchProducts` to each resulting argument; the result is labeled as the

$S1 \times S2$ MutationMatrix.

The probability of each gene tree pair can be obtained by taking the outer product of the tree probability vector with itself; this is labeled here as the $S1 \times S2$ TreeMatrix.

$P(T_x, T_y | S_1 \times S_2) P(T_x) P(T_y)$, for all T_x and T_y , labeled here as $S1 \times S2$ PMatrix, can then be obtained by taking the element-wise product of the $S1 \times S2$ MutationMatrix with the $S1 \times S2$ TreeMatrix.

The marginal probability of concordance, at a single locus, can then be obtained by summing the rows and columns corresponding to the concordant gene trees from this matrix (rows/columns 1 and 2), and dividing by the sum of all elements in the matrix. The marginal probability of discordance can be similarly obtained (rows/columns 3 and 4). The sum of marginal probabilities for both concordance and discordance is equivalent to the sum of all elements in the matrix, that is, the denominator of Eq. 1 from the main text.

The relative probability of concordance can then be compared to the expected probability of concordance for a random locus: $1 - \frac{2 e^{-t_2}}{3}$.

```
S1xS2MutationMatrix = Outer[GetBranchProducts[###, S1xS2] &, trees, trees];
S1xS2TreeMatrix = Outer[Times, tree$probabilities, tree$probabilities];
S1xS2PMatrix = S1xS2MutationMatrix * S1xS2TreeMatrix;
```

(*

Sanity check:

the probability of concordance for a random locus should be $1 - (2/3)e^{-t}$,
We should get this by taking the marginal probability of
concordance from the gene tree probability matrix

*)

```
FullSimplify[(1/2) *
  (Total[S1xS2TreeMatrix[[1]]] + Total[S1xS2TreeMatrix[[2]]] +
   Total[S1xS2TreeMatrix[[All, 1]]] + Total[S1xS2TreeMatrix[[All, 2]]])]
1 -  $\frac{2 e^{-t_2}}{3}$ 
```

```
S1xS2MarginalConcordant =
```

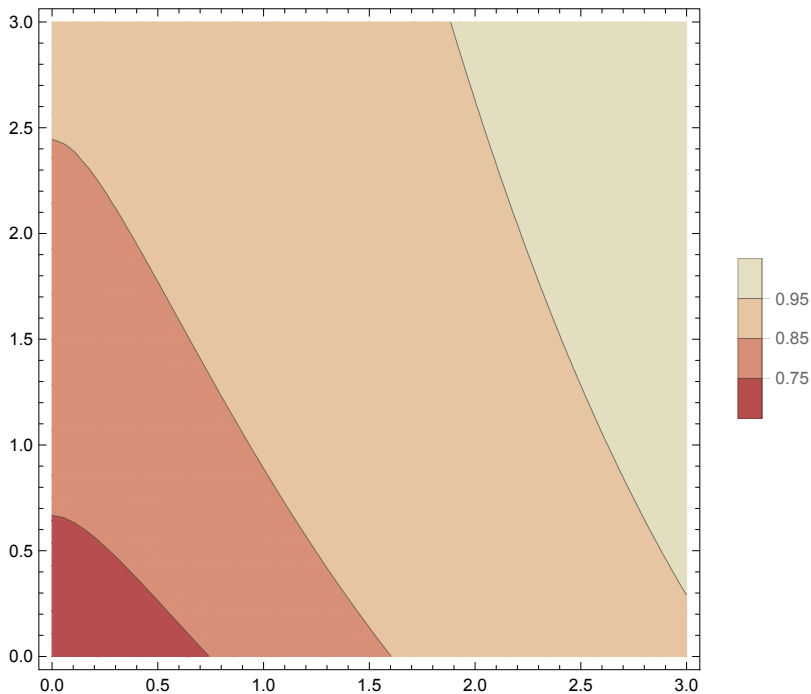
```
Simplify[(1/2) (Total[S1xS2PMatrix[[1]]] + Total[S1xS2PMatrix[[2]]] +
  Total[S1xS2PMatrix[[All, 1]]] + Total[S1xS2PMatrix[[All, 2]]])];
```

```
S1xS2MarginalDiscordant =
```

```
Simplify[(1/2) (Total[S1xS2PMatrix[[3]]] + Total[S1xS2PMatrix[[4]]] +
  Total[S1xS2PMatrix[[All, 3]]] + Total[S1xS2PMatrix[[All, 4]]])];
```

```
S1xS2$ConcordanceRatio = FullSimplify[
  S1xS2$MarginalConcordant / (S1xS2$MarginalConcordant + S1xS2$MarginalDiscordant)]
1 -  $\frac{2 e^{-t_2} (7 + 3 t_1 + 3 t_2)}{9 (2 + t_1)}$ 
```

```
ContourPlot[S1xS2$ConcordanceRatio /  $\left(1 - \frac{2 e^{-t_2}}{3}\right)$ ,
  {t2, 0, 3}, {t1, 0, 3}, PlotLegends -> Automatic,
  Contours -> {0.55, 0.65, 0.75, 0.85, 0.95, 1.05, 1.15, 1.25, 1.35, 1.45},
  ColorFunctionScaling -> False,
  ColorFunction -> ColorData[{"ThermometerColors", {1.45, 0.6}}]]
```



Calculating $P(T_x, T_y \mid S_1 \times S_2; S_1 \times S_3 \text{ incompatibility})$, labeled here as

S1xS2\$S1xS3\$PMatrix and

$P(T_x, T_y \mid S_1 \times S_2; S_2 \times S_3 \text{ incompatibility})$, labeled here as **S1xS2\$S2xS3\$PMatrix**

S1xS2\$S1xS3\$MutationMatrix =

```
Outer[GetBranchProducts[##, S1xS2$S1xS3] &, trees, trees];
```

```
S1xS2$S1xS3$TreeMatrix = Outer[Times, tree$probabilities, tree$probabilities];
```

```
S1xS2$S1xS3$PMatrix = S1xS2$S1xS3$MutationMatrix * S1xS2$S1xS3$TreeMatrix;
```

```

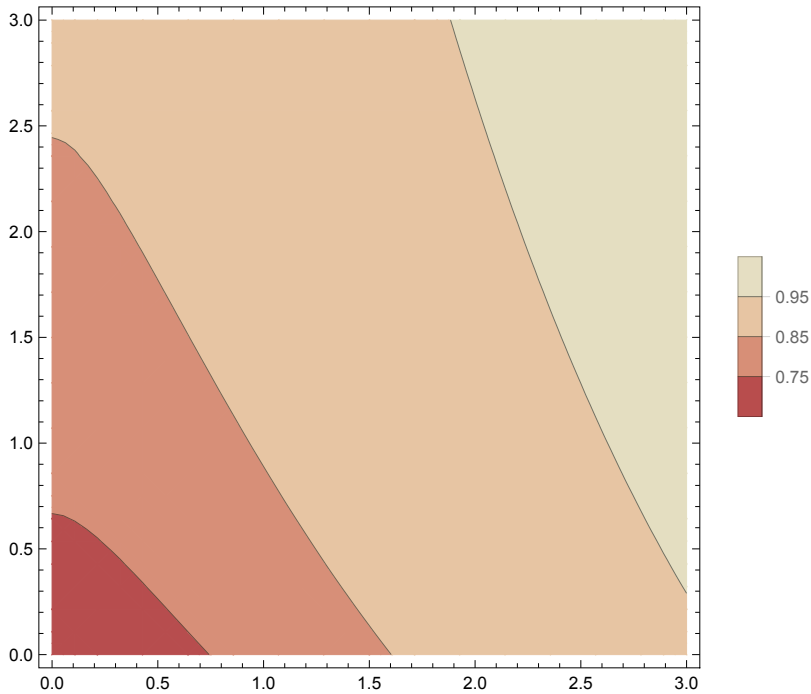
S1xS2$S1xS3$MarginalConcordant =
  Simplify[
    (1/2) (Total[S1xS2$S1xS3$PMatrix[[1]]] + Total[S1xS2$S1xS3$PMatrix[[2]]] +
      Total[S1xS2$S1xS3$PMatrix[[All, 1]]] +
      Total[S1xS2$S1xS3$PMatrix[[All, 2]]]);
S1xS2$S1xS3$MarginalDiscordant =
  Simplify[
    (1/2) (Total[S1xS2$S1xS3$PMatrix[[3]]] + Total[S1xS2$S1xS3$PMatrix[[4]]] +
      Total[S1xS2$S1xS3$PMatrix[[All, 3]]] + Total[S1xS2$S1xS3$PMatrix[[All, 4]]]);
S1xS2$S1xS3$ConcordanceRatio = FullSimplify[S1xS2$S1xS3$MarginalConcordant /
  (S1xS2$S1xS3$MarginalConcordant + S1xS2$S1xS3$MarginalDiscordant)]
1 -  $\frac{2 e^{-t_2} (7 + 3 t_1 + 3 t_2)}{9 (2 + t_1)}$ 

S1xS2$S2xS3$MutationMatrix =
  Outer[GetBranchProducts[###, S1xS2$S2xS3] &, trees, trees];
S1xS2$S2xS3$TreeMatrix = Outer[Times, tree$probabilities, tree$probabilities];
S1xS2$S2xS3$PMatrix = S1xS2$S2xS3$MutationMatrix * S1xS2$S2xS3$TreeMatrix;

S1xS2$S2xS3$MarginalConcordant =
  Simplify[
    (1/2) (Total[S1xS2$S2xS3$PMatrix[[1]]] + Total[S1xS2$S2xS3$PMatrix[[2]]] +
      Total[S1xS2$S2xS3$PMatrix[[All, 1]]] +
      Total[S1xS2$S2xS3$PMatrix[[All, 2]]]);
S1xS2$S2xS3$MarginalDiscordant =
  Simplify[
    (1/2) (Total[S1xS2$S2xS3$PMatrix[[3]]] + Total[S1xS2$S2xS3$PMatrix[[4]]] +
      Total[S1xS2$S2xS3$PMatrix[[All, 3]]] + Total[S1xS2$S2xS3$PMatrix[[All, 4]]]);
S1xS2$S2xS3$ConcordanceRatio = FullSimplify[S1xS2$S2xS3$MarginalConcordant /
  (S1xS2$S2xS3$MarginalConcordant + S1xS2$S2xS3$MarginalDiscordant)];
S1xS2$S2xS3$ConcordanceRatio = S1xS2$S1xS3$ConcordanceRatio
True

```

```
ContourPlot[S1xS2$S2xS3$ConcordanceRatio /  $\left(1 - \frac{2e^{-t_2}}{3}\right)$ ,
  {t2, 0, 3}, {t1, 0, 3}, PlotLegends -> Automatic,
  Contours -> {0.55, 0.65, 0.75, 0.85, 0.95, 1.05, 1.15, 1.25, 1.35, 1.45},
  ColorFunctionScaling -> False,
  ColorFunction -> ColorData[{"ThermometerColors", {1.45, 0.6}}]]
```



Calculating $P(T_x, T_y | S_1 \times S_3)$, labeled here as $S1xS3$PMatrix$

```
S1xS3$MutationMatrix = Outer[GetBranchProducts[###, S1xS3] &, trees, trees];
S1xS3$TreeMatrix = Outer[Times, tree$probabilities, tree$probabilities];
fv[x_] = E^-x / (1 - E^-t2);
fz[t_] = Simplify[Limit[Integrate[fv[x] / (t2 - x), x], x -> t] -
  Limit[Integrate[fv[x] / (t2 - x), x], x -> 0]];
g[t2_] = FullSimplify[Integrate[t * fv[t] * Integrate[fz[x], {x, t, t2}], {t, 0, t2}] +
  Integrate[t * fz[t] * Integrate[fv[x], {x, t, t2}], {t, 0, t2}]];
```

(*
Changes to the probability of an incompatibility due to mutation
ordering. Product from lengths of segment dg/eg and gh (also eh/dh and gh)
on $(S1, S2)S3$ int. tree subtracted. Replaced with calculations with
mutation order taken into account (see Supplemental Material).

*)

```

{Outer[Times, {1, 0, 0, 0}, {0, 1, 1, 1}] +
  Outer[Times, {0, 1, 1, 1}, {1, 0, 0, 0}] // MatrixForm,
  Outer[Times, {1, 0, 0, 0}, {1, 0, 0, 0}] // MatrixForm}
{

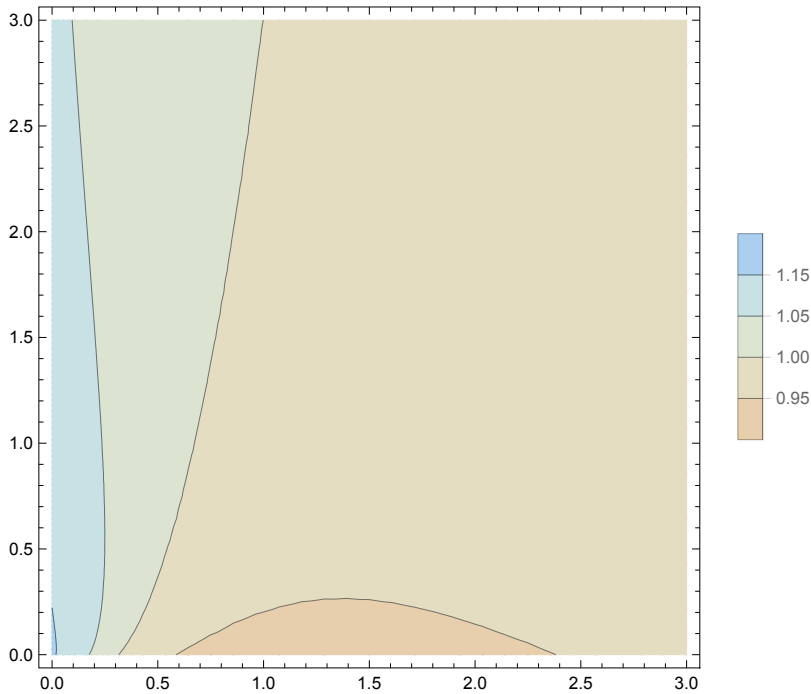
$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
}
S1xS3$MutationMatrix += (Outer[Times, {1, 0, 0, 0}, {0, 1, 1, 1}] +
  Outer[Times, {0, 1, 1, 1}, {1, 0, 0, 0}])
  (- (t2 - q[t2]) * t2 + (t2 - q[t2]) ((1/2) (q[t2] + t2)));
S1xS3$MutationMatrix += Outer[Times, {1, 0, 0, 0}, {1, 0, 0, 0}]
  (-2 * (t2 - q[t2]) * q[t2] + 2 * (t2 - q[t2]) * g[t2]);
S1xS3$PMatrix = S1xS3$MutationMatrix * S1xS3$TreeMatrix;
S1xS3$MarginalConcordant =
  Simplify[(1/2) (Total[S1xS3$PMatrix[[1]]] + Total[S1xS3$PMatrix[[2]]] +
    Total[S1xS3$PMatrix[[All, 1]]] + Total[S1xS3$PMatrix[[All, 2]]])];
S1xS3$MarginalDiscordant =
  Simplify[(1/2) (Total[S1xS3$PMatrix[[3]]] + Total[S1xS3$PMatrix[[4]]] +
    Total[S1xS3$PMatrix[[All, 3]]] + Total[S1xS3$PMatrix[[All, 4]]])];
S1xS3$ConcordanceRatio = Simplify[S1xS3$MarginalConcordant /
  (S1xS3$MarginalConcordant + S1xS3$MarginalDiscordant)];
S2xS3$MutationMatrix = Outer[GetBranchProducts[###, S2xS3] &, trees, trees];
S2xS3$TreeMatrix = Outer[Times, tree$probabilities, tree$probabilities];
S2xS3$MutationMatrix += (Outer[Times, {1, 0, 0, 0}, {0, 1, 1, 1}] +
  Outer[Times, {0, 1, 1, 1}, {1, 0, 0, 0}])
  (- (t2 - q[t2]) * t2 + (t2 - q[t2]) ((1/2) (q[t2] + t2)));
S2xS3$MutationMatrix += Outer[Times, {1, 0, 0, 0}, {1, 0, 0, 0}]
  (-2 * (t2 - q[t2]) * q[t2] + 2 * (t2 - q[t2]) * g[t2]);
S2xS3$PMatrix = S2xS3$MutationMatrix * S2xS3$TreeMatrix;
S2xS3$MarginalConcordant =
  Simplify[(1/2) (Total[S2xS3$PMatrix[[1]]] + Total[S2xS3$PMatrix[[2]]] +
    Total[S2xS3$PMatrix[[All, 1]]] + Total[S2xS3$PMatrix[[All, 2]]])];
S2xS3$MarginalDiscordant =
  Simplify[(1/2) (Total[S2xS3$PMatrix[[3]]] + Total[S2xS3$PMatrix[[4]]] +
    Total[S2xS3$PMatrix[[All, 3]]] + Total[S2xS3$PMatrix[[All, 4]]])];
S2xS3$ConcordanceRatio = Simplify[S2xS3$MarginalConcordant /
  (S2xS3$MarginalConcordant + S2xS3$MarginalDiscordant)];$$

```

`S1xS3$ConcordanceRatio == S2xS3$ConcordanceRatio`

True

```
ContourPlot[S1xS3$ConcordanceRatio /  $\left(1 - \frac{2 e^{-t_2}}{3}\right)$ ,
  {t2, 0, 3}, {t1, 0, 3}, PlotLegends -> Automatic, PlotRange -> All,
  Contours -> {0.55, 0.65, 0.75, 0.85, 0.95, 1, 1.05, 1.15, 1.25, 1.35, 1.45},
  ColorFunctionScaling -> False,
  ColorFunction -> ColorData[{"ThermometerColors", {1.45, 0.6}}]]
```



Calculating $P(T_x, T_y \mid S_1 \times S_3; S_2 \times S_3)$, labeled here as `S1xS3$S2xS3$PMatrix` and

`S1xS3$S2xS3$MutationMatrix =`

```
Outer[GetBranchProducts[###, S1xS3$S2xS3] &, trees, trees];
```

```
S1xS3$S2xS3$TreeMatrix = Outer[Times, tree$probabilities, tree$probabilities];
```

```
S1xS3$S2xS3$PMatrix = S1xS3$S2xS3$MutationMatrix * S1xS3$S2xS3$TreeMatrix;
```

`S1xS3$S2xS3$MarginalConcordant =`

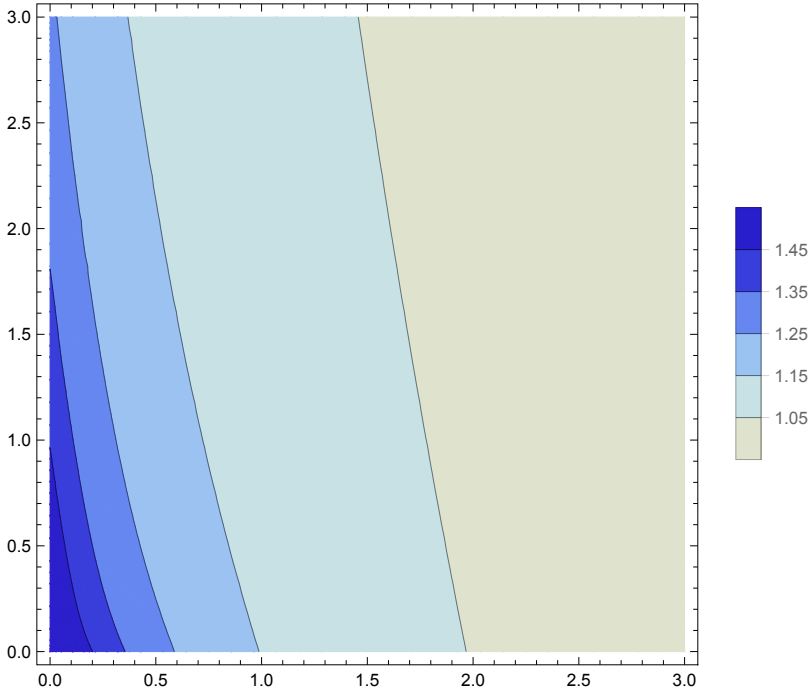
```
Simplify[
  (1/2) (Total[S1xS3$S2xS3$PMatrix[[1]]] + Total[S1xS3$S2xS3$PMatrix[[2]]] +
    Total[S1xS3$S2xS3$PMatrix[[All, 1]]] +
    Total[S1xS3$S2xS3$PMatrix[[All, 2]]]);
```

`S1xS3$S2xS3$MarginalDiscordant =`

```
Simplify[
  (1/2) (Total[S1xS3$S2xS3$PMatrix[[3]]] + Total[S1xS3$S2xS3$PMatrix[[4]]] +
    Total[S1xS3$S2xS3$PMatrix[[All, 3]]] + Total[S1xS3$S2xS3$PMatrix[[All, 4]]]);
```

```
S1xS3$S2xS3$ConcordanceRatio = Simplify[
  S1xS3$S2xS3$MarginalConcordant /
  (S1xS3$S2xS3$MarginalConcordant + S1xS3$S2xS3$MarginalDiscordant)];
```

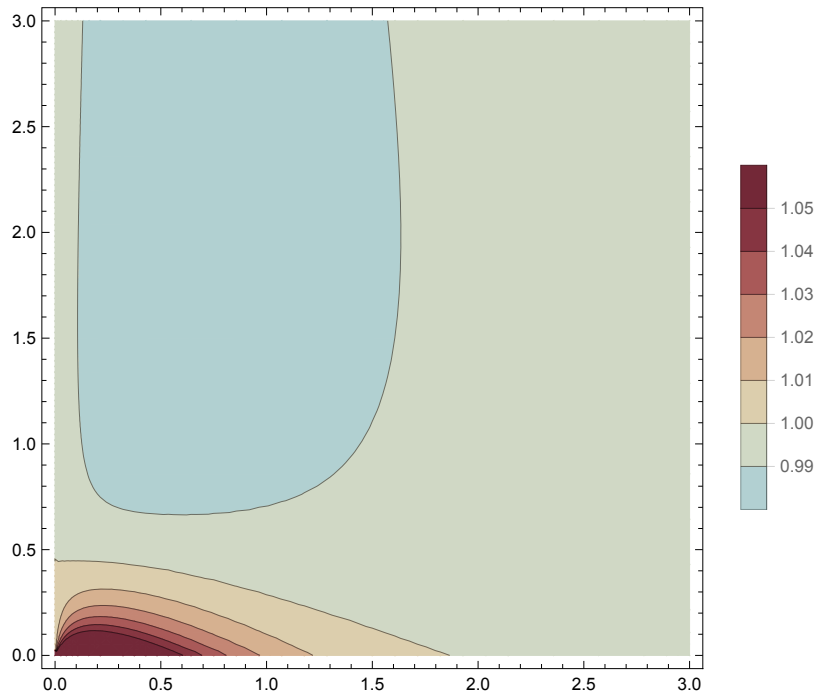
```
ContourPlot[S1xS3$S2xS3$ConcordanceRatio / (1 - 2 e^-t2),
  {t2, 0, 3}, {t1, 0, 3}, PlotLegends -> Automatic, PlotRange -> All,
  Contours -> {0.55, 0.65, 0.75, 0.85, 0.95, 1.05, 1.15, 1.25, 1.35, 1.45},
  ColorFunctionScaling -> False,
  ColorFunction -> ColorData[{"ThermometerColors", {1.45, 0.6}}]]
```



```
P$concord = Simplify[(S1xS2$ConcordanceRatio * Total[S1xS2$PMatrix, 2] +
  S1xS3$ConcordanceRatio * Total[S1xS3$PMatrix, 2] +
  S2xS3$ConcordanceRatio * Total[S2xS3$PMatrix, 2] +
  S1xS3$S2xS3$ConcordanceRatio * Total[S1xS3$S2xS3$PMatrix, 2] +
  S1xS2$S1xS3$ConcordanceRatio * Total[S1xS2$S1xS3$PMatrix, 2] +
  S1xS2$S2xS3$ConcordanceRatio * Total[S1xS2$S2xS3$PMatrix, 2]) /
  (Total[S1xS2$PMatrix, 2] + Total[S1xS3$PMatrix, 2] +
  Total[S2xS3$PMatrix, 2] + Total[S1xS3$S2xS3$PMatrix, 2] +
  Total[S1xS2$S1xS3$PMatrix, 2] + Total[S1xS2$S2xS3$PMatrix, 2])];
```

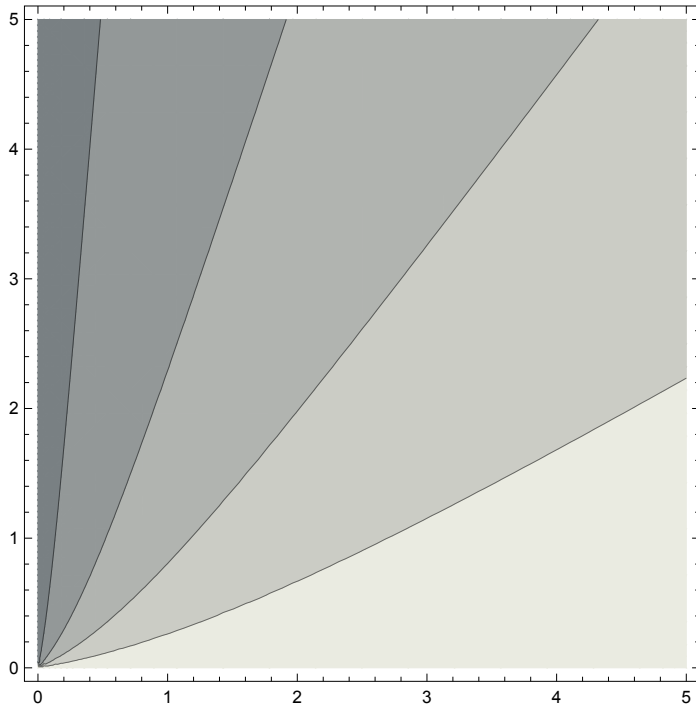


```
ContourPlot[P$concord /  $\left(1 - \frac{2e^{-t_2}}{3}\right)$ , {t2, 0, 3}, {t1, 0, 3}, PlotLegends → Automatic,
  ColorFunction → ColorData[{"RedBlueTones", {1.05, 0.95}}],
  PlotRange → All, ColorFunctionScaling → False,
  Contours → {0.94, 0.95, 0.96, 0.97, 0.98, 0.99, 1.00, 1.01, 1.02, 1.03, 1.04, 1.05}]
```

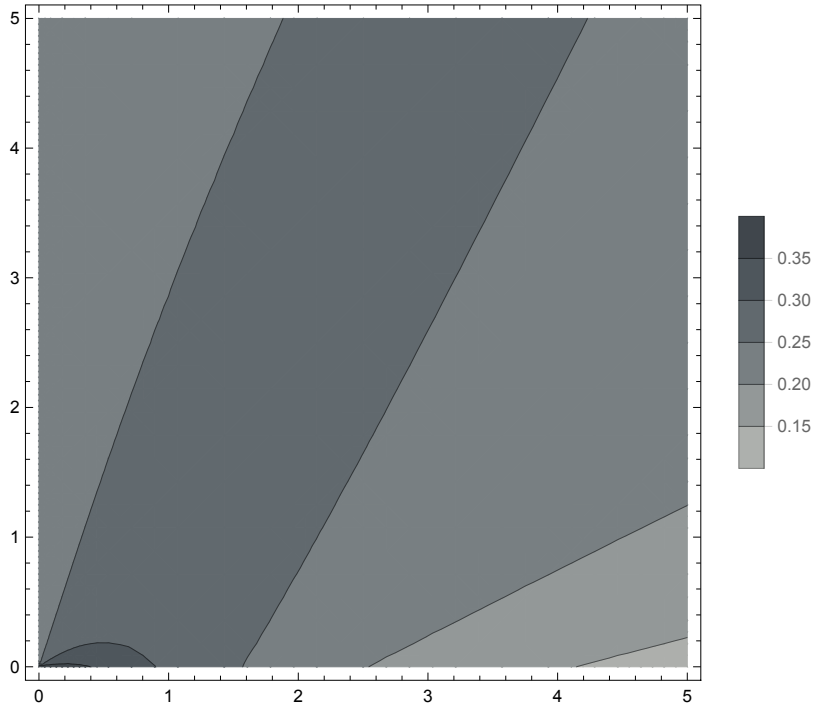


```
TotalP = Total[S1xS2$PMatrix, 2] + Total[S1xS3$PMatrix, 2] +
  Total[S2xS3$PMatrix, 2] + Total[S1xS3$S2xS3$PMatrix, 2] +
  Total[S1xS2$S1xS3$PMatrix, 2] + Total[S1xS2$S2xS3$PMatrix, 2];
```

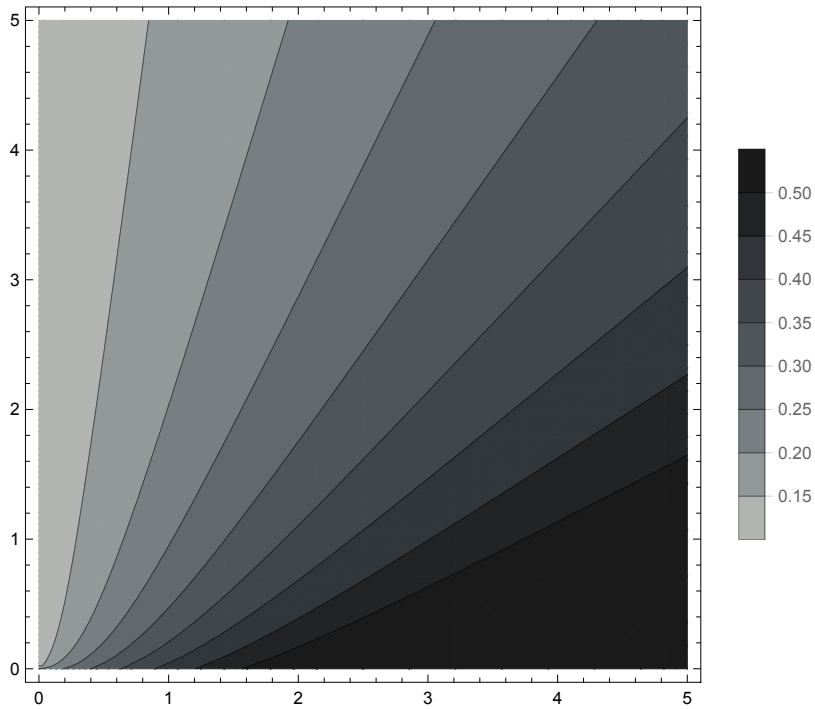
```
ContourPlot[(Total[S1xS2$PMatrix, 2]) / TotalP, {t2, 0, 5}, {t1, 0, 5},  
ColorFunctionScaling -> False,  
Contours -> {0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5},  
ColorFunction -> ColorData[{"GrayTones", {0.5, 0}}]]
```



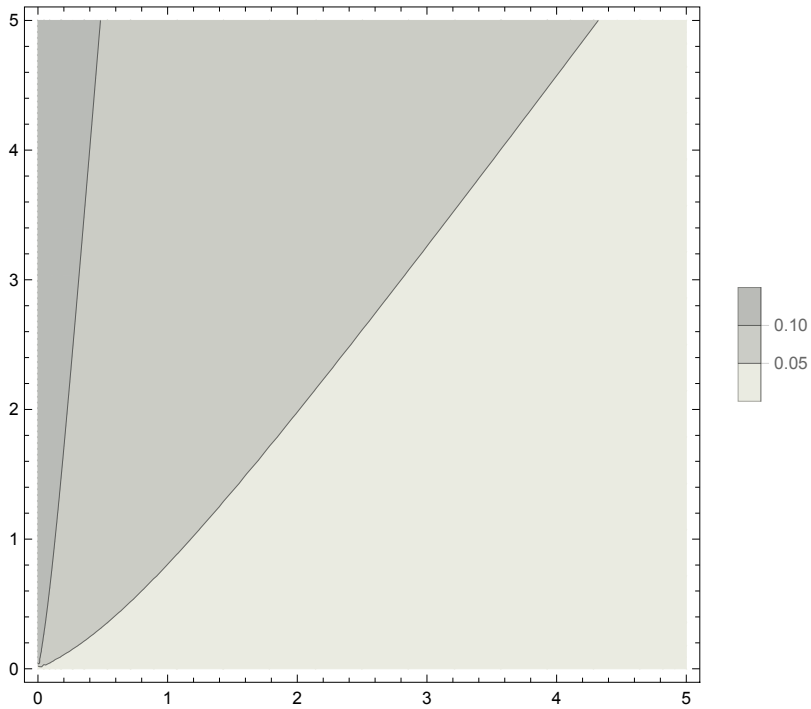
```
ContourPlot[(Total[S1xS3$PMatrix, 2]) / TotalP, {t2, 0, 5}, {t1, 0, 5},  
ColorFunctionScaling -> False, PlotRange -> All,  
Contours -> {0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5},  
ColorFunction -> ColorData[{"GrayTones", {0.5, 0}}],  
PlotLegends -> Automatic]
```



```
ContourPlot[(Total[S1xS3$S2xS3$PMatrix, 2]) / TotalP, {t2, 0, 5}, {t1, 0, 5},  
ColorFunctionScaling -> False, PlotRange -> All,  
Contours -> {0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5},  
ColorFunction -> ColorData[{"GrayTones", {0.5, 0}}],  
PlotLegends -> Automatic]
```



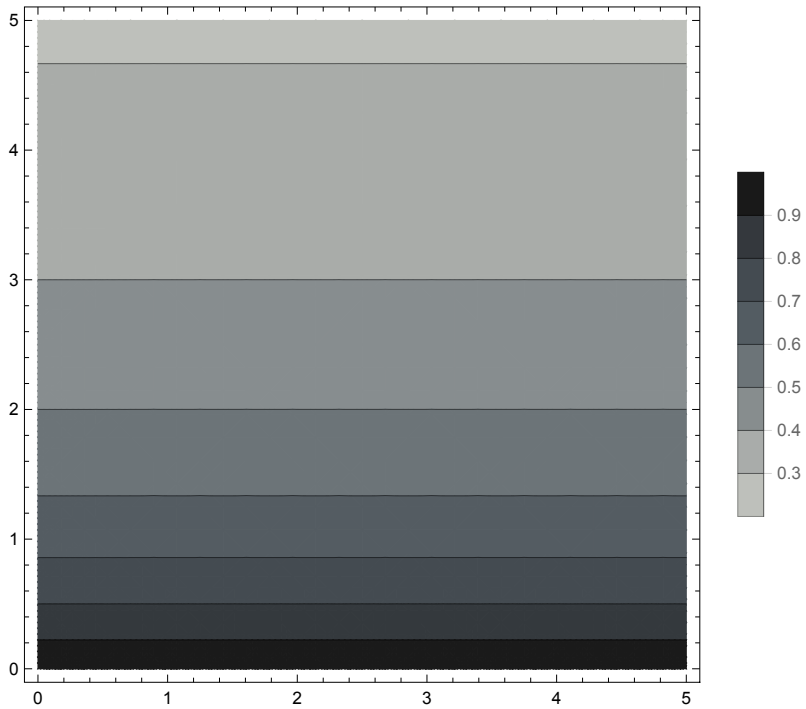
```
ContourPlot[(Total[S1xS2$S2xS3$PMatrix, 2]) / TotalP, {t2, 0, 5}, {t1, 0, 5},
  ColorFunctionScaling -> False, PlotRange -> All,
  Contours -> {0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5},
  ColorFunction -> ColorData[{"GrayTones", {0.5, 0}}],
  PlotLegends -> Automatic]
```



```
S1xS2$AncestralP =
```

```
Outer[GetAncestralProducts[###, S1xS2] &, trees, trees] * S1xS2$TreeMatrix;
```

```
ContourPlot[Total[S1xS2$AncestralP, 2] / Total[S1xS2$PMatrix, 2],
  {t2, 0, 5}, {t1, 0, 5}, ColorFunctionScaling -> False, PlotRange -> All,
  Contours -> {0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9},
  ColorFunction -> ColorData[{"GrayTones", {1, 0.1}}],
  PlotLegends -> Automatic]
```



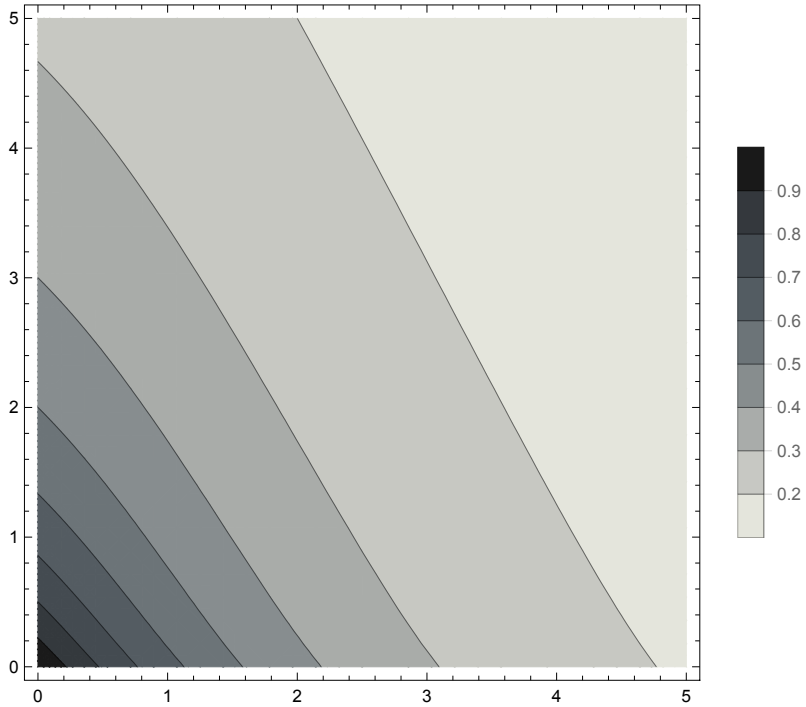
```
S1xS3$AncestralP =
```

```
  Outer[GetAncestralProducts[###, S1xS3] &, trees, trees] * S1xS3$TreeMatrix;
```

```
S2xS3$AncestralP =
```

```
  Outer[GetAncestralProducts[###, S2xS3] &, trees, trees] * S1xS3$TreeMatrix;
```

```
ContourPlot[Total[S1xS3$AncestralP, 2] / Total[S1xS3$PMatrix, 2],
  {t2, 0, 5}, {t1, 0, 5}, ColorFunctionScaling -> False, PlotRange -> All,
  Contours -> {0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9},
  ColorFunction -> ColorData[{"GrayTones", {1, 0.1}}],
  PlotLegends -> Automatic]
```



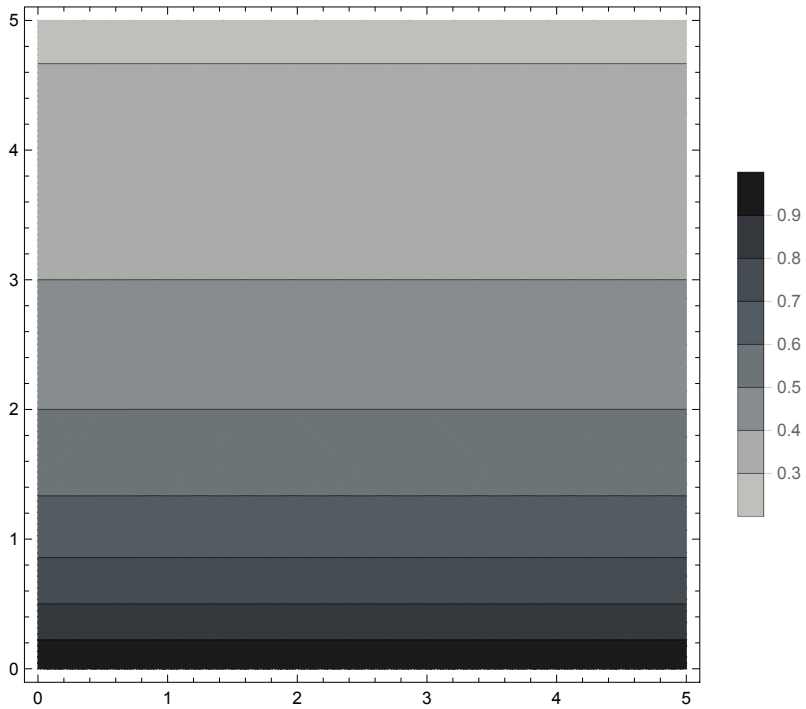
```
S1xS2$S1xS3$AncestralP =
```

```
  Outer[GetAncestralProducts[###, S1xS2$S1xS3] &, trees, trees] * S1xS3$TreeMatrix;
```

```
S1xS2$S2xS3$AncestralP =
```

```
  Outer[GetAncestralProducts[###, S1xS2$S1xS3] &, trees, trees] * S1xS3$TreeMatrix;
```

```
ContourPlot[Total[S1xS2$S1xS3$AncestralP, 2] / Total[S1xS2$S1xS3$PMatrix, 2],
  {t2, 0, 5}, {t1, 0, 5}, ColorFunctionScaling -> False, PlotRange -> All,
  Contours -> {0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9},
  ColorFunction -> ColorData[{"GrayTones", {1, 0.1}}],
  PlotLegends -> Automatic]
```

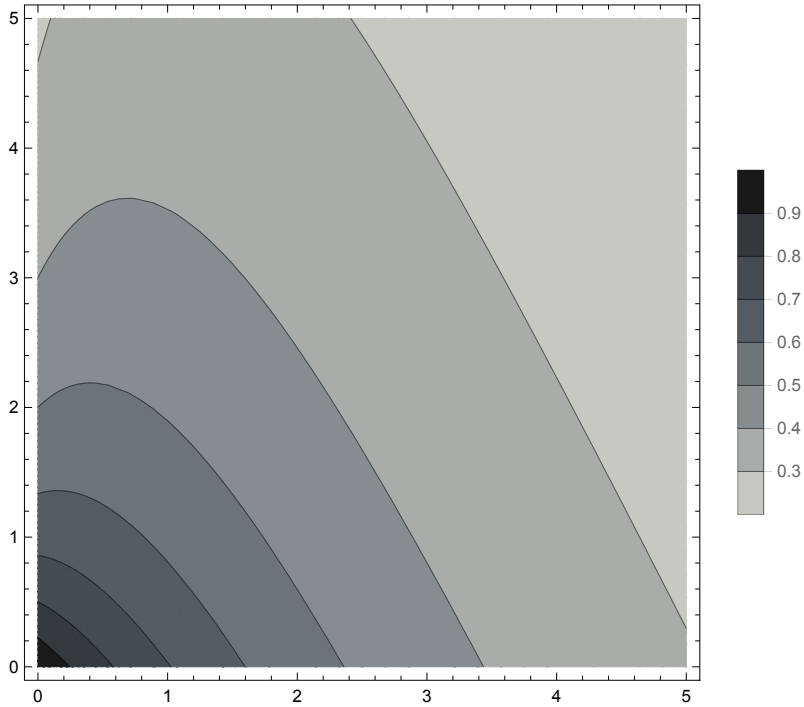


```
S1xS3$S2xS3$AncestralP =
```

```
Outer[GetAncestralProducts[###, S1xS3$S2xS3] &, trees, trees] * S1xS3$TreeMatrix;
```



```
ContourPlot[Total[S1xS3$S2xS3$AncestralP, 2] / Total[S1xS3$S2xS3$PMatrix, 2],
  {t2, 0, 5}, {t1, 0, 5}, ColorFunctionScaling -> False, PlotRange -> All,
  Contours -> {0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9},
  ColorFunction -> ColorData[{"GrayTones", {1, 0.1}}],
  PlotLegends -> Automatic]
```



```
P$ancestral = Simplify[(Total[S1xS2$AncestralP, 2] +
  Total[S1xS3$AncestralP, 2] +
  Total[S2xS3$AncestralP, 2] +
  Total[S1xS3$S2xS3$AncestralP, 2] +
  Total[S1xS2$S1xS3$AncestralP, 2] +
  Total[S1xS2$S2xS3$AncestralP, 2]) /
  (Total[S1xS2$PMatrix, 2] + Total[S1xS3$PMatrix, 2] +
  Total[S2xS3$PMatrix, 2] + Total[S1xS3$S2xS3$PMatrix, 2] +
  Total[S1xS2$S1xS3$PMatrix, 2] + Total[S1xS2$S2xS3$PMatrix, 2])];
```

```
ContourPlot[P$ancestral, {t2, 0, 3}, {t1, 0, 3},  
ColorFunctionScaling → False, PlotRange → All,  
Contours → {0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9},  
ColorFunction → ColorData[{"GrayTones", {1, 0.1}}],  
PlotLegends → Automatic]
```

