

1001 **Recurrent computations for visual pattern completion**

1002 **Supporting Information Appendix**

1003

1004 Hanlin Tang^{1,4*}, Martin Schrimpf^{2,4*}, William Lotter^{1,3,4*}, Charlotte Moerman⁴, Ana
1005 Paredes⁴, Josue Ortega Caro⁴, Walter Hardesty⁴, David Cox³, Gabriel Kreiman⁴

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1014 **1. Supplementary Materials and Methods**

1015 **Psychophysics experiments**

1016 A total of 106 volunteers (62 female, ages 18-34 y) with normal or corrected to
1017 normal vision participated in the psychophysics experiments reported in this study.

1018 All subjects gave informed consent and the studies were approved by the

1019 Institutional Review Board at Children's Hospital, Harvard Medical School. In 67

1020 subjects, eye positions were recorded during the experiments using an infrared

1021 camera eye tracker at 500 Hz (Eyelink D1000, SR Research, Ontario, Canada). We

1022 performed a main experiment (reported in **Figure 1F-G**) and three variations

1023 (reported in **Figures 1I-J, 2, S1 and S8-9**).

1024

1025 *Backward masking.* Multiple lines of evidence from behavioral (e.g. (1, 2)),

1026 physiological (e.g. (3-6)), and computational studies (e.g. (7-11)) suggest that

1027 recognition of whole isolated objects can be approximately described by rapid,

1028 largely feed-forward, mechanisms. Despite the success of these feed-forward

1029 architectures in describing the initial steps in visual recognition, each layer has

1030 limited spatial integration of its inputs. Additionally, feed-forward algorithms lack

1031 mechanisms to integrate temporal information or to take advantage of the rich
1032 temporal dynamics characteristic of neural circuits that allow comparing signals
1033 within and across different levels of the visual hierarchy. It has been suggested that
1034 backward masking can interrupt recurrent and top-down signals: when an image is
1035 rapidly followed by a spatially overlapping mask: the new high-contrast mask
1036 stimulus interrupts any additional, presumably recurrent, processing of the original
1037 image (3, 12-20). Thus, the psychophysical experiments tested recognition under
1038 both unmasked and backward masked conditions.

1039

1040 *Main experiment.* Both spatial and temporal integration are likely to play an
1041 important role in pattern completion mechanisms (21-27). A scheme of the
1042 experiment designed to study the spatial and temporal integration during
1043 recognition of occluded or partially visible objects is shown in **Figure 1**. Twenty-one
1044 subjects were asked to categorize images into one of 5 possible semantic groups (5-
1045 alternative forced choice) by pressing buttons on a gamepad. Stimuli consisted of
1046 contrast-normalized gray scale images of 325 objects belonging to five categories
1047 (animals, chairs, human faces, fruits, and vehicles). Each object was only presented
1048 once in each condition. Each trial was initiated by fixating on a cross for at least 500
1049 ms. After fixation, subjects were presented with the image of an object for a variable
1050 time (25 ms, 50 ms, 75 ms, 100 ms, or 150 ms), referred to as the stimulus onset
1051 asynchrony (SOA). The image was followed by either a noise mask (**Figure 1B**) or a
1052 gray screen (**Figure 1A**), with a duration of 500 ms, after which a choice screen
1053 appeared requiring the subject to respond. We use the term “pattern completion” to
1054 indicate successful categorization of partial images in the 5-alternative forced choice
1055 task used here and we do not mean to imply that subjects are forming any mental
1056 image of the entire object, which we did not test. The noise mask was generated by
1057 scrambling the phase of the images, while retaining the spectral coefficients. The
1058 images (256 x 256 pixels) subtended approximately 5 degrees of the visual field. In
1059 approximately 15% of the trials, the objects were presented in unaltered fashion
1060 (the ‘Whole’ condition, **Figure 1C** left). In the other 85% of the trials, the objects
1061 were rendered partially visible by presenting visual features through Gaussian

1062 bubbles (28) (the ‘Partial condition’, standard deviation = 14 pixels, **Figure 1C**
1063 right). Each subject performed an initial training session to familiarize themselves
1064 with the task and the stimuli. They were presented with 40 trials of whole objects,
1065 then 80 calibration trials of occluded objects. During the calibration trials, the
1066 number of bubbles was titrated using a staircase procedure to achieve an overall
1067 task difficulty of 80% correct rate. The number of bubbles (but not their positions)
1068 was then kept constant for the rest of the experiment. Results from the
1069 familiarization and calibration phase were not included in the analyses. Despite
1070 calibrating the number of bubbles, there was a wide range of degrees of occlusion
1071 because the positions of the bubbles were randomized in every trial. Each image
1072 was only presented once in the masked condition and once in the unmasked
1073 condition.

1074

1075 *Physiology-based psychophysics experiment.* In the physiology-based psychophysics
1076 experiment (**Figure 2**, n = 33 subjects), stimuli consisted of 650 images from five
1077 categories for which we had previously recorded neural responses (see below). In
1078 the neurophysiological recordings (25), bubble positions were randomly selected in
1079 each subject and therefore each subject was presented with different images (except
1080 for the fully visible ones). The main difference between the physiology-based
1081 psychophysics experiment and the Main experiment is that here we used the exact
1082 same images that were used in the physiological recordings (see description under
1083 “Neurophysiological Recordings” below).

1084

1085 *Occlusion experiment.* In the occlusion experiment (**Figure 1I**, **Figure S1**, n=14
1086 subjects in the partial objects experiment and n =15 subjects in the occlusion
1087 experiment), we generated occluded images that revealed the same sets of features
1088 as the partial objects, but contained an explicit occluder (**Figure 1D**) to activate
1089 amodal completion cues. The stimulus set consisted of 16 objects from 4 different
1090 categories. For comparison, we also collected performance with partial objects from
1091 this reduced stimulus set.

1092

1093 *Novel objects experiment.* The main set of experiments required categorization of
1094 images containing pictures of animals, chairs, faces, fruits and vehicles. None of the
1095 subjects involved in the psychophysics or neurophysiological measurements had
1096 had any previous exposure to the *specific pictures* in these experiments, let alone
1097 with the partial images rendered through bubbles. Yet, it can be surmised that all
1098 the subjects had had extensive previous experience with *other* images of objects
1099 from those categories, including occluded versions of other animals, chairs, faces,
1100 fruits and vehicles. In order to evaluate whether experience with occluded instances
1101 of objects from a specific category is important to recognize novel instances of
1102 partially visible objects from the same category, we conducted a new psychophysics
1103 experiment with novel objects. We used 500 unique novel objects belonging to 5
1104 categories, all the novel objects were chosen from the Tarr Lab stimulus repository
1105 (29). An equal amount of stimuli were chosen from each category. One exemplar
1106 from each category is shown in **Figure S8A**. In the Cognitive Science community, the
1107 first three categories are known as “Fribbles” and the last two categories as
1108 “Greebles” and “Yufos” (29). In our experiments, each category was assigned a Greek
1109 letter name (**Figure S8A**) so as not to influence the subjects with potential meanings
1110 of an invented name.

1111 The experiment followed the same protocol as the main experiment (**Figure**
1112 **1**). Twenty-three new subjects (11 female, 20 to 34 years old) participated in this
1113 experiment. Since the subjects had no previous exposure to these stimuli, they
1114 underwent a short training session where they were presented with 2 fully visible
1115 exemplars from each category so that they could learn the mapping between
1116 categories and response buttons. In order to start the experiment, subjects were
1117 required to get 8 out of 10 correct responses, 5 times in a row using these practice
1118 stimuli. On average, reaching this level of accuracy required 80 ± 40 trials. Those 2
1119 stimuli from each category were not used in the subsequent experiments. Therefore,
1120 whenever we refer to “novel” objects, what we mean is objects from 5 categories
1121 where subjects were exposed to ~ 80 trials of 2 fully visible exemplars per category,
1122 different from the ones used in the psychophysics tests. This regime represented
1123 our compromise of ensuring that subjects knew which button they had to press,

1124 while at the same time keeping only minimal initial training. Importantly, this initial
1125 training only involved whole objects and subjects had no exposure to partial novel
1126 objects before the onset of the psychophysics measurements. Halfway through the
1127 experiment, we repeated 3 runs of the recognition test with the same 2 initial fully
1128 visible exemplars as a control to ensure that subjects were still performing the task
1129 correctly, and all subjects passed this control (>80% performance in just 3
1130 consecutive runs).

1131 During the experiment, subjects were presented with 1,000 uniquely
1132 rendered stimuli from 500 contrast-normalized gray scale novel objects, resized to
1133 256x256 pixels, subtending approximately 5° of visual angle. All images were
1134 contrast normalized using the `histMatch` function from the SHINE toolbox (30).
1135 This function equates the luminance histogram of sets of images. For each subject,
1136 1,000 unique renderings were obtained by applying different bubbles to the original
1137 images, resulting in a total of 23,000 different stimuli across subjects.

1138 The SOAs and other parameters were identical to those used in the main
1139 experiment. The analyses and models for the novel object experiments follow those
1140 in the main experiment (**Figures S8B-D** are the analogs of **Figure 1F-H**, **Figure S9A**
1141 is the analog of **Figure 3A**, **Figure S9B-D** are the analogs of **Figure 4B-D**).

1142

1143 **Neurophysiology experiments**

1144 The neurophysiological data analyzed in **Figures 2** and **3** were taken from
1145 the study by Tang *et al* (25), to which we refer for further details. Briefly, subjects
1146 were patients with pharmacologically intractable epilepsy who had intracranial
1147 electrodes implanted for clinical purposes. These electrodes record intracranial field
1148 potential signals, which represent aggregate activity from large numbers of neurons.
1149 All studies were approved by the hospital's Institutional Review Board and were
1150 carried out with the subjects' informed consent. Images of partial or whole objects
1151 were presented for 150 ms, followed by a gray screen for 650 ms. Subjects
1152 performed a five-alternative forced choice categorization task as described in
1153 **Figure 1** with the following differences: (i) the physiological experiment did not
1154 include the backward mask condition; (ii) 25 different objects were used in the

1155 physiology experiment; (iii) the SOA was fixed at 150 ms in the physiology
1156 experiment.

1157 Bubbles were randomly positioned in each trial. In order to compare models,
1158 behavior and physiology on an image-by-image basis, we had to set up a stimulus
1159 set based on the exact images (same bubble locations) presented to a given subject
1160 in the physiology experiment. To construct the stimulus set for the physiology-
1161 based psychophysics experiment (**Figure 2**), we chose two electrodes according to
1162 the following criteria: (i) those two electrodes had to come from different
1163 physiology subjects (to ensure that the results were not merely based on any
1164 peculiar properties of one individual physiology subject), (ii) the electrodes had to
1165 respond both to whole objects and partially visible objects (to ensure a robust
1166 response where we could estimate latencies in single trials), and (iii) the electrodes
1167 had to show visual selectivity (to compare the responses to the preferred and non-
1168 preferred stimuli). The electrode selection procedure was strictly dictated by these
1169 criteria and was performed before even beginning the psychophysics experiment.
1170 We extracted the images presented during the physiological recordings in $n = 650$
1171 trials for psychophysical testing. For the preferred category for each electrode, only
1172 trials where the amplitude of the elicited neural response was in the top 50th
1173 percentile were included, and trials were chosen to represent a distribution of
1174 neural response latencies. After constructing this stimulus set, we performed
1175 psychophysical experiments with $n = 33$ new subjects (Physiology-based
1176 psychophysics experiment) to evaluate the effect of backward masking for the exact
1177 same images for which we had physiological data.

1178 For the physiological data, we focused on the neural latency, defined as the
1179 time of the peak in the physiological response, as shown in **Figure 2B**. These
1180 latencies were computed in single trials (see examples in **Figure 2C**). Because these
1181 neural latencies per image are defined in single trials, there are no measures of
1182 variation in the x-axis in **Figure 2F** or **Figure 3C-D**. A more extensive analysis of the
1183 physiological data, including extensive discussion of many ways of measuring neural
1184 latencies, was presented in (25).

1185

1186 **Behavioral and neural data analysis**

1187 *Masking Index.* To quantify the effect of backward masking, we defined the masking
1188 index as 100%-pAUC, where pAUC is the percent area under the curve when
1189 plotting performance as a function of SOA (e.g. **Figure 2E**). To evaluate the
1190 variability in the masking index, we used a half-split reliability measure by
1191 randomly partitioning the data into two halves and computing the masking index
1192 separately in each half. **Figure S2** provides an example of such a split. Error bars in
1193 **Figure 2F** constitute half-split reliability values.

1194

1195 *Correlation between masking index and neural latency.* To determine the correlation
1196 between masking index and neural response latency, we combined data from the
1197 two recording sites by first standardizing the latency measurements (z-score,
1198 **Figure 2F**). We then used a linear regression on neural response latency with
1199 masking index, percent visibility, and recording site as predictor factors to avoid any
1200 correlations dictated by task difficulty or differences between recording sites.
1201 We used only trials from the preferred category for each recording site and reported
1202 the correlation and statistical significance in **Figure 2F**. There was no significant
1203 correlation between the masking index and neural latency when considering trials
1204 from the non-preferred category.

1205

1206 *Correlation between model distance and neural response latency.* As described below,
1207 we simulated the activity of units in several computational models in response to
1208 the same images used in the psychophysics and physiology experiments. To
1209 correlate the model responses with neural response latency, we computed the
1210 Euclidean distance between the model representation of partial and whole objects.
1211 We computed the distance between each partial object in the physiology-based
1212 psychophysics experiment stimulus set and the centroid of the whole images from
1213 the same category (distance-to-category). We then assessed significance by using a
1214 linear regression on the model distance versus neural response latency while
1215 controlling for masking index, percent visibility, and recording site as factors.

1216

1217 **Feed-forward Models**

1218 We considered the ability to recognize partially visible images by state-of-
1219 the-art feed-forward computational models of vision (**Figure 3A, Figure S3** and
1220 **Figure S4**). First, we evaluated whether it was possible to perform recognition
1221 purely based on pixel intensities. Next, in the main text we evaluated the
1222 performance of the AlexNet model (31). AlexNet is an eight-layer deep convolutional
1223 neural network consisting of convolutional, max-pooling and fully-connected layers
1224 with a large number of weights trained in a supervised fashion for object
1225 recognition on ImageNet, a large collection of labeled images from the web (31, 32).
1226 We used a version of AlexNet trained using *caffe* (33), a deep learning library. Two
1227 layers within the AlexNet were tested: pool5 and fc7. Pool5 is the last convolutional
1228 (retinotopic) layer in the architecture. fc7 is the last layer before the classification
1229 step and is fully connected, that is, every unit in fc7 is connected to every unit in the
1230 previous layer. The number of features used to represent each object was
1231 $256 \times 256 = 65536$ for pixels, 9216 for pool5 and 4096 for fc7.

1232 We also considered many other similar feed-forward models: VGG16 block5,
1233 fc1 and fc2 (25088, 4096 and 4096 features respectively) (34), VGG19 fc1 and fc2
1234 (4096 features each) (34), layers 40 to 49 of ResNet50 (200704 to 2048 features)
1235 (35), and InceptionV3 mixed 10 layer (131072 features) (36). In all of these cases,
1236 we used models pre-trained for the ImageNet 2012 data set and randomly
1237 downsampled the number of features to 4096 as in AlexNet. Results for all of these
1238 models are shown in **Figure S4**; more layers and models can be found in the
1239 accompanying web site:

1240 http://klab.tch.harvard.edu/resources/Tangetal_RecurrentComputations.html

1241 Classification performance for each model was evaluated on a stimulus set
1242 consisting of 13,000 images of partial objects (generated from 325 objects from 5
1243 categories). These were the same partial objects used to collect human performance
1244 in the main experiment (**Figure 1**). We used a support vector machine (SVM) with a
1245 linear kernel to perform classification on the features computed by each model. We
1246 used 5-fold cross-validation across the 325 objects. Each split contained 260 objects
1247 for training, and 65 objects split for validation and testing, such that each object was

1248 used exactly in one validation and testing split, and such that there was an equal
1249 number of objects from each category in each split. Decision boundaries were fit on
1250 the training set using the SVM with the C parameter determined through the
1251 validation set by considering the following possible C values: 10^{-4} , 10^{-3} , ..., 10^3 , 10^4 .
1252 The SVM boundaries were fit using images of whole objects and tested on images of
1253 partial objects. Final performance numbers for partial objects were calculated on
1254 the full data set of 13,000 images -- that is, for each split, classification performance
1255 was evaluated on the partial objects corresponding to the objects in the test set,
1256 such that, over all splits, each partial object was evaluated exactly once.

1257 As indicated above, all the results shown on **Figure 3A**, **Figure S3** and
1258 **Figure S4** are based on models that were trained on the ImageNet 2012 data set
1259 and then tested using our stimulus set. We also tested a model created by fine-
1260 tuning the AlexNet network. We fine-tuned AlexNet using the set of whole objects in
1261 our data set and then re-examined the model's performance under the low visibility
1262 conditions in **Figure S5**. We fine-tuned AlexNet by replacing the original 1000-way
1263 fully-connected classifier layer (fc8) trained on ImageNet with a 5-way fully-
1264 connected layer (fc8') over the categories in our dataset and performing back-
1265 propagation over the entire network. We again performed cross validation over
1266 objects, choosing final weights by monitoring validation accuracy. To be consistent
1267 with previous analysis, after fine-tuning the representation, we used an SVM
1268 classifier on the resulting fc7 activations.

1269 To graphically display the representation of the images based on all 4096
1270 units in the fc7 layer of the model in a 2D plot (**Figure 4C**), we used stochastic
1271 neighborhood embedding (t-SNE) (37). We note that this was done exclusively for
1272 display purposes and all the analyses, including distances, classification and
1273 correlations, are based on the model representation with all the units in the
1274 corresponding layer as described above. For each model and each image, we
1275 computed the Euclidian distance between the model's representation and the mean
1276 point across all whole objects within the corresponding category. This distance-to-
1277 category corresponds to the y-axis in **Figure 3B-C**.

1278

1279 **Recurrent Neural Network Models**

1280 A recurrent neural network (RNN) was constructed by adding all-to-all
1281 recurrent connections to different layers of the bottom-up convolutional networks
1282 described in the previous section (for example, to the fc7 layer of AlexNet in **Figure**
1283 **4A**). We first describe here the model for AlexNet; a similar procedure was followed
1284 for the other computational models. An RNN consists of a state vector that is
1285 updated according to the input at the current time step and its value at the previous
1286 time step. Denoting \mathbf{h}_t as the state vector at time t and \mathbf{x}_t as the input into the
1287 network at time t , the general form of the RNN update equation is $\mathbf{h}_t = f(\mathbf{W}_h \mathbf{h}_{t-1}, \mathbf{x}_t)$
1288 where f introduces a non-linearity as defined below. In our model, \mathbf{h}_t represents the
1289 fc7 feature vector at time t and \mathbf{x}_t represents the feature vector for the previous
1290 layer, fc6, multiplied by the transition weight matrix $\mathbf{W}_{6 \rightarrow 7}$. For simplicity, the first
1291 six layers of AlexNet were kept fixed to their original feed-forward versions.

1292 We chose the weights \mathbf{W}_h by constructing a Hopfield network (38), RNN_h , as
1293 implemented in MATLAB's `newhop` function, which is a modified version of the
1294 original description by Hopfield (39). Since this implementation is based on binary
1295 unit activity, we first converted the scalar activities in \mathbf{x} to $\{-1, +1\}$ by mapping those
1296 values greater than 0 to +1 and all other values to -1. Depending on the specific layer
1297 and model, this binarization step in some cases led to either an increase or a
1298 decrease in performance (even before applying the attractor network dynamics); all
1299 the results shown in the Figures report the results after applying the Hopfield
1300 dynamics. The weights in RNN_h are symmetric ($W_{ij} = W_{ji}$) and are dictated by the

1301 Hebbian learning rule $W_{ij} = \frac{1}{n_p} \sum_{p=1}^{n_p} x_i^p x_j^p$ where the sum goes over the n_p patterns of

1302 whole objects to be stored (in our case $n_p=325$) and x_i^p represents the activity of
1303 unit i in response to pattern p . This model does not have any free parameters that
1304 depend on the partial objects and the weights are uniquely specified by the activity
1305 of the feed-forward network in response to the whole objects. After specifying \mathbf{W}_h ,
1306 the activity in RNN_h was updated according to $\mathbf{h}_0 = \mathbf{x}$ and $\mathbf{h}_t = \text{satlins}(\mathbf{W}_h \mathbf{h}_{t-1} + \mathbf{b})$ for
1307 $t > 0$ where *satlins* represents the saturating linear transfer function,

1308 $\text{satlins}(z) = \max(\min(1, z), -1)$ and \mathbf{b} introduces a constant bias term. The activity in
1309 RNN_h was simulated until convergence, defined as the first time point where there
1310 was no change in the sign of any of the features between two consecutive time
1311 points.

1312 To evaluate whether the increase in performance obtained in the RNN_h was
1313 specific to the AlexNet architecture, we also implemented recurrent connections
1314 added onto other networks. **Figure S7** shows a comparison between performance of
1315 the VGG16 network layer fc1 (34) and a VGG16 fc1 model endowed with additional
1316 recurrent connections in the same format as used with AlexNet. We used the time
1317 steps of the Hopfield network that yielded maximal performance. The
1318 VGG16+Hopfield model also showed performance improvement with respect to the
1319 purely bottom-up VGG16 counterpart. Several additional models were tested for
1320 other layers of AlexNet, VGG16, VGG19, ResNet and InceptionV3, showing a
1321 distribution with different degrees of consistent improvement upon addition of the
1322 recurrent connectivity (shown in the accompanying web material at
1323 http://klab.tch.harvard.edu/resources/Tangetal_RecurrentComputations.html).

1324 We ran an additional simulation with the RNN models to evaluate the effects
1325 of backward masking (**Figure 4F**). For this purpose, we simulated the response of
1326 the feed-forward AlexNet model to the same masks used for the psychophysical
1327 experiments to determine the fc6 features for each mask image. Next, we used this
1328 mask as the fixed input \mathbf{x}_t into the recurrent network, at different time points after
1329 the initial image input.

1330

1331 **2. Supplementary Discussion**

1332

1333 **Partially visible versus occluded objects**

1334 In most of the experiments, we rendered objects partially visible by
1335 presenting them through “bubbles” (**Fig. 1C**) in an attempt to distill the basic
1336 mechanisms required for spatial integration during pattern completion. It was
1337 easier to recognize objects behind a real occluder (**Fig. 1D, S1, (40)**). The results

1338 presented here were qualitatively similar (**Fig. S1**) when using explicit occluders
1339 (**Fig. 1D**): recognition of occluded objects was also disrupted by backward masking
1340 (**Fig. 1I, S1**). As expected, performance was higher for the occlusion versus the
1341 bubbles condition.

1342

1343 **“Unfolding” recurrent neural networks into feed-forward neural networks**

1344 Before examining computational models including recurrent connections, we
1345 analyzed bottom-up architectures and showed that they were *not* robust to
1346 extrapolating from whole objects to partial objects (**Figure 4**). However, there exist
1347 infinitely many possible bottom-up models. Hence, even though we examined state-
1348 of-the-art models that are quite successful in object recognition, the failure to
1349 account for the behavioral and physiological results in the bottom-up models
1350 examined here (as well as similar failures reported in other studies, e.g. (41, 42))
1351 should be interpreted with caution. We do not imply that it is impossible for *any*
1352 bottom-up architecture to recognize partially visible objects. In fact, it is possible to
1353 unfold a recurrent network with a finite number of time steps into a bottom-up
1354 model by creating an additional layer for each additional time step. However, there
1355 are several advantages to performing those computations with a recurrent
1356 architecture including a drastic reduction in the number of units required as well as
1357 in the number of weights that need to be trained and the fact that such unfolding is
1358 applicable only when we know *a priori* the fixed number of computational steps
1359 required, in contrast with recurrent architectures that allow an arbitrary and
1360 variable number of computations.

1361

1362 **Recurrent computations and “slower” integration**

1363 A related interpretation of the current findings is that more challenging
1364 tasks, such as recognizing objects from minimal pixel information, may lead to
1365 “slower processing” throughout the ventral visual stream. According to this idea,
1366 each neuron would receive weaker inputs and require a longer time for integration,
1367 leading to the longer latencies observed experimentally at the behavioral and
1368 physiological level. It seems unlikely that the current observations could be fully

1369 accounted by longer integration times at all levels of the visual hierarchy. First, all
1370 images were contrast normalized to avoid any overall intensity effects. Second,
1371 neural delays for poor visibility images were not observed in early visual areas (25).
1372 Third, the correlations between the effects of backward masking and neural delays
1373 persisted even after accounting for difficulty level (**Fig. 3**). Fourth, none of the state-
1374 of-the-art purely bottom-up computational models were able to account for human
1375 level performance (see further elaboration of this point below). These arguments
1376 rule out slower processing throughout the entire visual system due to low intensity
1377 signals in the lower visibility conditions. However, the results presented here are
1378 still compatible with the notion that the inputs to higher-level neurons in the case of
1379 partial objects could be weaker and could require further temporal integration. This
1380 possibility is consistent with the model proposed here. Because the effects of
1381 recurrent computations are delayed with respect to the bottom-up inputs, we
1382 expect that any such slow integration would have to interact with the outputs of
1383 recurrent signals.

1384

1385 **Extensions to the proposed proof-of-concept architecture**

1386 A potential challenge with attractor network architectures is the pervasive
1387 presence of spurious attractor states, particularly prominent when the network is
1388 near capacity. Furthermore, the simple instantiation of a recurrent architecture
1389 presented here still performed below humans, particularly under very low visibility
1390 conditions. It is conceivable that more complex architectures that take into account
1391 the known lateral connections in every layer as well as top-down connections in
1392 visual cortex might improve performance even further. Additionally, future
1393 extensions will benefit from incorporating other cues that help in pattern
1394 completion such as relative positions (front/behind), segmentation, movement,
1395 source of illumination, and stereopsis, among others.

1396

1397 **Mixed training regime**

1398 All the computational results shown in the main text and discussed thus far
1399 involve training models *exclusively* with whole objects and testing performance with

1400 images of partially visible objects. Here we discuss a “mixed training” regime where
 1401 the models are trained with access to partially visible objects. As emphasized in the
 1402 main text, these are weaker models since they show less extrapolation (from
 1403 partially visible objects to other partially visible objects as opposed to from whole
 1404 objects to partially visible objects) and they depart from the typical ways of
 1405 assessing invariance to object transformations (e.g. training at one rotation and
 1406 testing at other rotations). Furthermore, humans do not require this type of
 1407 additional training as described in the novel object experiments reported in **Figures**
 1408 **S8** and **S9**. Despite these caveats, the mixed training regime is interesting to explore
 1409 because it seems natural to assume that, at least in some cases, humans may be
 1410 exposed to both partially visible objects and their whole counterparts while learning
 1411 about objects. We emphasize that we cannot directly compare models that are
 1412 trained only with whole objects and models that are trained with both whole objects
 1413 and partially visible ones.

1414 We considered two different versions of RNN models that were trained to
 1415 reconstruct the feature representations of the whole objects from the feature
 1416 representations of the corresponding partial objects. These models were based on a
 1417 mixed training regime whereby both whole objects and partial objects were used
 1418 during training. The state at time $t > 0$ was computed as the activation of the
 1419 weighted sum of the previous state and the input from the previous
 1420 layer: $\mathbf{h}_t = \text{ReLU}(\mathbf{W}_h \mathbf{h}_{t-1}, \mathbf{x}_t)$ where $\text{ReLU}(z) = \max(0, z)$. The loss function was the
 1421 mean squared Euclidean distance between the features from the partial objects and
 1422 the features from the whole objects. Specifically, the RNN was iterated for a fixed
 1423 number of time steps ($t_{\max} = 4$) after the initial feed-forward pass, keeping the input
 1424 from fc6 constant. Thus, letting $\mathbf{h}_{t_{\max}}^i$ be the RNN state at the last time step for a given
 1425 image i and ${}_{\text{whole}}\mathbf{h}_{t_0}^i$ be the feed-forward feature vector of the corresponding whole
 1426 image, the loss function has the form

$$1427 \quad E = \frac{1}{T_l} \sum_{i=1}^{T_l} \left[\frac{1}{T_u} \sum_{j=1}^{T_u} (h_{t_{\max}}^i[j] - {}_{\text{whole}}h_{t_0}^i[j])^2 \right]$$

1428 where j goes over all the T_u units in fc7 and i goes over all the T_l images in the
1429 training set. The RNN was trained in a cross validated fashion (5 folds) using the
1430 same cross validation scheme as with the feed-forward models and using the
1431 RMSprop algorithm for optimization. In RNN₅, the weights of the RNN were trained
1432 with 260 objects for each fold. All of the partial objects from the psychophysics
1433 experiment for the given 260 objects, as well as one copy of the original 260 images,
1434 were used to train the RNN for the corresponding split. In the case where the input
1435 to the RNN was the original image itself, the network did not change its
1436 representation over the recurrent iterations. Given the high number of weights to
1437 be learned by the RNN as compared to the number of training examples, the RNNs
1438 overfit fairly quickly. Therefore, early stopping (10 epochs) was implemented as
1439 determined from the validation set, i.e., we used the weights at the time step where
1440 the validation error was minimal.

1441 To evaluate the extent of extrapolation across categories, we considered an
1442 additional version, RNN₁. In RNN₁, the recurring weights were trained using objects
1443 from only one category and the model was tested using objects from the remaining
1444 4 categories. In all RNN versions, once \mathbf{W}_h was fixed, classification performance was
1445 assessed using a linear SVM, as with the feed-forward models. Specifically, the SVM
1446 boundaries were trained using the responses from the feed-forward model to the
1447 whole objects and performance was evaluated using the representation at different
1448 time steps of recurrent computation.

1449 The RNN₅ model had 40962 recurrent weights trained on a subset of the
1450 objects from all five categories. The RNN₅ model matched or surpassed human
1451 performance (**Figure S11**). Considering all levels of visibility, the RNN₅ model
1452 performed slightly above human levels ($p=3 \times 10^{-4}$, Chi-squared test). While the RNN₅
1453 model can extrapolate across objects and categorize images of partial objects that it
1454 has not seen before, it does so by exploiting features that are similar for different
1455 objects within the 5 categories in the experiment. RNN₁, a model where the
1456 recurrent weights were trained using solely objects from one of the categories and
1457 performance was evaluated using objects from the remaining 4 categories, did not
1458 perform any better than the purely feed-forward architecture ($p=0.05$, Chi-squared

1459 test). Upon inspection of the fc7 representation, we observed that several of the
1460 features were sparsely represented across categories. Therefore, the recurrent
1461 weights in RNN₁ only modified a fraction of all the possible features, missing many
1462 important features to distinguish the other objects. Thus, the improvement in
1463 RNN₅ is built upon a sufficiently rich dictionary of features that are shared among
1464 objects within a category. These results show that recurrent neural networks
1465 trained with subsets of the partially visible objects can achieve human level
1466 performance, extrapolating across objects, as long as they are trained with a
1467 sufficiently rich set of features.

1468 We also evaluated the possibility of training the bottom-up model (AlexNet)
1469 using the mixed training regime and the same loss function as with RNN₅ and RNN₁,
1470 i.e. the Euclidean distance between features of whole and occluded images. Using
1471 the fc7 representation of the AlexNet model trained with partially visible objects
1472 also led to a model that either matched or surpassed human level performance at
1473 most visibility levels (**Figure S11**). The bottom-up model in the mixed training
1474 regime showed slightly worse performance than humans at very high visibility
1475 levels, including whole objects, perhaps because of the extensive fine-tuning with
1476 partially visible objects (note performance above humans at extremely low visibility
1477 levels). Within the mixed-training regimes, the RNN₅ model slightly outperformed
1478 the bottom-up model (**Figure S11**).

1479 A fundamental distinction between the models presented in the text,
1480 particularly RNN_h, and the models introduced here, is that the mixed training
1481 models require training with partial objects from the same categories in which they
1482 will be evaluated. Although the specific photographs of objects used in the
1483 psychophysics experiments presented here were new to the subjects, humans have
1484 extensive experience in recognizing similar objects from partial information. It
1485 should also be noted that there is a small number of partially visible images in
1486 ImageNet, albeit not with such low visibility levels as the ones explored here, and all
1487 the models considered here were pre-trained using ImageNet. Yet, the results
1488 shown in **Figures S8-S9** demonstrate that humans can recognize objects shown
1489 under low visibility conditions even when they have had no experience with partial

1490 objects of a specific category and have had only minimal experience with the
1491 corresponding whole objects.

1492

1493 **Temporal scale for recurrent computations**

1494 The models presented here, and several discussions in the literature,
1495 schematically and conceptually separate feed-forward computations from within-
1496 layer recurrent computations. Physiological signals arising within ~ 150 ms after
1497 stimulus onset have been interpreted to reflect largely feed-forward processing (1,
1498 3, 5, 8, 10, 11, 43), whereas signals arising in the following 50 to 100 ms may reflect
1499 additional recurrent computations (27, 44, 45). This distinction is clearly an
1500 oversimplification: the dynamics of recurrent computations can very well take place
1501 quite rapidly and well within ~ 150 ms of stimulus onset (46). Rather than a
1502 schematic initial feed-forward path followed by recurrent signals within the last
1503 layer in discrete time steps as implemented in RNN_h , cortical computations are
1504 based on continuous time and continuous interactions between feed-forward and
1505 within-layer signals (in addition to top-down signals). A biologically plausible
1506 implementation of a multi-layered spiking network including both feed-forward and
1507 recurrent connectivity was presented in ref. (46), where the authors estimated that
1508 recurrent signaling can take place within ~ 15 ms of computation per layer. Those
1509 time scales are consistent with the results shown here. Recurrent signals offer
1510 dynamic flexibility in terms of the amount of computational processing. Under noisy
1511 conditions (an injected noise term added to modify the input to each layer in (46),
1512 more occlusion in our case, and generally any internal or external source of noise),
1513 the system can dynamically use more computations to solve the visual recognition
1514 challenge.

1515 **Figures 4C-F, S10, S11, and S12** show dynamics evolving over tens of
1516 discrete recurrent time steps. The RNN_h model performance and correlation with
1517 humans saturate within approximately 10-20 recurrent steps (**Fig. 4C-F**).
1518 Membrane time constants of 10-15 ms (47) and one time constant per recurrent
1519 step would necessitate hundreds of milliseconds. Instead, the behavioral and
1520 physiological delays accompanying recognition of occluded objects occur within a

1521 delay of 50 to 100 ms (**Fig. 1-2, S12**) (25, 48), which are consistent with a
1522 continuous time implementation of recurrent processing (46).

1523

1524 **3. Supplementary Figures Legends**

1525

1526 **Figure S1: Robust performance with occluded stimuli**

1527 We measured categorization performance with masking (solid lines) or without
1528 masking (dashed lines) for **(A)** partial and **(B)** occluded stimuli on a set of 16
1529 exemplars belonging to 4 categories (chance = 25%, dashed lines). There was no
1530 overlap between the 14 subjects that participated in **(A)** and the 15 subjects that
1531 participated in **(B)**. The effect of backward masking was consistent across both
1532 types of stimuli. The black lines indicate whole objects and the gray lines indicate
1533 the partial and occluded objects. Error bars denote SEM.

1534

1535 **Figure S2: Example half-split reliability of psychophysics data**

1536 **Figure 2E** in the main text reports the masking index, a measure of how much
1537 recognition of each individual image is affected by backward masking. This measure
1538 is computed by averaging performance across subjects. In order to evaluate the
1539 variability in this metric, we randomly split the data into two halves and computed
1540 the masking index for each image for each half of the data. This figure shows one
1541 such split and how well one split correlates with the other split. **Figure 2F** shows
1542 error bars defined by computing standard deviations of the masking indices from
1543 100 such random splits.

1544

1545 **Figure S3: Bottom-up models can recognize minimally occluded images**

1546 **A.** Extension to **Figure 3A** showing that bottom-up models successfully recognize
1547 objects when more information is available (**Figure 3A** showed visibility values up
1548 to 35% whereas this figure extends visibility up to 100%). The format and
1549 conventions are the same as those in **Figure 3A**. The black dotted line shows
1550 interpolated human performance between the psychophysics experimental values
1551 measured at 35% and 100% visibility levels.

1552 **(B)** Stochastic neighborhood embedding dimensionality reduction (t-SNE, **Methods**)
1553 to visualize the fc7 representation in the AlexNet model for whole objects (open
1554 circles) and partial objects (closed circles). Different categories are separable in this
1555 space, but the boundaries learned on whole objects did not generalize to the space of
1556 partial objects. The black arrow shows a schematic example of model distance
1557 definition, from an image of a partial face (green circle) to the average face centroid
1558 (black cross).

1559

1560 **Figure S4: All of the purely feed-forward models tested were impaired under**
1561 **low visibility conditions**

1562 The human, AlexNet-pool5 and AlexNet-fc curves are the same ones shown in
1563 **Figure 3A** and are reproduced here for comparison purposes. This figure shows
1564 performance for several other models: VGG16-fc2, VGG19-fc2, ResNet50-flatten,
1565 inceptionV3-mixed10, VGG16-block5 (see text for references). In all cases, these
1566 models were pre-trained to optimize performance under ImageNet 2012 and there
1567 was no additional training (see also **Figure S5**). An expanded version of this figure
1568 with many other layers and models can be found on our web site:

1569 http://klab.tch.harvard.edu/resources/Tangetal_RecurrentComputations.html

1570

1571 **Figure S5: Fine-tuning did not improve performance under heavy occlusion**

1572 The human and fc7 curves are the same ones shown in **Figure 3A** and are
1573 reproduced here for comparison purposes. The pre-trained AlexNet network used
1574 in the text was fine tuned using back-propagation with the set of *whole* images from
1575 the psychophysics experiment (in contrast with the pre-trained Alexnet network
1576 which was trained using the Imagenet 2012 data set). The fine-tuning involved all
1577 layers (**Methods**).

1578

1579 **Figure S6: Correlation between RNN_h model and human performance for**
1580 **individual objects as a function of time**

1581 At each time step in the recurrent neural network model (RNN_h), the scatter plots
1582 show the relationship between the model's performance on individual partial

1583 exemplar objects and human performance. Each dot is an individual exemplar
1584 object. In **Figure 4E** we report the average correlation coefficient across all
1585 categories.

1586

1587 **Figure S7: Adding recurrent connectivity to VGG16 also improved**
1588 **performance**

1589 This Figure parallels the results shown in **Figure 4B** for AlexNet, here using the
1590 VGG16 network, implemented in keras (**Methods**). The results shown here are
1591 based on using 4096 units from the fc1 layer. The red curve (vgg16-fc1)
1592 corresponds to the original model without any recurrent connections. The
1593 implementation of the RNN_h model here (VGG16-fc1-Hopfield) is similar to the one
1594 in **Figure 4B**, except that here we use the VGG16 fc1 activations instead of the
1595 AlexNet fc7 activations. An expanded version of this figure with similar results for
1596 several other layers and models can be found on our web site:

1597 http://klab.tch.harvard.edu/resources/Tangetal_RecurrentComputations.html

1598

1599 **Figure S8: Robust recognition of *novel* objects under low visibility conditions**

1600 **A.** Single exemplar from each of the 5 novel object categories (**Methods**).
1601 **(B-C)** Behavioral performance for the unmasked (**B**) and masked (**C**) trials. The
1602 experiment was identical to the one in **Figure 1** and the format of this figure follows
1603 that in **Figure 1F-G**. The colors denote different SOAs. Error bars=SEM. Dashed line
1604 = chance level (20%). Bin size=2.5%. Note the discontinuity in the x-axis to report
1605 performance for whole objects (100% visibility). **(D)** Average recognition
1606 performance as a function of the stimulus onset asynchrony (SOA) for partial objects
1607 (same data and conventions as **B-C**, excluding 100% visibility). Error bars=SEM.
1608 Performance was significantly degraded by masking (solid) compared to the
1609 unmasked trials (dotted) ($p < 0.0001$, Chi-squared test, d.f.=4).

1610

1611 **Figure S9: The performance of feed-forward and recurrent computational**
1612 **models for *novel* objects was similar to those for known object categories**

1613 **A.** Performance of feed-forward computational models (format as in **Figure 3A**) for
1614 novel objects.
1615 **B.** Performance of the recurrent neural network RNN_h (format as in **Figure 4B**) for
1616 novel objects.
1617 **C.** Temporal evolution of the feature representation for RNN_h (format as in **Figure**
1618 **4C**). The colors and greek letters denote the five object categories (see examples in
1619 **Figure S8A**).
1620 **D.** Performance of RNN_h as a function of recurrent time for novel objects (format as
1621 in **Figure 4D**).

1622

1623 **Figure S10: Side-by-side comparison of neurophysiological signals,**
1624 **psychophysics and computational model**

1625 **A.** Adaptation of Figure 6C from Tang et al 2014. This figure shows the dynamics of
1626 decoding object information for whole objects and (black) and partial objects (gray)
1627 from neurophysiological recordings as a function of time post stimulus onset (see
1628 Tang et al 2014 for details).

1629 **B.** Reproduction of **Figure 1H** (behavior).

1630 **C.** Reproduction of **Figure 4F** (RNN_h model).

1631 Above each subplot, the experiment schematic highlights that part **A** involves no
1632 masking and fixed SOA = 150 ms whereas parts **B** and **C** involve masking and
1633 variable SOAs. The inset in part **C** directly overlays the results of the RNN_h model in
1634 part **C** onto the results of the psychophysics experiment in part **B**. In order to create
1635 this plot, we mapped 0 time steps to 25ms, 256 time steps to 150 ms and linearly
1636 interpolated the time steps in between.

1637

1638 **Figure S11: Mixed training regimes.**

1639 **A.** This figure follows the format of **Fig3A, 4B** and **S3, S4, S5, S7, S9A-B**. The black
1640 line shows human performance and is copied from **Fig. 3A**. The green and blue lines
1641 show the recurrent model (RNN_5) and bottom-up model (AlexNet fc7), respectively,
1642 trained in a mixed regime that included the occluded objects with visibility levels
1643 within the gray rectangle (the same ones used to evaluate human psychophysics

1644 performance). In the RNN5 model, there were ~16 million weights trained (all-to-all
1645 in the fc7 layer) whereas in the Alexnet fc7 model, there were ~60 million weights
1646 trained (all the weights across layers in the Alexnet model). Cross-validated test
1647 performance is shown here as well as in the other figures throughout the
1648 manuscript. As noted in the text, we emphasize that this figure involves a different
1649 training regime from the ones in the previous figures and therefore one cannot
1650 directly compare performance with the previous figures.

1651 **B.** This figure follows the format of **Fig. 4E**. The green and blue bars show the
1652 correlation between human and model for the recurrent model and bottom-up
1653 model, respectively, both trained using occluded objects. The gray rectangle shows
1654 human-human correlation, see **Fig. 4E** for details..

1655

1656 **Figure S12: Image-by-image comparison between RNNh model performance** 1657 **and human performance in the masked condition**

1658 Expanding on **Figure 4E**, this figure shows the correlation coefficient between
1659 human recognition performance in the masked condition (**Figure 1B**) at a given
1660 SOA (y-axis) and RNN_h model performance at a given time step (x-axis). The top row
1661 shows the unmasked condition (**Figure 1A**). In this figure, there is no mask for the
1662 model (see **Figure 4F** for model performance with a mask). The computation of the
1663 correlation coefficient follows the same procedure illustrated in **Figure S6** and **4E**.
1664 The color scale for the correlation coefficient is shown on the right. As an upper
1665 bound and as shown in **Figure 4E**, the correlation coefficient between different
1666 human subjects was 0.41 for the unmasked condition. The yellow boxes highlight
1667 the highest correlation for a given SOA value.

1668

1669 **4. Author contributions**

1670 Conceptualization: HT, BL, MS, DC, GK

1671 Physiology experiment design: HT, GK

1672 Physiological data collection and analyses: HT

1673 Psychophysics experiment design: HT, BL, MS, CM, GK

1674 Psychophysics data collection: HT, BL, MS, AP, JO, WH, CM

1675 Computational models: HT, BL, MS, DC, CM, GK

1676 Resources: DC, GK

1677 Manuscript writing: HT, BL, MS, GK

1678

1679 **5. Data availability**

1680 All relevant data and code (including image databases, behavioral measurements,
1681 physiological measurements and computational algorithms) are publicly available
1682 through the lab's website and through the lab's GitHub page:

1683 http://klab.tch.harvard.edu/resources/Tangetal_RecurrentComputations.html

1684

1685 **6. References**

1686

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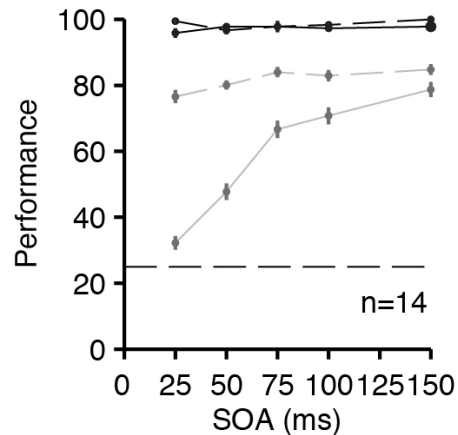
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- 1800

Supplementary Figure 1

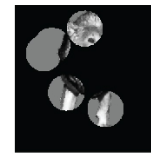
A



Partial



B



Occluded

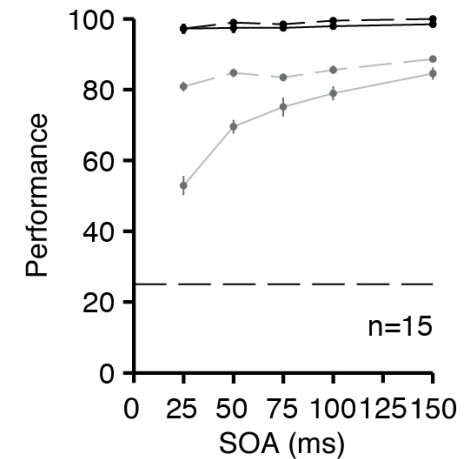


Figure S1: Robust performance with occluded stimuli

We measured categorization performance with masking (solid lines) or without masking (dashed lines) for **(A)** partial and **(B)** occluded stimuli on a set of 16 exemplars belonging to 4 categories (chance = 25%, dashed lines). There was no overlap between the 14 subjects that participated in **(A)** and the 15 subjects that participated in **(B)**. The effect of backward masking was consistent across both types of stimuli. The black lines indicate whole objects and the gray lines indicate the partial and occluded objects. Error bars denote SEM.

Supplementary Figure 2

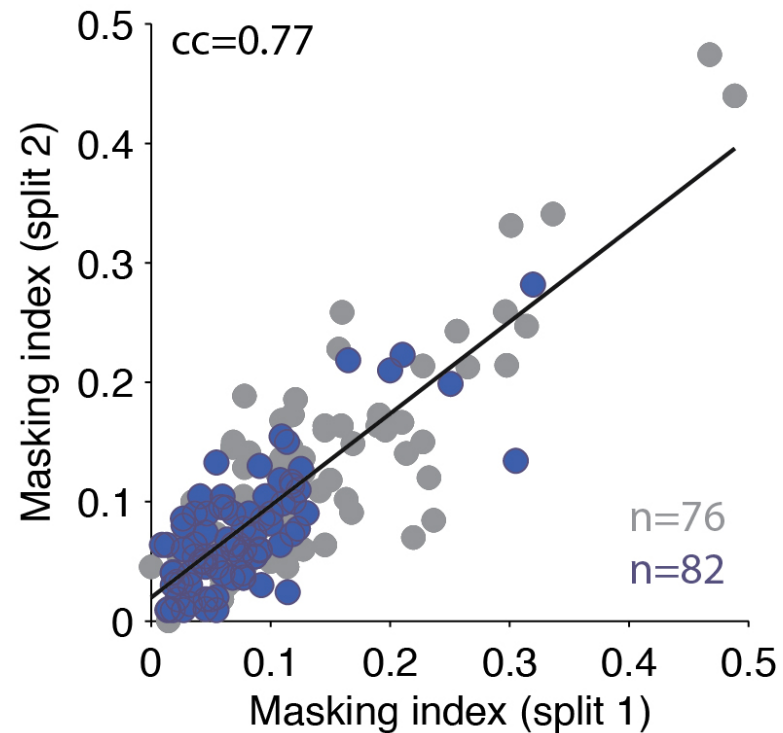


Figure S2: Example half-split reliability of psychophysics data

Figure 2E in the main text reports the masking index, a measure of how much recognition of each individual image is affected by backward masking. This measure is computed by averaging performance across subjects. In order to evaluate the variability in this metric, we randomly split the data into two halves and computed the masking index for each image for each half of the data. This figure shows one such split and how well one split correlates with the other split. **Figure 2F** shows error bars defined by computing standard deviations of the masking indices from 100 such random splits.

Supplementary Figure 3

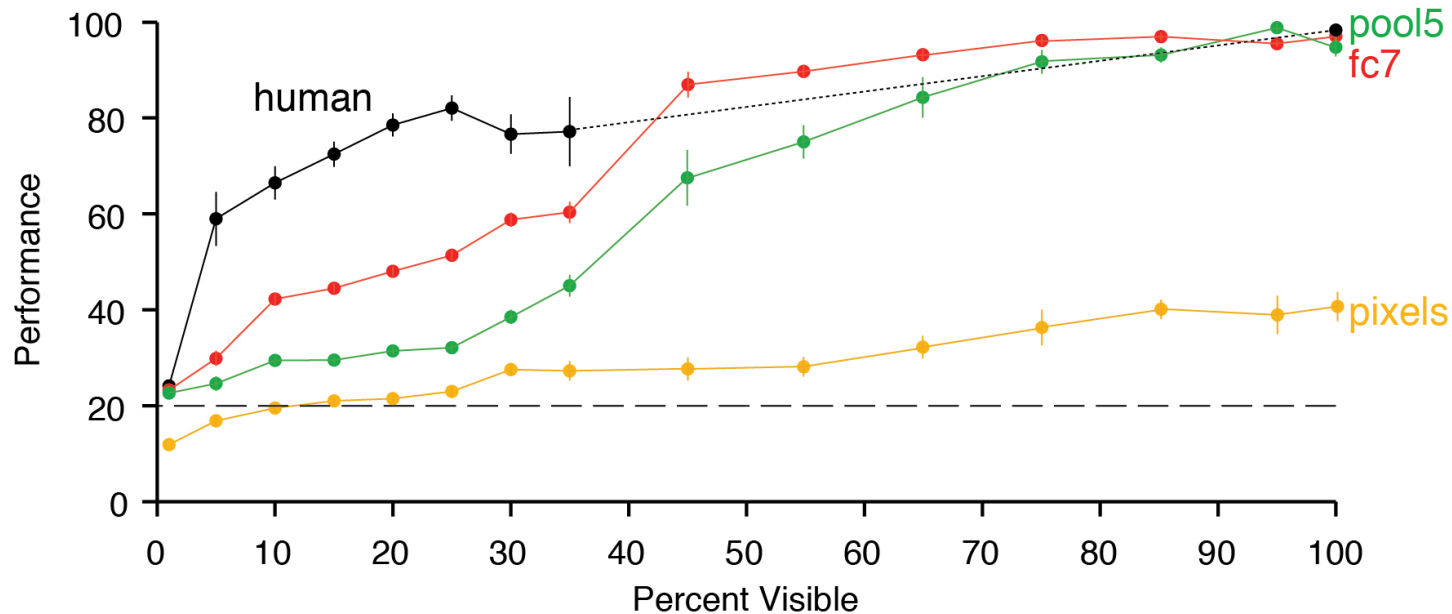


Figure S3: Bottom-up models can recognize minimally occluded images

Extension to **Fig. 3A** showing that bottom-up models successfully recognize objects when more information is available (**Fig. 3A** showed visibility values up to 35% whereas this figure extends visibility up to 100%). The format and conventions are the same as those in **Fig. 3A**. The black dotted line shows interpolated human performance between the psychophysics experimental values measured at 35% and 100% visibility levels.

Supplementary Figure 4

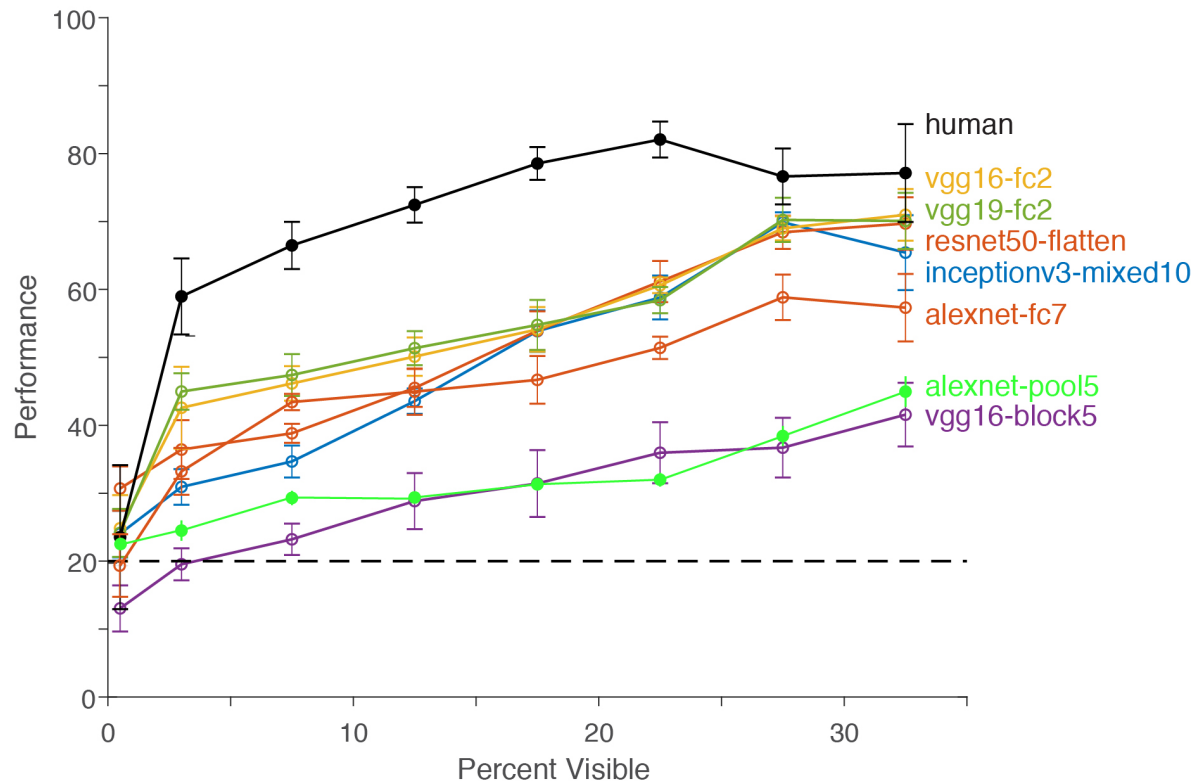


Figure S4: All of the purely feed-forward models tested were impaired under low visibility conditions

The human, AlexNet-pool5 and AlexNet-fc curves are the same ones shown in **Figure 3A** and are reproduced here for comparison purposes. This figure shows performance for several other models: VGG16-fc2, VGG19-fc2, ResNet50-flatten, inceptionV3-mixed10, VGG16-block5 (see text for references). In all cases, these models were pre-trained to optimize performance under ImageNet 2012 and there was no additional training (see also **Figure S5** for fine tuning results). An expanded version of this figure with many other layers and models can be found on our web site:

http://klab.tch.harvard.edu/resources/Tangetal_RecurrentComputations.html

Supplementary Figure 5

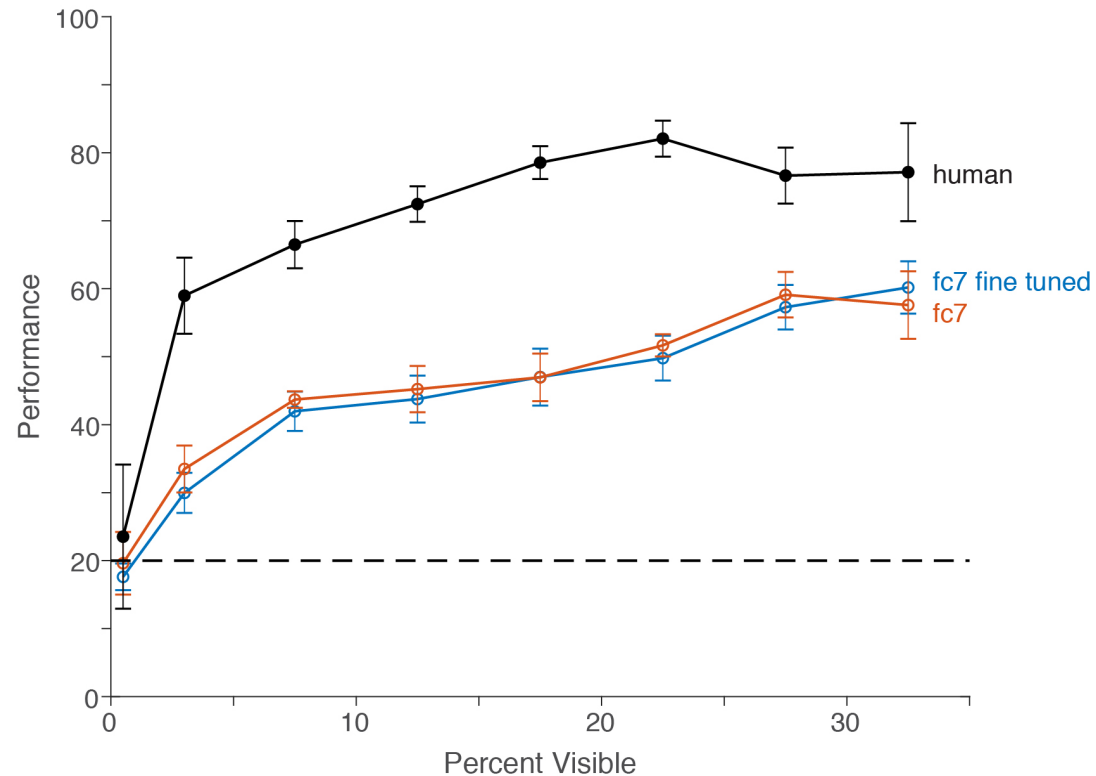


Figure S5: Fine-tuning did not improve performance under heavy occlusion

The human and fc7 curves are the same ones shown in **Figure 3A** and are reproduced here for comparison purposes. The pre-trained AlexNet network used in the text was fine tuned using back-propagation with the set of *whole* images from the psychophysics experiment (in contrast with the pre-trained Alexnet network which was trained using the Imagenet 2012 data set). The fine-tuning involved all layers (**Methods**).

Supplementary Figure 6

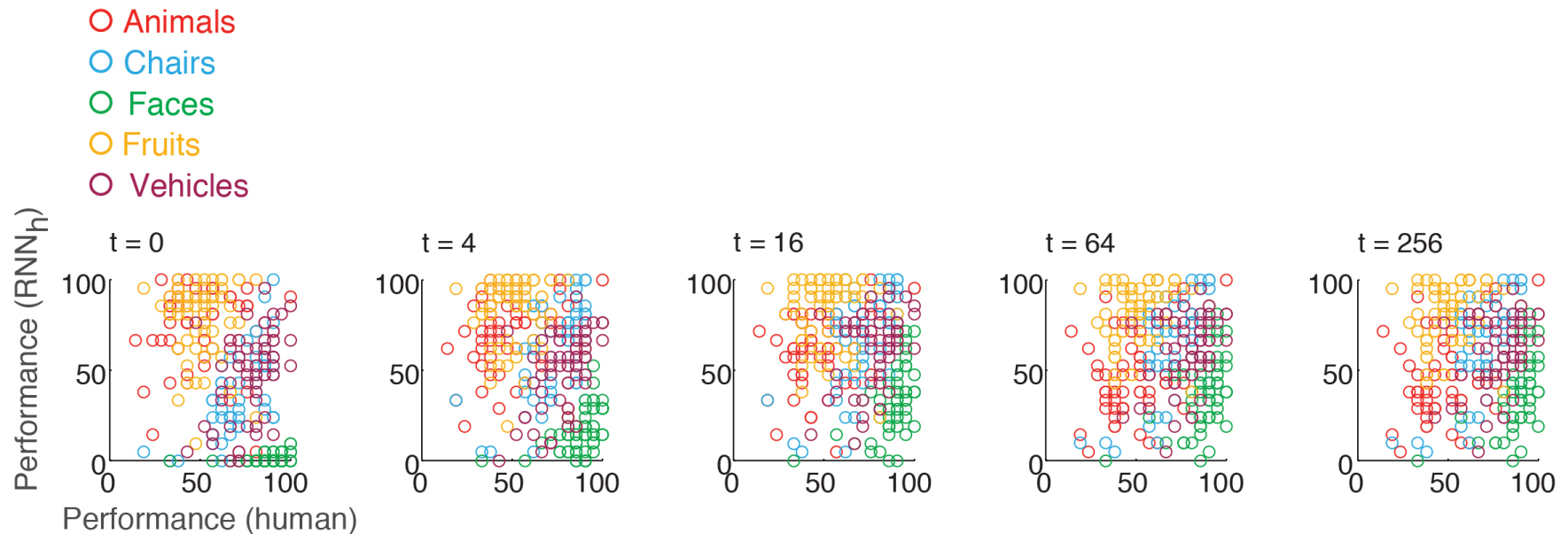


Figure S6: Correlation between RNN_h model and human performance for individual objects as a function of time

At each time step in the recurrent neural network model (RNN_h), the scatter plots show the relationship between the model's performance on individual partial exemplar objects and human performance. Each dot is an individual exemplar object. In **Fig. 4E** we report the average correlation coefficient across all categories.

Supplementary Figure 7

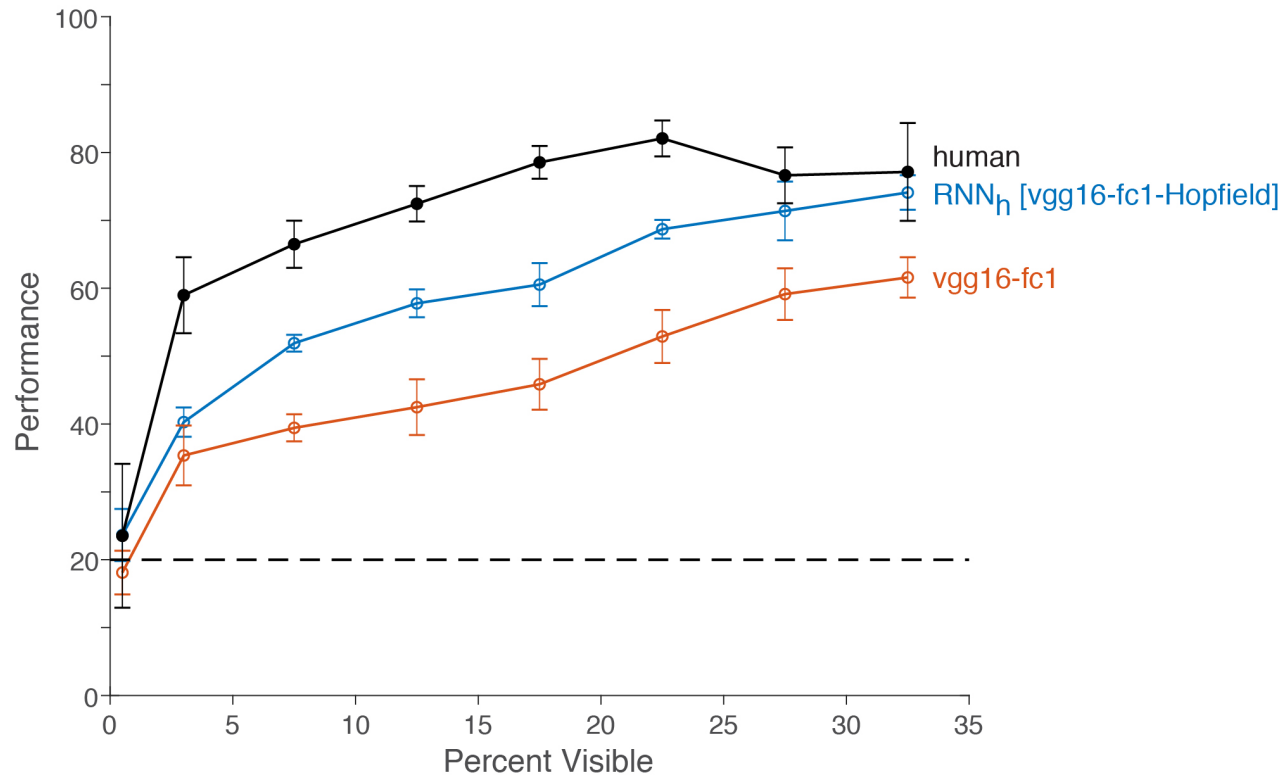


Figure S7: Adding recurrent connectivity to VGG16 also improved performance

This Figure parallels the results shown in **Figure 4B** for AlexNet, here using the VGG16 network, implemented in keras (**Methods**). The results shown here are based on using 4096 units from the fc1 layer. The red curve (vgg16-fc1) corresponds to the original model without any recurrent connections. The implementation of the RNN_h model here (VGG16-fc1-Hopfield) is similar to the one in **Figure 4B**, except that here we use the VGG16 fc1 activations instead of the AlexNet fc7 activations. An expanded version of this figure with similar results for several other layers and models can be found on our web site: http://klab.tch.harvard.edu/resources/Tangetal_RecurrentComputations.html

Supplementary Figure 8

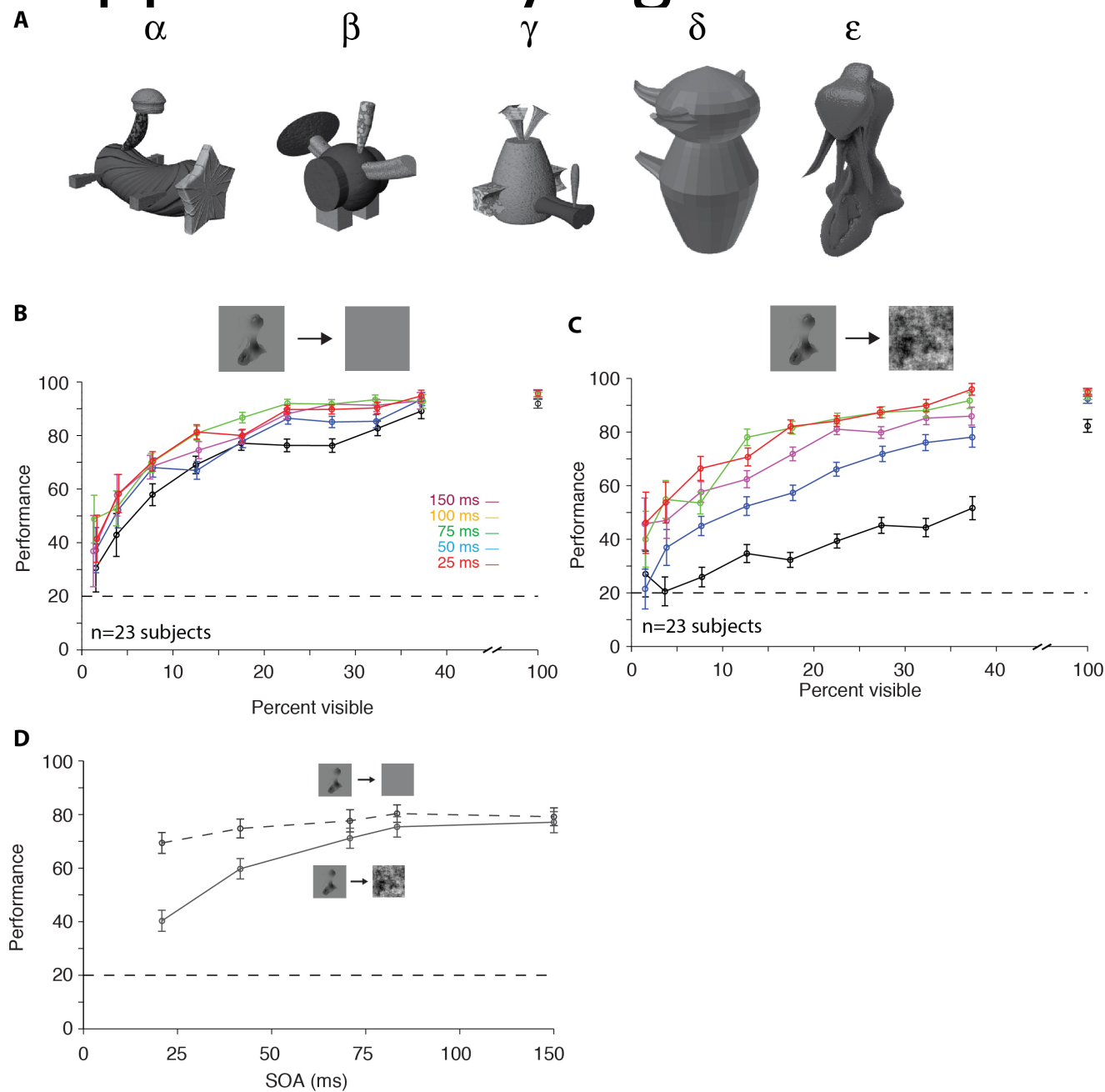


Figure S8: Robust recognition of novel objects under low visibility conditions

A. Single exemplar from each of the 5 novel object categories (**Methods**). **(B-C)** Behavioral performance for the unmasked (**B**) and masked (**C**) trials. The experiment was identical to the one in **Figure 1** and the format of this figure follows that in **Figure 1F-G**. The colors denote different SOAs. Error bars=SEM. Dashed line = chance level (20%). Bin size=2.5%. Note the discontinuity in the x-axis to report performance for whole objects (100% visibility). **(D)** Average recognition performance as a function of the stimulus onset asynchrony (SOA) for partial objects (same data and conventions as **B-C**, excluding 100% visibility). Error bars=SEM. Performance was significantly degraded by masking (solid) compared to the unmasked trials (dotted) ($p < 0.0001$, Chi-squared test, $d.f. = 4$).

Supplementary Figure 9

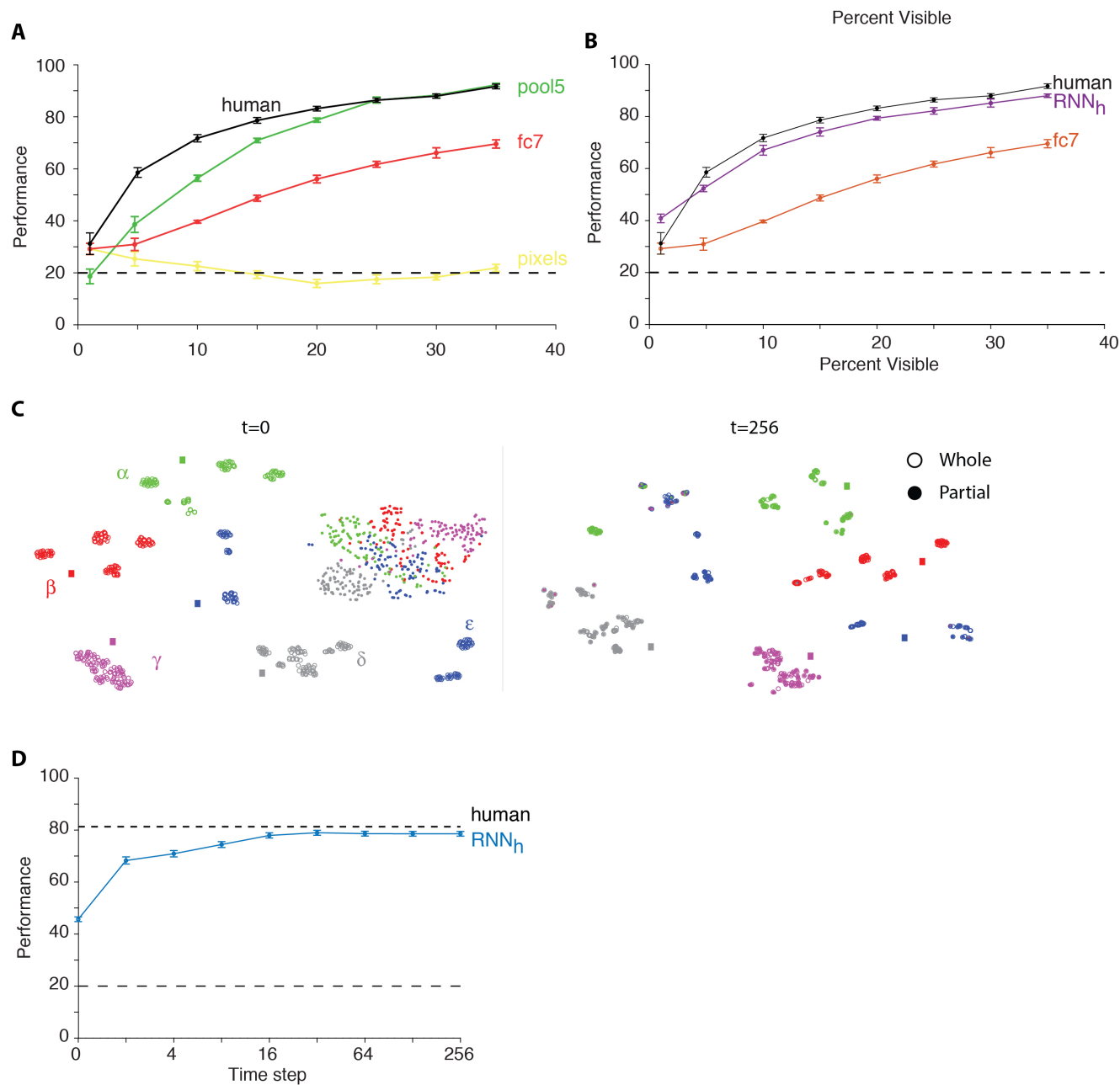


Figure S9: The performance of feed-forward and recurrent computational models for novel objects was similar to those for known object categories

A. Performance of feed-forward computational models (format as in **Figure 3A**) for novel objects.

B. Performance of the recurrent neural network RNN_h (format as in **Figure 4B**) for novel objects.

C. Temporal evolution of the feature representation for RNN_h (format as in **Figure 4C**). The colors and greek letters denote the five object categories (see examples in **Figure S8A**).

D. Performance of RNN_h as a function of recurrent time for novel objects (format as in **Figure 4D**).

Supplementary Figure 10

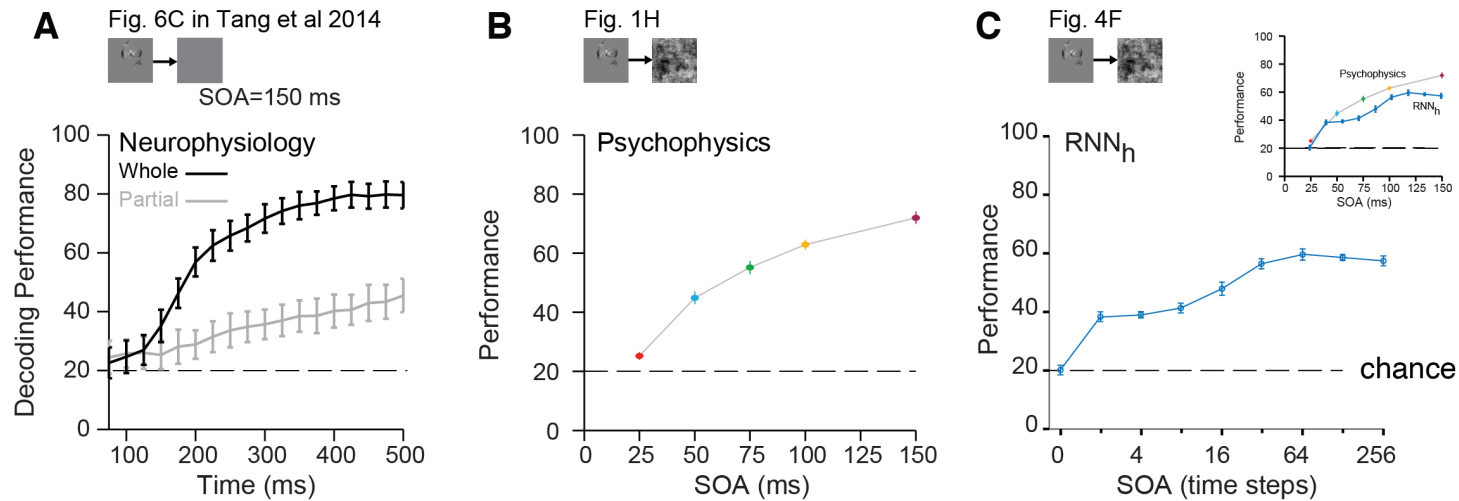


Figure S10: Side-by-side comparison of neurophysiological signals, psychophysics and computational model

A. Reproduction of Figure 6C from Tang et al 2014. This figure shows the dynamics of decoding object information for whole objects and (black) and partial objects (gray) from neurophysiological recordings as a function of time post stimulus onset (see Tang et al 2014 for details).

B. Reproduction of **Figure 1H** (behavior).

C. Reproduction of **Figure 4F** (RNN_h model).

Above each subplot, the experiment schematic highlights that **A** involves no masking and fixed SOA = 150 ms whereas **B**, **C** involve masking and variable SOAs. The inset in part **C** directly overlays the results of the RNN_h model in **C** onto the results of the psychophysics experiment in **B**. In order to create this plot, we mapped 0 time steps to 25ms, 256 time steps to 150 ms and linearly interpolated the time steps in between.

Supplementary Figure 11

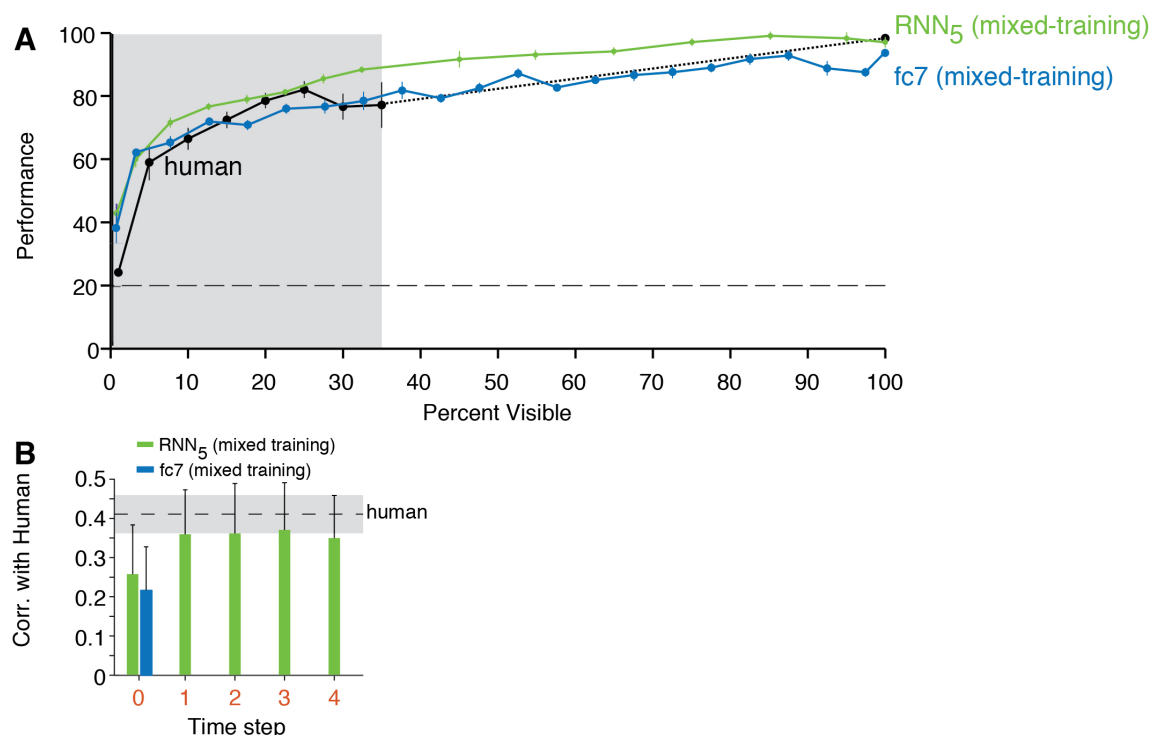


Figure S11: Mixed training regimes.

A. This figure follows the format of **Fig3A, 4B** and **S3A, S4, S5, S7, S9A-B**. The black line shows human performance and is copied from **Fig. 3A** for comparison purposes. The green and blue lines show the recurrent model (RNN₅) and bottom-up model (AlexNet fc7), respectively, trained in a mixed regime that included the occluded objects with visibility levels within the gray rectangle (the same ones used to evaluate human psychophysics performance). In the RNN₅ model, there were ~16 million weights trained (all-to-all in the fc7 layer) whereas in the Alexnet fc7 model, there were ~60 million weights trained (all the weights across layers in the Alexnet model). Cross-validated test performance is shown here as well as in the other figures throughout the manuscript. As noted in the text, we emphasize that this figure involves a different training regime from the ones in the previous figures (here the models are trained with occluded objects) and, therefore, one cannot directly compare performance in this figure with the previous figures.

B. This figure follows the format of **Fig. 4E**. The green and blue bars show the correlation between human and model for the recurrent model and bottom-up model, respectively, both trained using occluded objects. The gray rectangle shows human-human correlation, see **Fig. 4E** for details..

Supplementary Figure 12

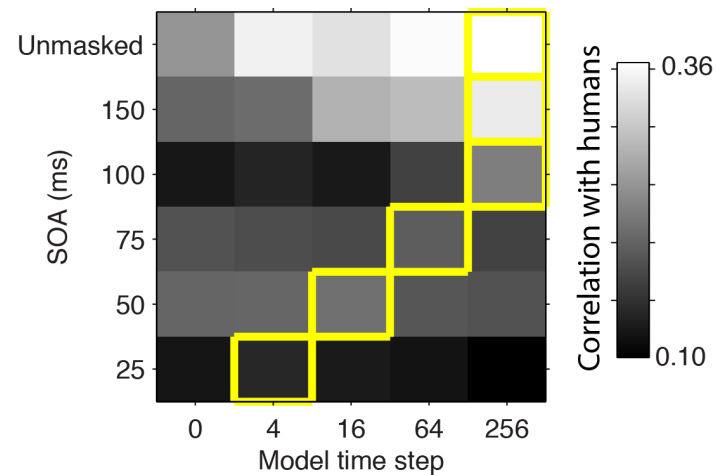


Figure S12: Image-by-image comparison between RNNh model performance and human performance in the masked condition

Expanding on **Figure 4E**, this figure shows the correlation coefficient between human recognition performance in the masked condition (**Figure 1B**) at a given SOA (y-axis) and RNNh model performance at a given time step (x-axis). The top row shows the unmasked condition (**Figure 1A**). In this figure, there is no mask for the model (see **Figure 4F** for model performance with a mask). The computation of the correlation coefficient follows the same procedure illustrated in **Figure S6** and **4E**. The color scale for the correlation coefficient is shown on the right. As an upper bound and as shown in **Figure 4E**, the correlation coefficient between different human subjects was 0.41 for the unmasked condition. The yellow boxes highlight the highest correlation for a given SOA value.