Supplementary Materials

Table S1: Parameters
Parameters used in our model.

Table S2: Key calibration targets and calibration values achieved
Calibration targets used in our model.

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This figure shows the total population size over the simulated period. Bars indicate target calibration range.

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Figure S3: Sensitivity analysis of TB incidence to recovery rate on IPT
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Figure S4: Sensitivity analysis of TB incidence to TB activation rate in HIV-positive individuals
This figure shows sensitivity of TB incidence to the rate at which HIV-positive individuals with latent TB progress to active TB disease.

Figure S5: Sensitivity analysis to the rate at which individuals who have recovered from TB cease IPT treatment
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Figure S7: Sensitivity analysis to the rate of HIV infection
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Figure S10: Effects of general adolescent IPT on TB incidence in HIV-positive population
This figure shows the impact of adolescent IPT on TB incidence in the HIV-positive population.

Figure S11: Effects of general adolescent IPT on TB incidence in HIV-positive adult population
This figure shows the impact of adolescent IPT on TB incidence in the adult HIV-positive population.

Figure S12: Conditions under which TB incidence could increase as a result of expanded IPT
This figure shows a parameter set in which it could be possible for IPT to increase TB incidence.

Table S3: Summary of one-way sensitivities of TB incidence in 2032
These tables summarize the numerical values of TB incidence in year 2032 of the model under key sensitivity analyses.

Figure S13: Impact of IPT access by demographic group
This figure compares the impact on general population TB incidence of IPT access in A) adolescents only, B) adults only, C) both adolescents and adults.

Figure S14: Multiple-way sensitivity analyses
This figure shows multi-way sensitivity analyses between key variables.

Figure S15: Effects of IPT in adolescents on various demographic groups
This figure shows the impact of IPT access in adolescents on TB incidence in A) adolescents, and B) adults

Equations
The equations of the model are included here.

Table S1: Parameters

TB Parameters[1]

Parameter	Description	Calibrated Value	Source
Transmission Rate	Rate of TB transmission	0.0000004	Sensitivity Analysis
Latency Rate Upon Infection in HIV-	Rate at which HIV- individuals infected with TB remain latently, rather than actively, infected	0.9	Abu-Raddad et al. 2009[1]*
Latency Rate Upon Infection in HIV+	Rate at which HIV+ individuals infected with TB remain latently, rather than actively, infected	0.5	Lawn et al. 2009[2]
Latency Rate Upon Infection in HIV+ on ART	Rate at which HIV+ individuals on ART infected with TB remain latently, rather than actively, infected	0.9	Abu-Raddad et al. 2009[1]*
TB Activation Rate in HIV-	Rate at which HIV- individuals experience latent TB activation	0.005	Horsburgh et al. 2004[3]
TB Activation Rate in HIV+	Rate at which HIV+ individuals experience latent TB activation	0.09	Sensitivity Analysis
TB Activation Rate in ART	Rate at which HIV+ individuals on ART experience latent TB activation	0.005	Horsburgh et al. 2004[3]
TB Treatment Rate	Rate at which TB is treated with combination therapy	0.9	WHO 2013[4]
Recovery Rate for Combination Therapy	Rate at which individuals with TB recover when treated with combination therapy	0.9	WHO 2013[4], Bacaer et al. 2008[5]*
Return to Latency After Combination Therapy	Rate at which individuals receiving combination therapy for active TB return to latency	0.08	Suen et al. 2014[6]*
Recovery Rate for IPT	Rate at which individuals recover on IPT	0.47	Zelner et al. 2014[7]
TB Activation Rate on IPT for HIV-	Rate at which HIV- individuals latently infected with TB experience TB activation	0.0001	Houben et al. 2014[8]*

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[1] With regard to the units of the TB and HIV parameters, each of these parameters is reported as the fraction of the population within a given compartment flowing out of that compartment into another specified by the model diagram per unit time

*Parameter taken as either an input or an output from a modeling publication

TB Activation Rate on IPT for HIV+	Rate at which HIV+ individuals latently infected with TB experience TB activation	0.0001	Houben et al. 2014[8]*, Selwyn et al. 1992[9], Lin et al. 2010[10]
Non-Cure on Combination Therapy	Rate at which individuals who have received combination therapy are not cured	0.02	Suen et al. 2014[6]*
Natural Cure Rate	Rate at which individuals with active TB experience spontaneous resolution to latent TB	0.1	Tiemersma et al. 2011[11]
Incorrect Diagnosis Rate in HIV-	Rate at which HIV- individuals who are actively infected with TB are incorrectly diagnosed as latently infected	0.00001	Boehme et al. 2011[12]
Incorrect Diagnosis Rate in HIV+	Rate at which HIV+ individuals not on ART who are actively infected with TB are incorrectly diagnosed as latently infected	0.0001	Boehme et al. 2011[12]
Incorrect Diagnosis Rate in HIV+ on ART	Rate at which HIV+ individuals on ART who are actively infected with TB are incorrectly diagnosed as latently infected	0.00001	Boehme et al. 2011[12]
Correct Move from IPT to Combination Therapy	Rate at which individuals who had incorrectly been placed on combination therapy are redirected to IPT	0.9	Boehme et al. 2011[12]
Re-infection adjustment	Factor to adjust for the possibility of re-infection with TB following treatment and cure	0.21	Andrews et al. 2012[13]*
IPT Access Rate	Rate at which individuals latently infected with TB are placed on IPT	0.05/0.5/0.9	Sensitivity Analysis
Default rate on IPT	Rate at which individuals on IPT default	0	Sensitivity Analysis
Rate of ceasing IPT	Rate at which individuals on IPT stop treatment	0	Sensitivity Analysis

HIV Parameters[2]

Parameter	Description	Value	Source
HIV Infection in Adolescents	Rate at which adolescents are infected with HIV	0.0000000039	Auvert et al. 2005[14]
HIV Infection in Adults	Rate at which adults are infected with HIV	0.0000000039	Calibration
Progression of CD4 Count	Rate at which individuals infected with HIV progress to ART eligibility (experience CD4 count decline)	0.13	Mellors et al. 1997[15]
Access to ART	Rate at which eligible HIV+ individuals are placed on ART	0.02/ 0.1	Calibration
Recovery of CD4 Count on ART	Rate at which HIV+ individuals on ART experience CD4 count recovery	0.01	Calibration

Demographic Parameters[3]

Parameter	Description	Value
Child Natural Mortality	Mortality rate among children not infected with TB or HIV	0.001
Adolescent Natural Mortality	Mortality rate among adolescents not infected with TB or HIV	0.009
Adult Natural Mortality	Mortality rate among adults not infected with TB or HIV	0.02
HIV+ Pre-ART Adolescent Mortality	Mortality rate among adolescents who are HIV+ and not yet eligible for ART	0.02
HIV+ Pre-ART Adult Mortality	Mortality rate among adults who are HIV+ and not yet eligible for ART	0.03
HIV+ ART-Eligible Adolescent Mortality	Mortality rate among adolescents who are HIV+ and eligible for ART	0.05
HIV+ ART-Eligible Adult Mortality	Mortality rate among adults who are HIV+ and eligible for ART	0.05
HIV+ on ART Low CD4 Count Adolescent Mortality	Mortality rate among adolescents who are HIV+ and on ART with low CD4 count	0.09

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[2] With regard to the units of the TB and HIV parameters, each parameter is reported as the fraction of the population within a given compartment flowing out of that compartment into another specified by the model diagram per unit time

[3] All demographic parameters were generated via calibration to the population structure

HIV+ on ART Low CD4 Count Adult Mortality	Mortality rate among adults who are HIV+ and on ART with low CD4 count	0.02
HIV+ on ART High CD4 Count Adolescent Mortality	Mortality rate among adolescents who are HIV+ and on ART with high CD4 count	0.09
HIV+ on ART High CD4 Count Adult Mortality	Mortality rate among adults who are HIV+ and on ART with high CD4 count	0.02
TB Adolescent Mortality	Mortality rate among adolescents who are infected with TB, and are HIV-	0.058
TB Adult Mortality	Mortality rate among adults who are infected with TB, and are HIV-	0.068
TB & HIV+ Pre-ART Adolescent Mortality	Mortality rate among adolescents who are infected with TB, and are HIV+, not yet eligible for ART	0.12
TB & HIV+ Pre-ART Adult Mortality	Mortality rate among adults who are infected with TB, and are HIV+, not yet eligible for ART	0.23
TB & HIV+ ART-Eligible Adolescent Mortality	Mortality rate among adolescents who are infected with TB, and are HIV+, and eligible for ART	0.17
TB & HIV+ ART-Eligible Adult Mortality	Mortality rate among adults who are infected with TB, and are HIV+, and eligible for ART	0.3
TB & HIV+ Low CD4 Count Adolescent Mortality	Mortality rate among adolescents who are infected with TB, and are HIV+, on ART with low CD4 count	0.07
TB & HIV+ Low CD4 Count Adult Mortality	Mortality rate among adults who are infected with TB, and are HIV+, on ART with low CD4 count	0.1
TB & HIV+ High CD4 Count Adolescent Mortality	Mortality rate among adolescents who are infected with TB, and are HIV+, on ART with high CD4 count	0.062
TB & HIV+ High CD4 Count Adult Mortality	Mortality rate among adults who are infected with TB, and are HIV+, on ART with high CD4 count	0.072
Births	Birth rate	0.03
Aging Rate from Children to Adolescents	Rate at which children age into the adolescent group	0.0769
Aging Rate from Adolescents to Adults	Rate at which adolescents age into the adult group	0.2

*Parameter is reported as an input or output from a modeling publication
Note: All parameters subject to adjustment for calibration

Table S2: Key calibration targets and calibration values achieved

Calibration Indicator	Target Value	Calibration Value Achieved	Source
Incidence	1117	1113.13	Global TB Report
Mortality	220	268.33	Global TB Report
HIV Prevalence in Adult Population	15	14.99	South African National HIV Incidence, Prevalence and Behavior Survey
ART Coverage among HIV+	31	30.91	South African National HIV Incidence, Prevalence and Behavior Survey
HIV-Positivity in TB Deaths	74	40.82	Global TB Report

Table S3: Summary of one-way sensitivities of TB incidence in 2032

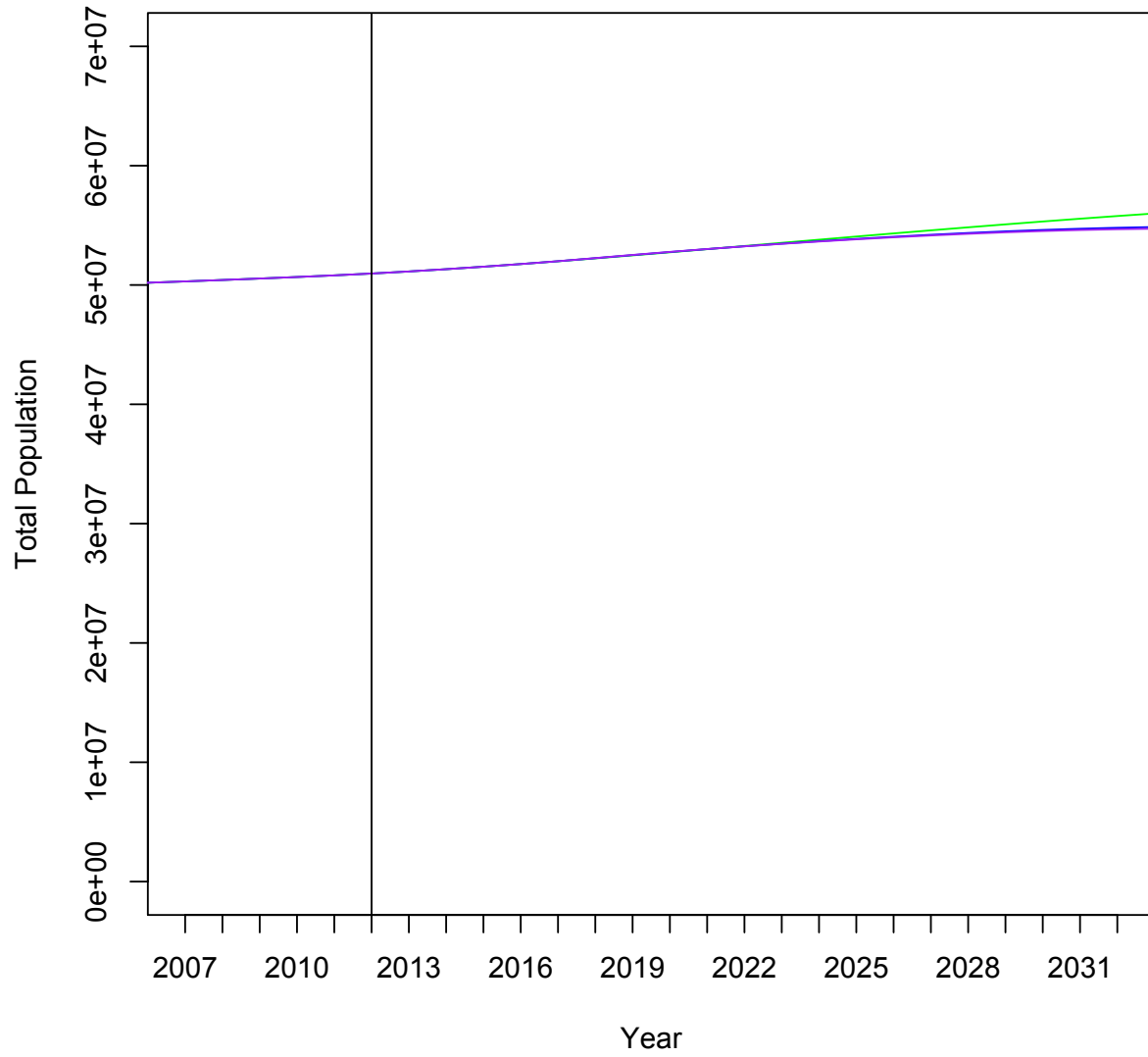| Sensitivity Analysis of TB Incidence to TB Transmission Rate	*Transmission rate*	0.0000001	0.0000002	0.0000003	0.0000005	0.0000006
	TB Incidence	1221	956	820	847	894
Sensitivity Analysis of TB Incidence to TB Transmission Rate	*Transmission rate*	0.0000007	0.0000008	0.0000009	0.000001	0.0000015
	TB Incidence	948	1008	1074	1148	1640

Sensitivity Analysis of TB Incidence to Recovery Rate on IPT	*Recovery rate*	0.1	0.3	0.6	0.9
	TB Incidence	1036.1274	838.5098	807.5356	799.5961

Sensitivity Analysis of TB Incidence to TB Activation Rate in HIV-positive individuals	*TB Activation rate in HIV-positive individuals*	0.005	0.05	0.2	0.5	0.9
	TB Incidence	726.4634	793.7456	822.9659	812.6559	808.0295

Sensitivity analysis to the rate at which individuals who have recovered from TB cease IPT treatment	*Rate of ceasing IPT*	0.05	0.1	0.15
	TB Incidence	1191.5818	1462.8285	1679.0699

Sensitivity analysis to the rate of HIV infection	HIV Infection rate	1E-10	1E-9	2E-9	3E-9	4E-9	5E-9	5.5E-09
	TB Incidence	707	699	694	724	829	1002	1090

Sensitivity analysis to birthrate	Birth rate	0.001	0.01	0.02	0.04
	TB Incidence	207.5082	338.2676	509.442	1559.0206

Effects to Expanding IPT to adults	TB Incidence	
		815.0284

Figure S1: Total population size

Total Population Size

Figure S2: Sensitivity analysis of TB incidence to TB transmission rate
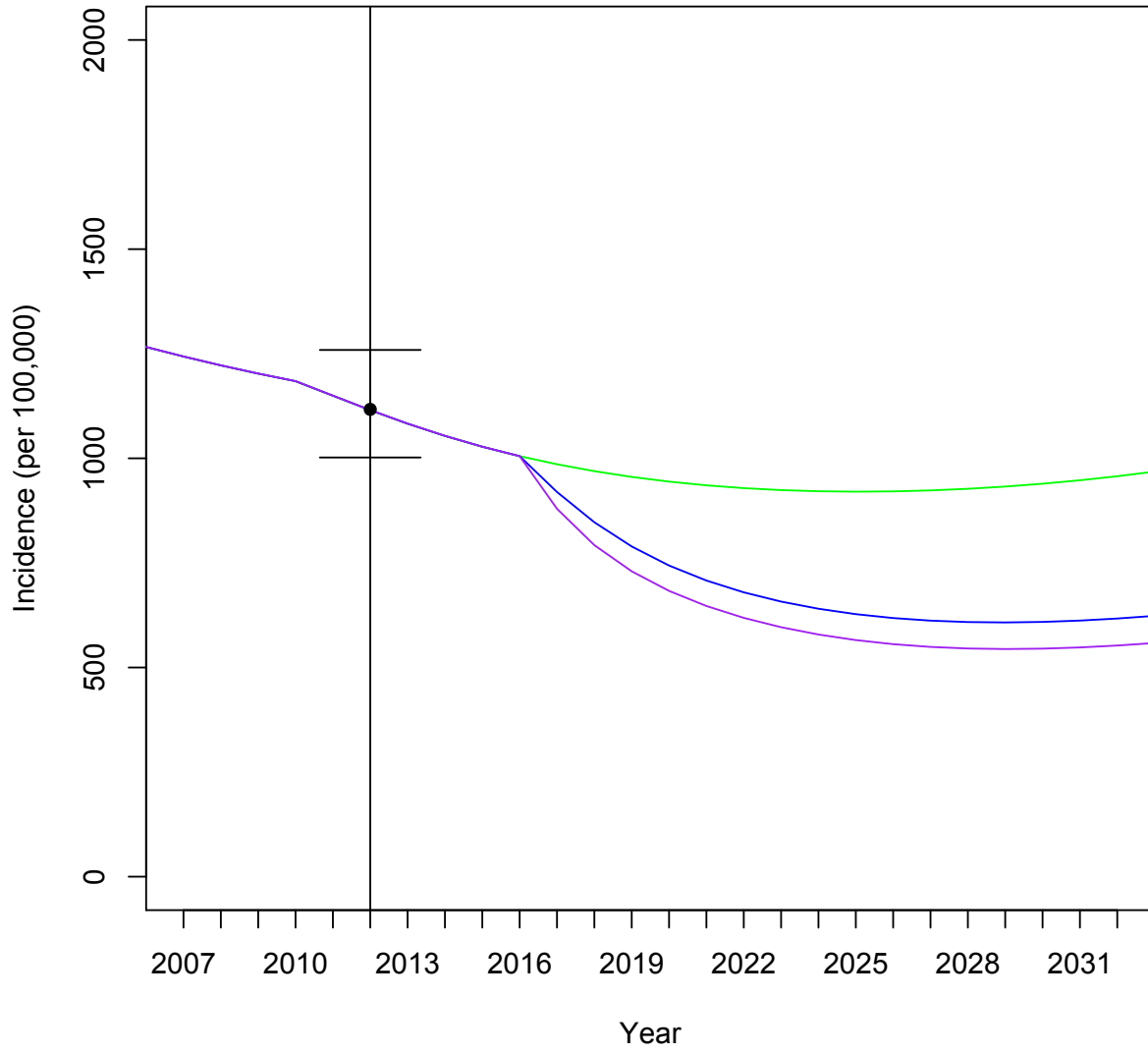
Transmission Rate of 0.0000001
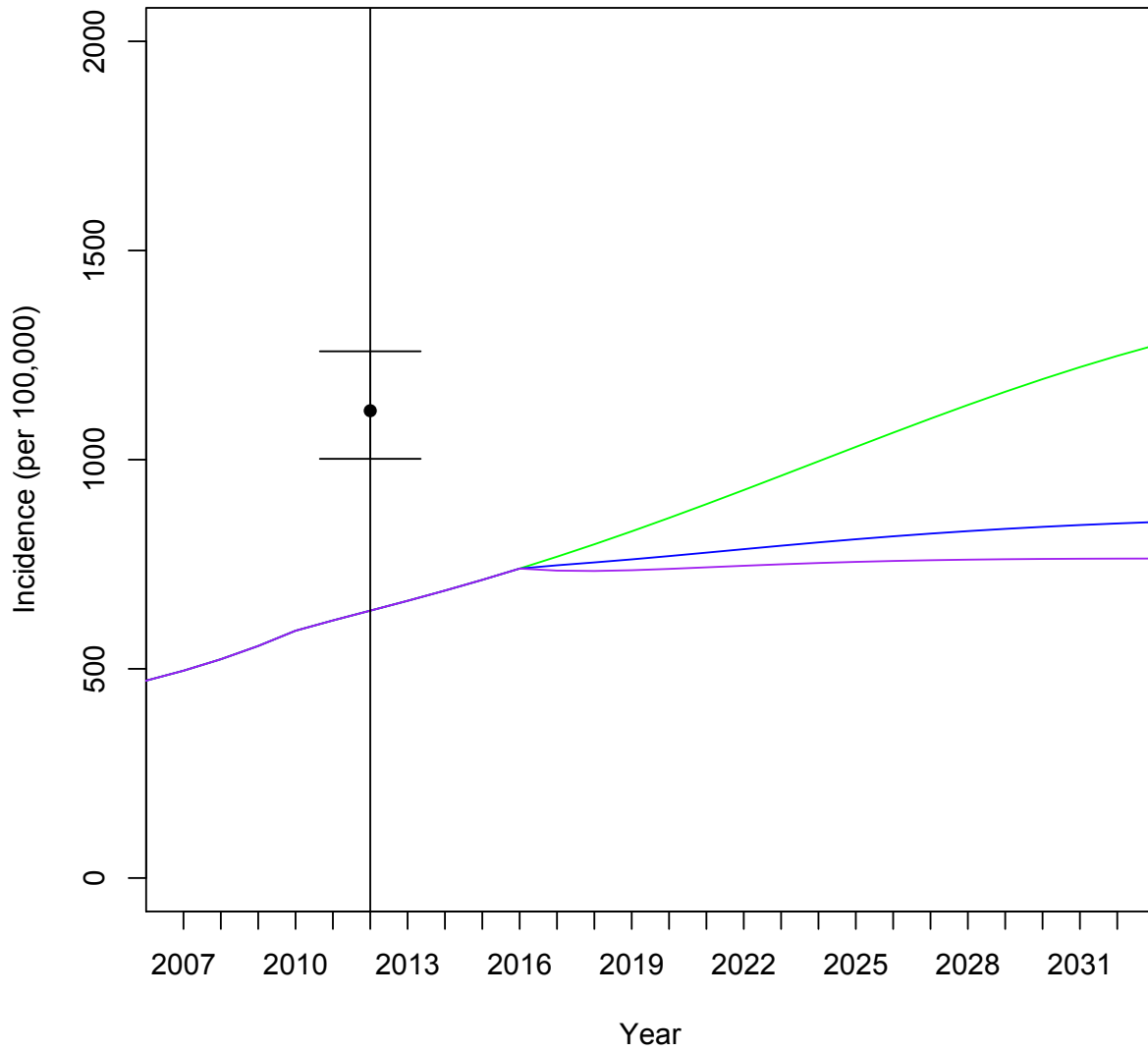
Transmission Rate of 0.0000002
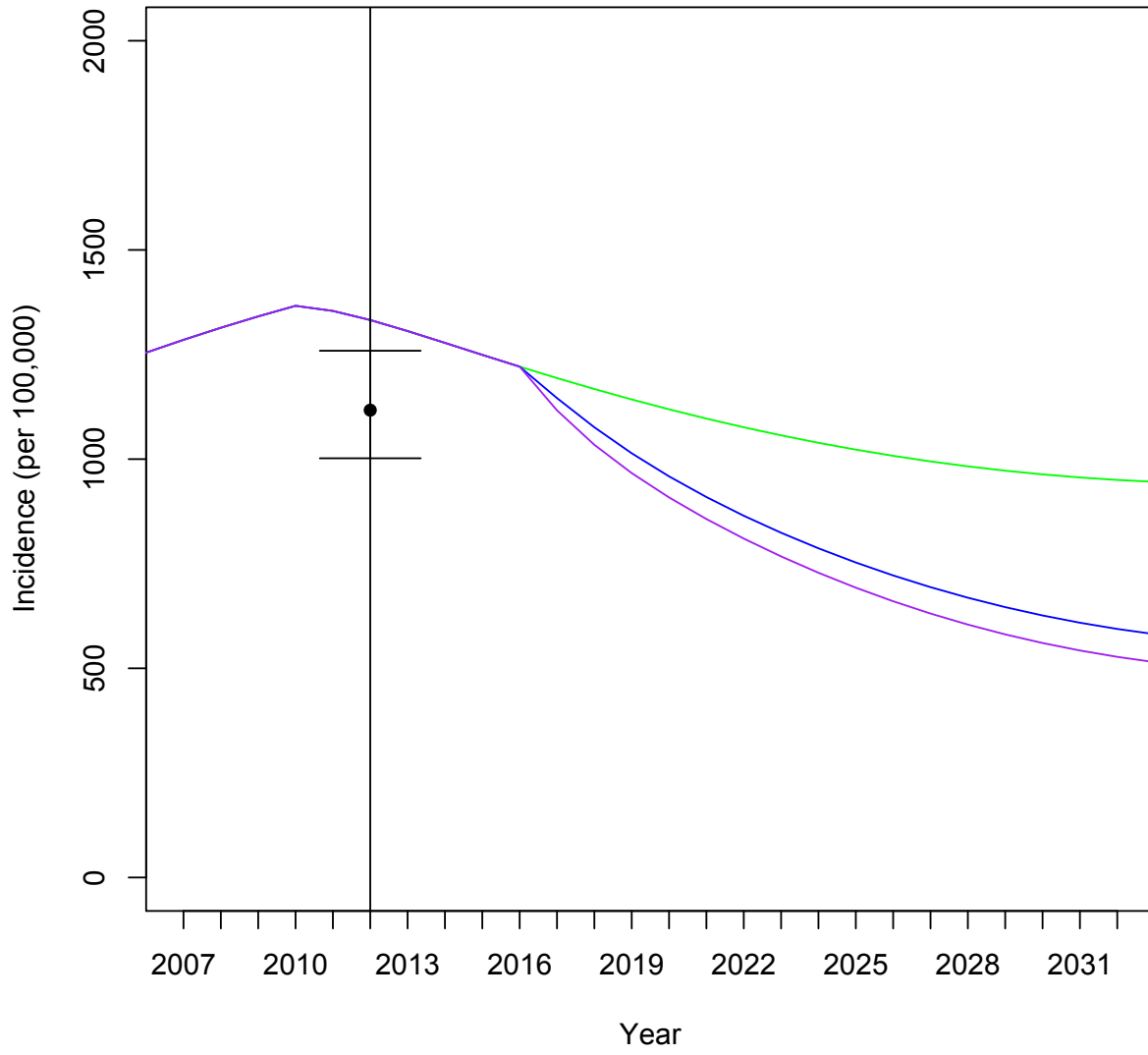
Transmission Rate of 0.0000003
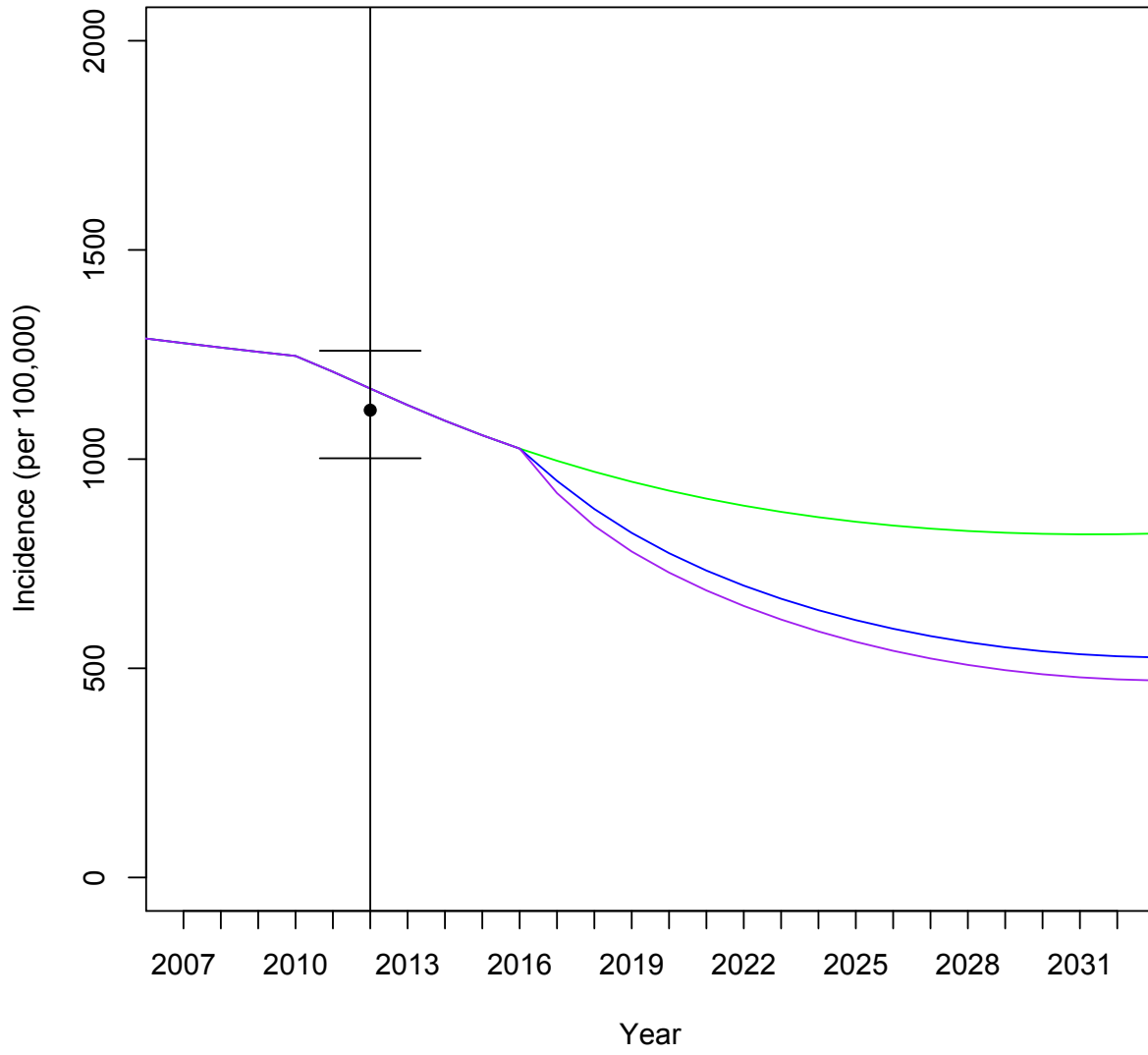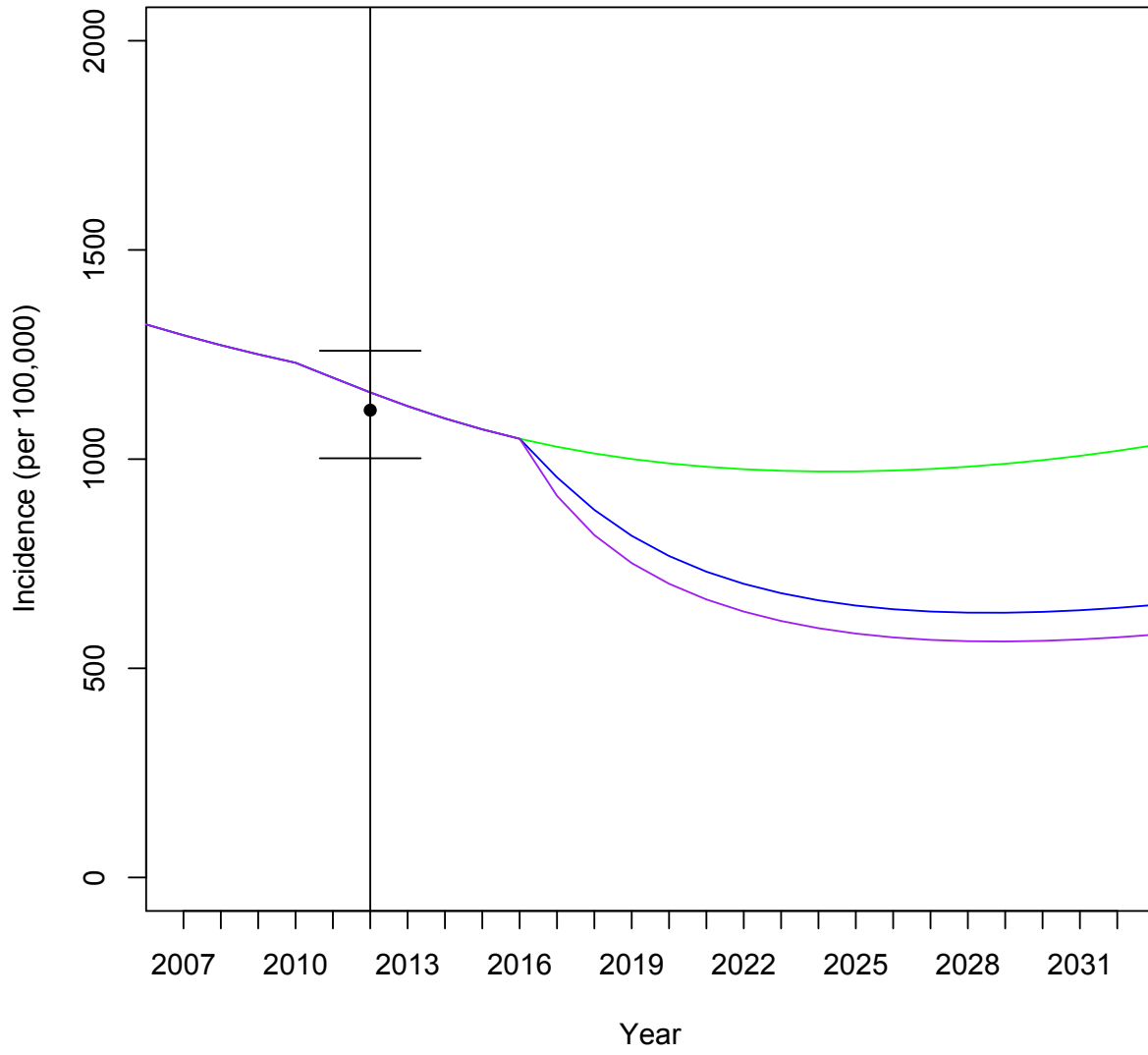
Transmission Rate of 0.0000005
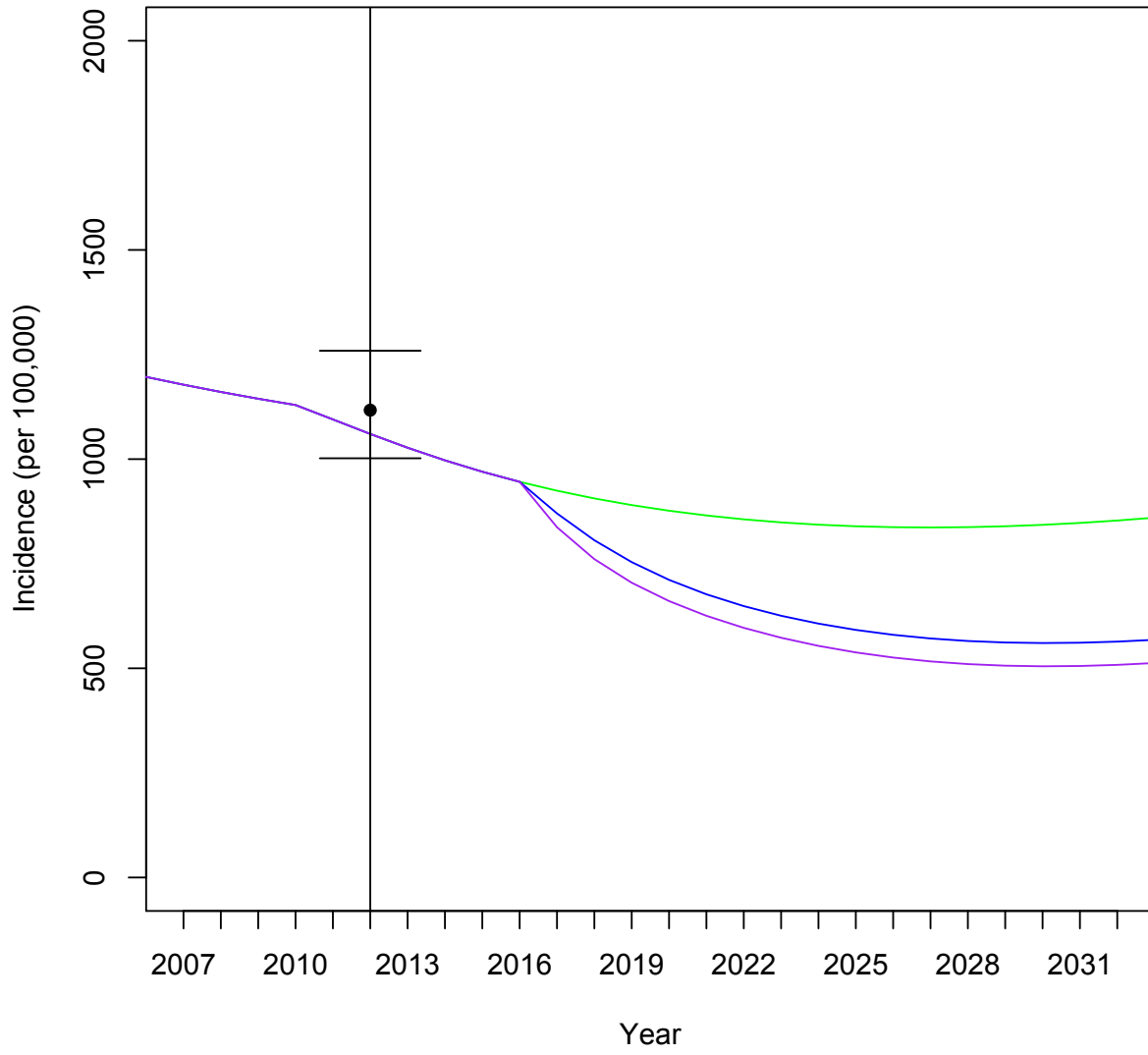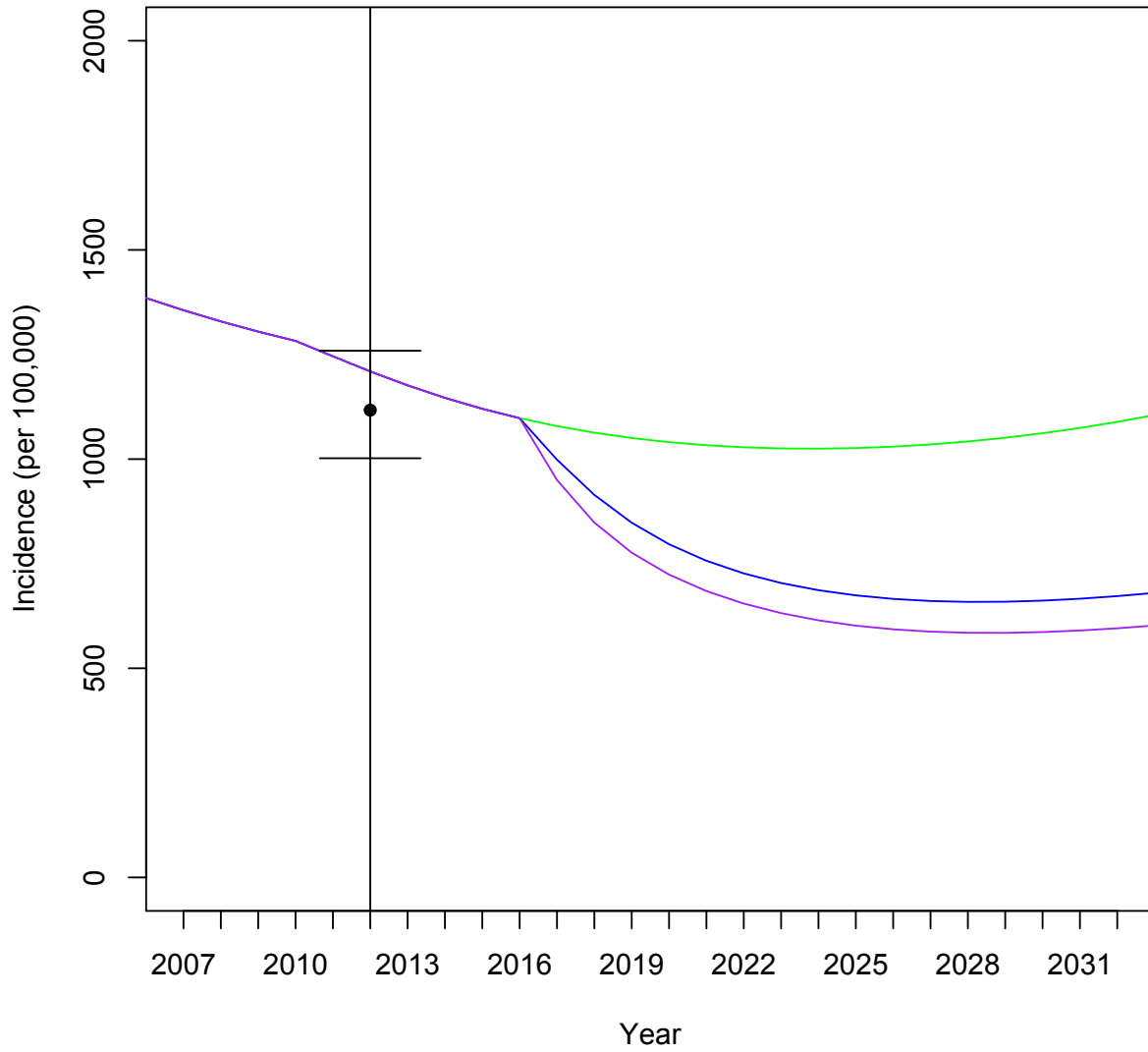
Transmission Rate of 0.0000006
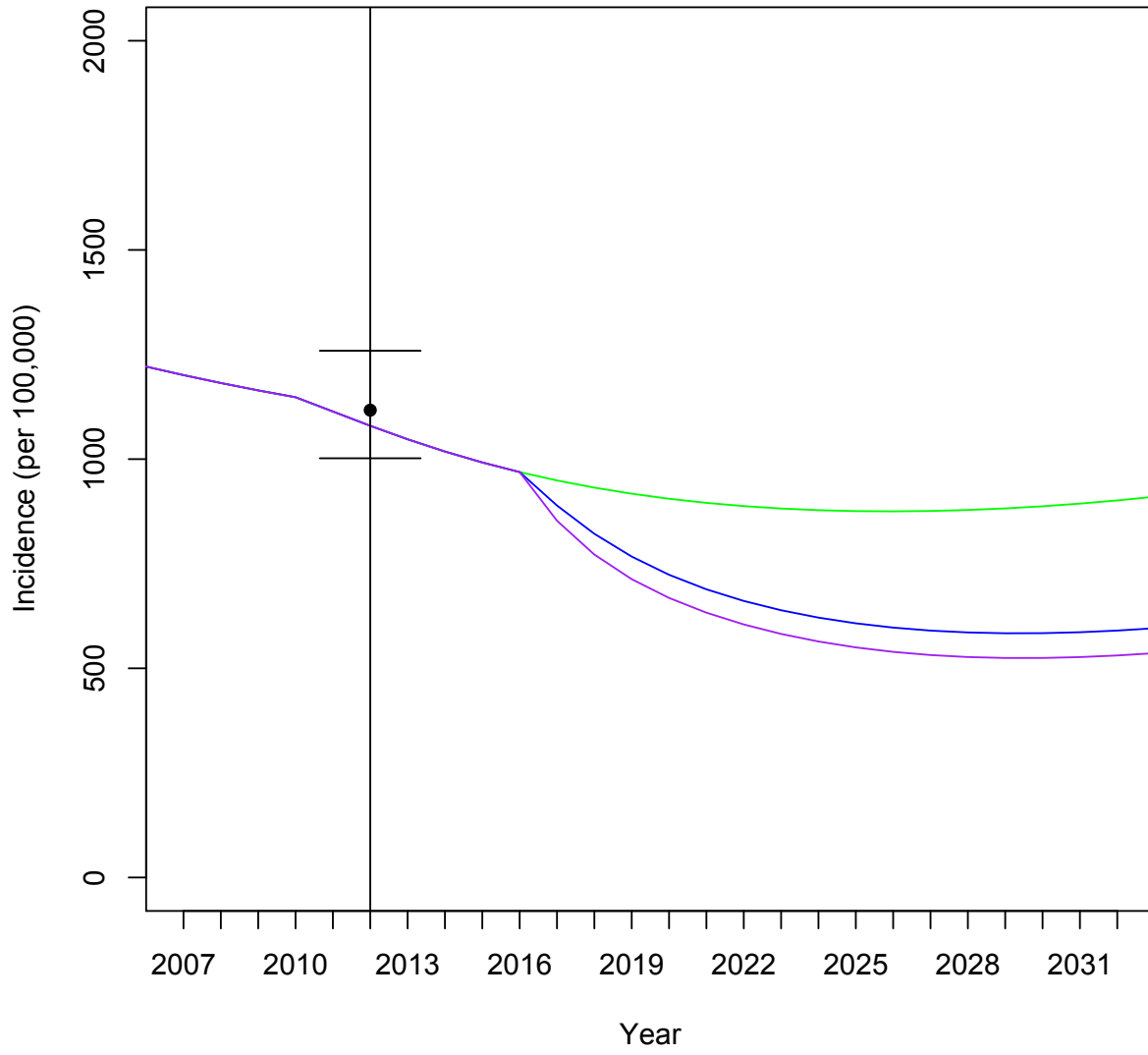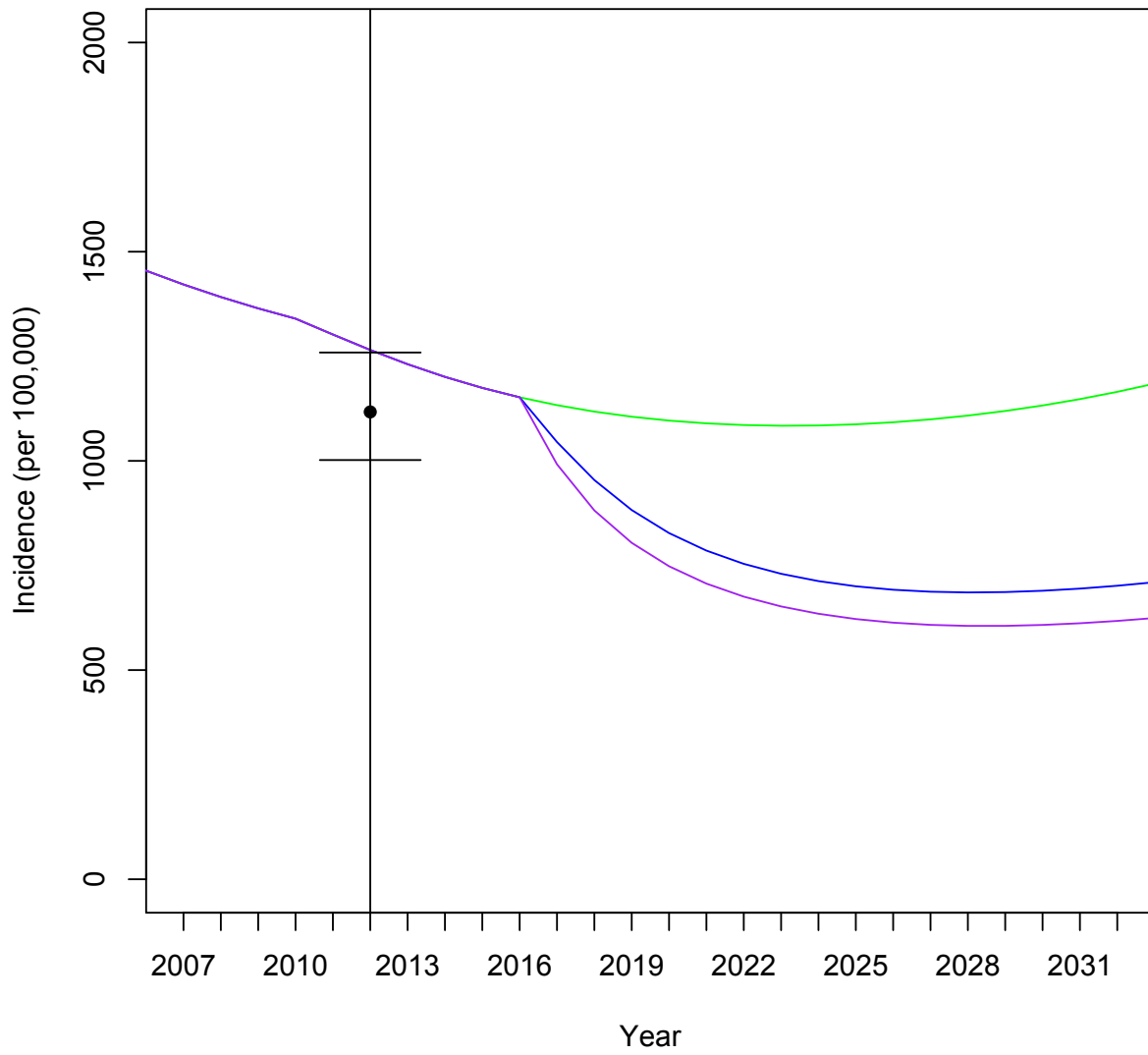
Transmission Rate of 0.0000007

Transmission Rate of 0.0000008

Transmission Rate of 0.0000009
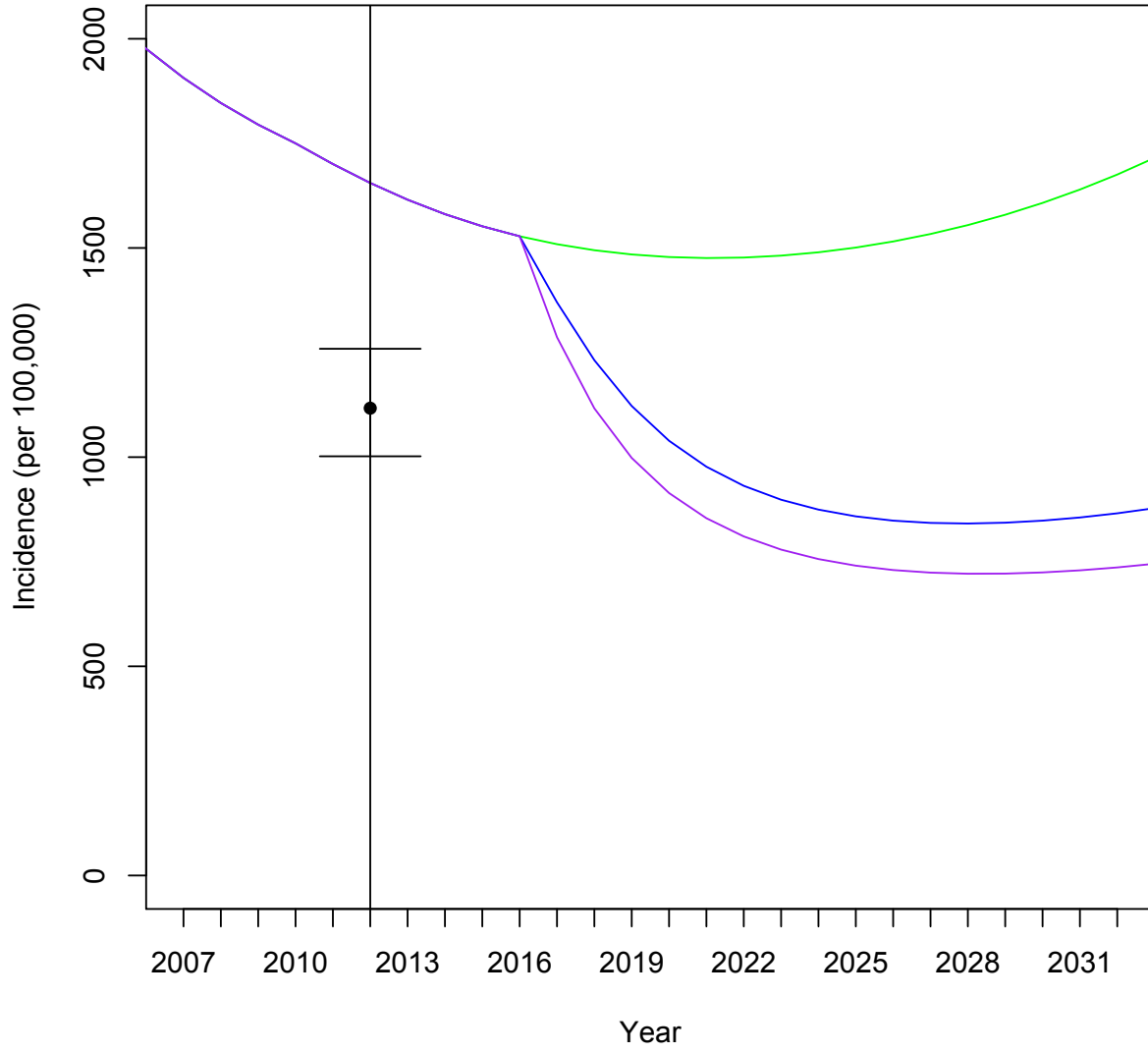
Transmission Rate of 0.000001

Transmission Rate of 0.0000015

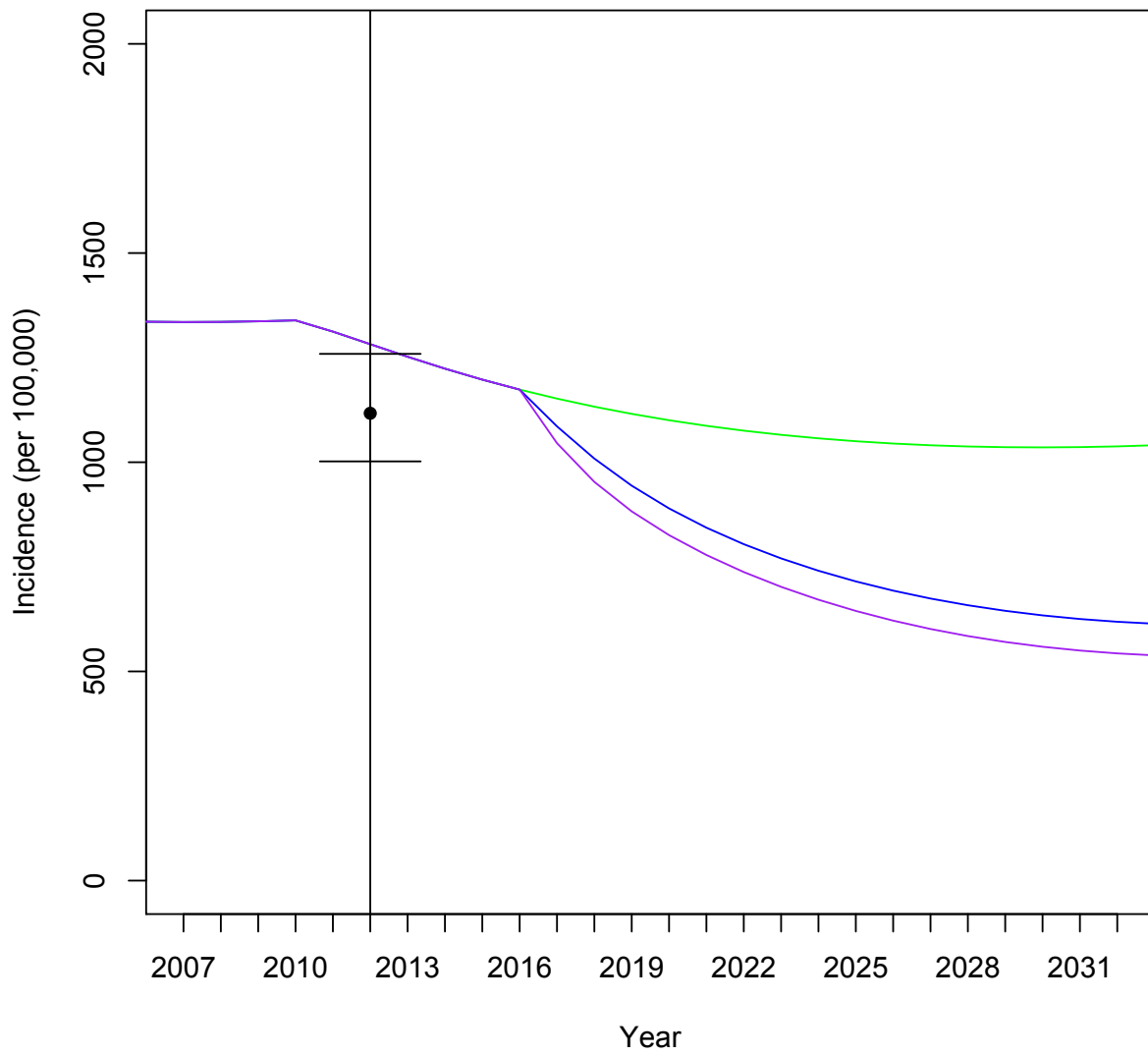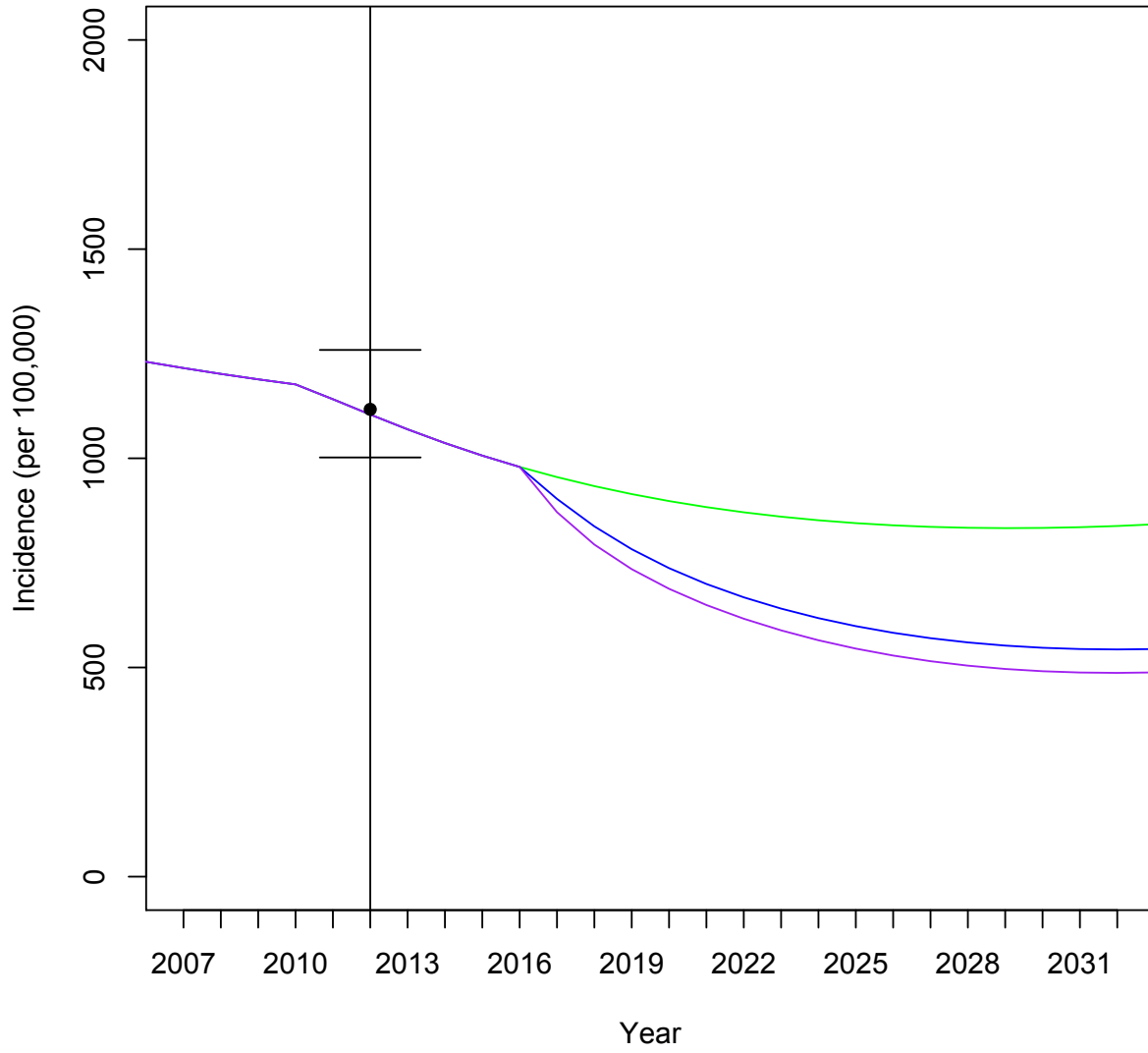*Figure S3: Sensitivity analysis of TB incidence to recovery rate on IPT*

IPT Recovery Rate of 0.1

IPT Recovery Rate: 0.6
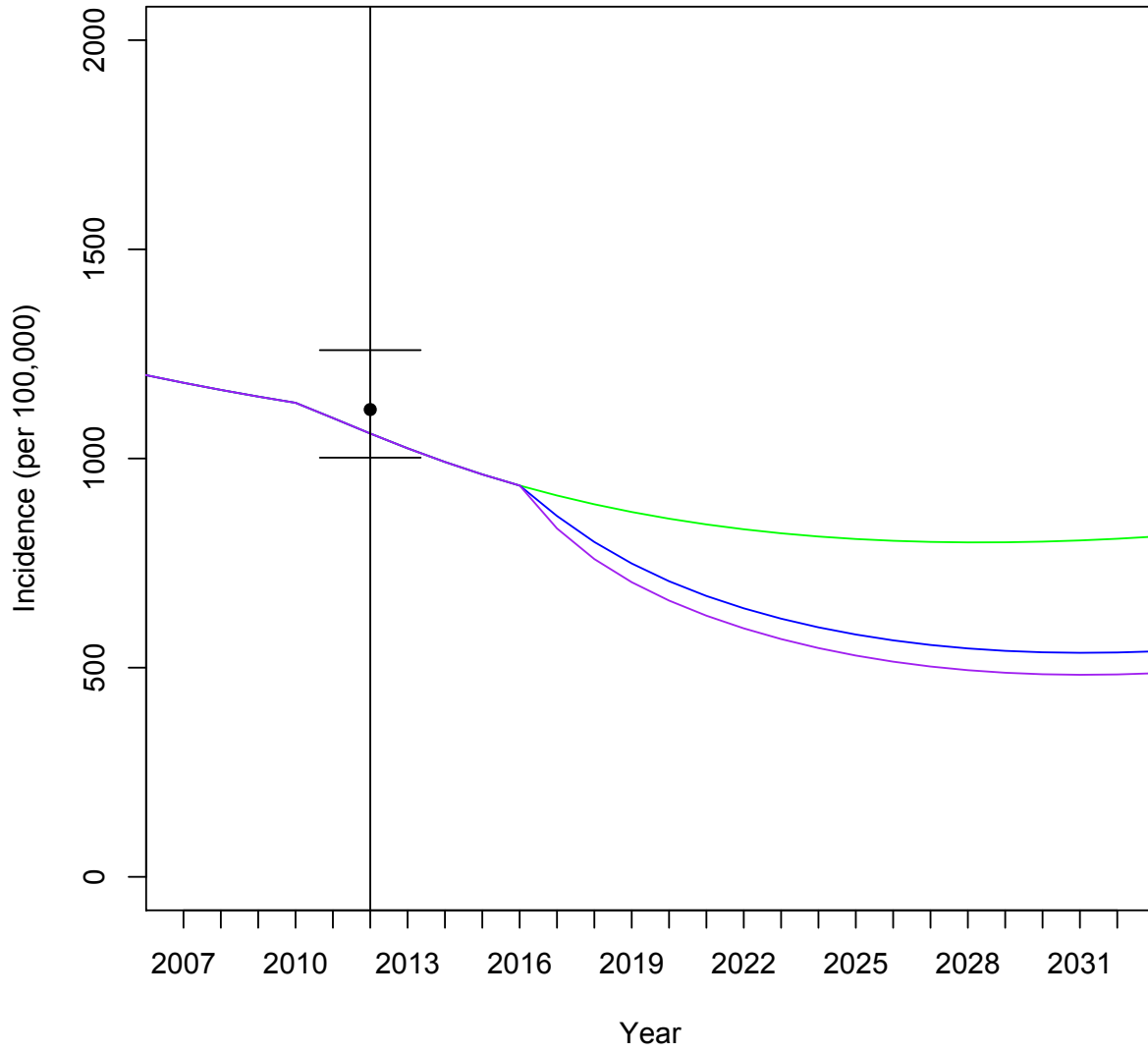
IPT Recovery of 0.9
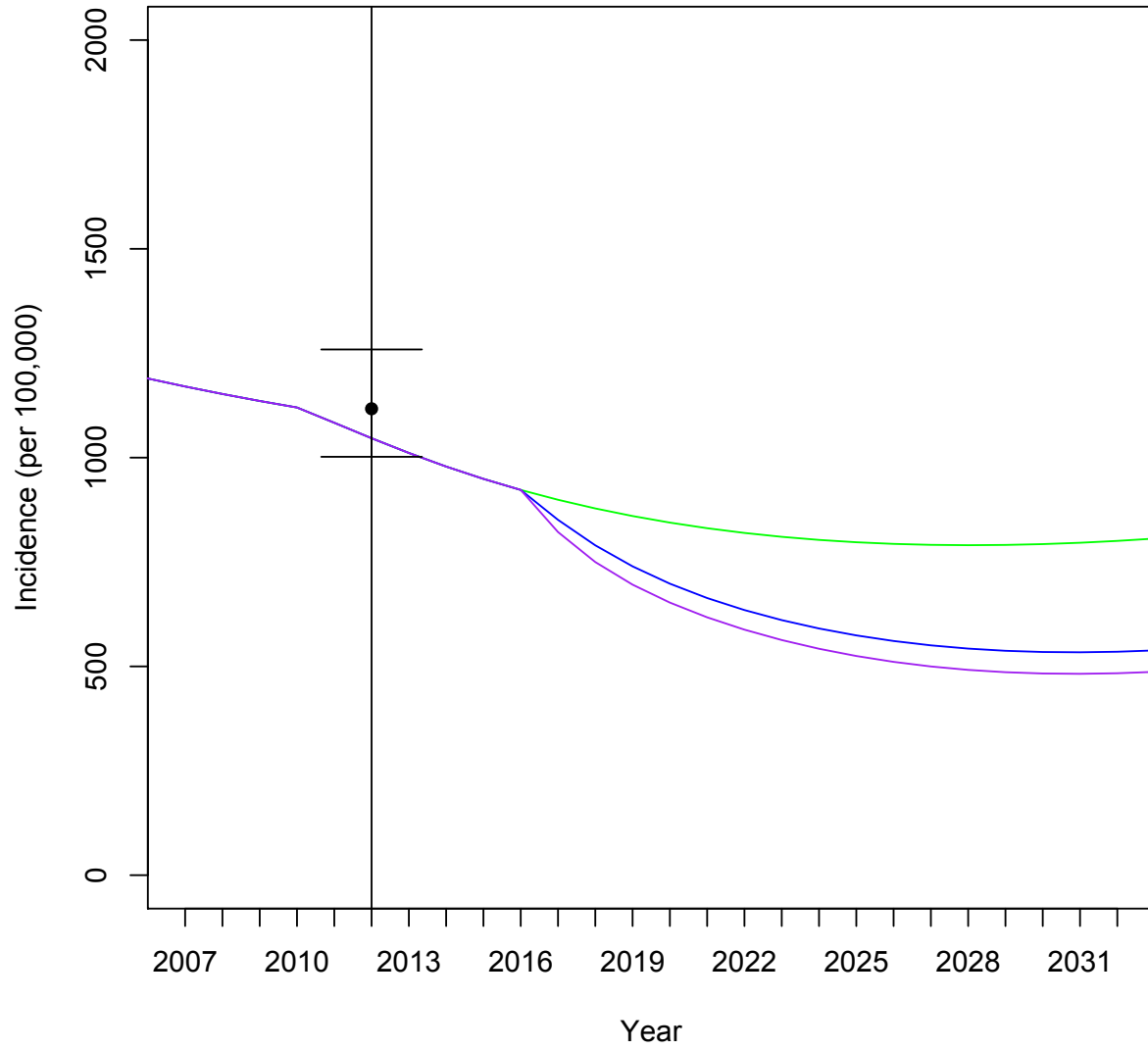
Figure S4: Sensitivity analysis of TB incidence to TB activation rate in HIV-positive individuals

TB Activation in HIV+ of 0.005
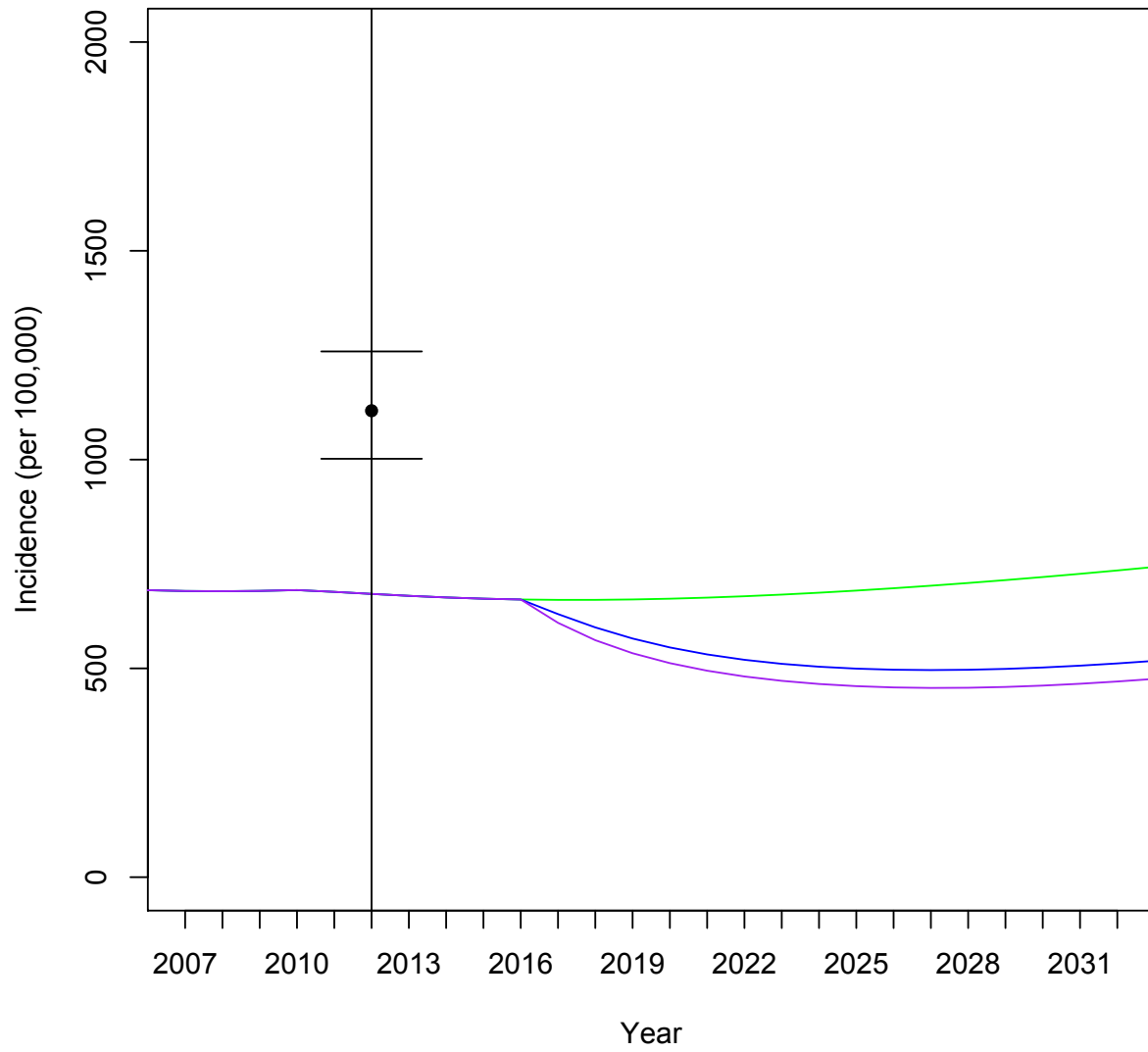
TB Activation in HIV+ of 0.05
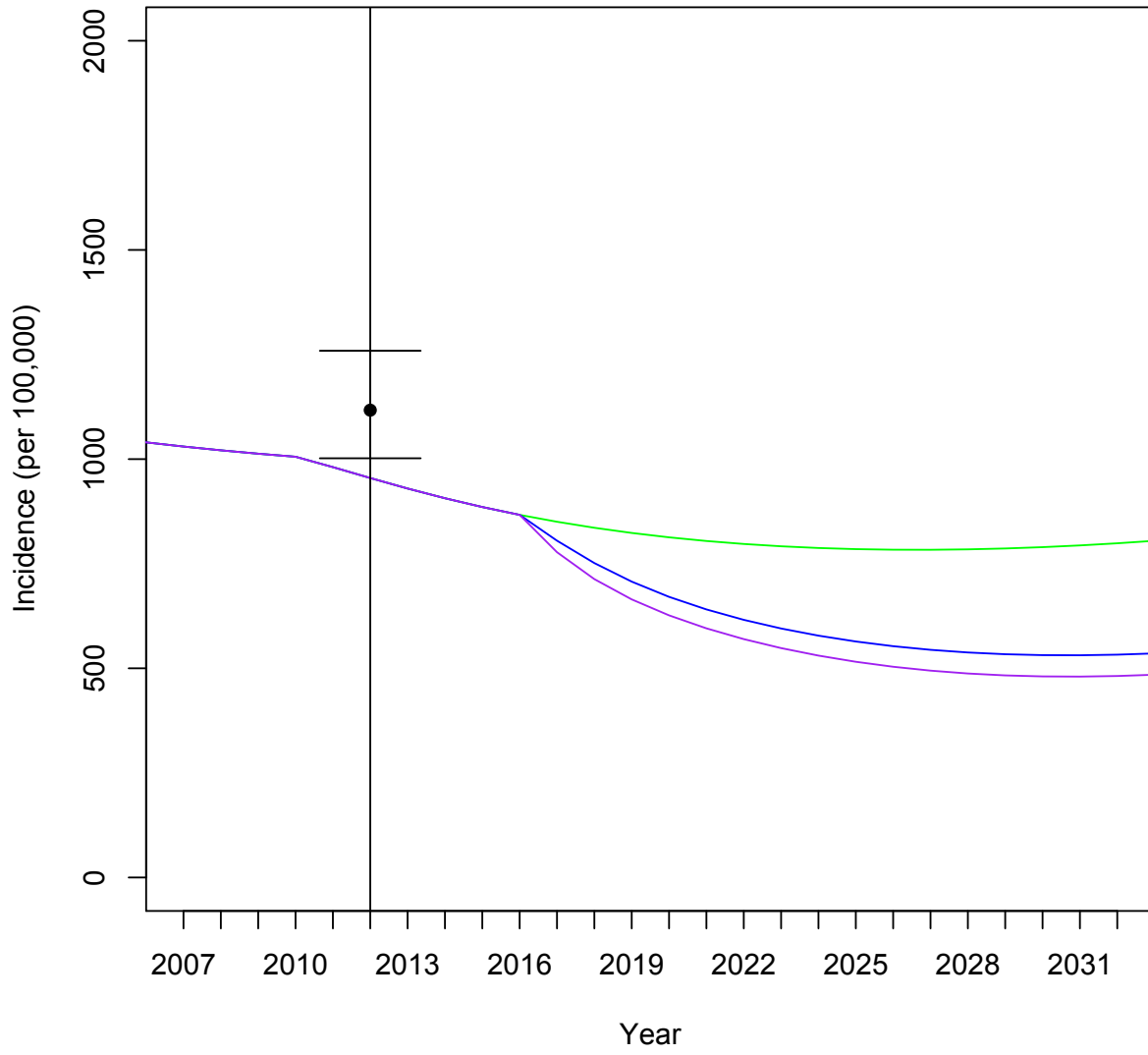
TB Activation in HIV+ of 0.2
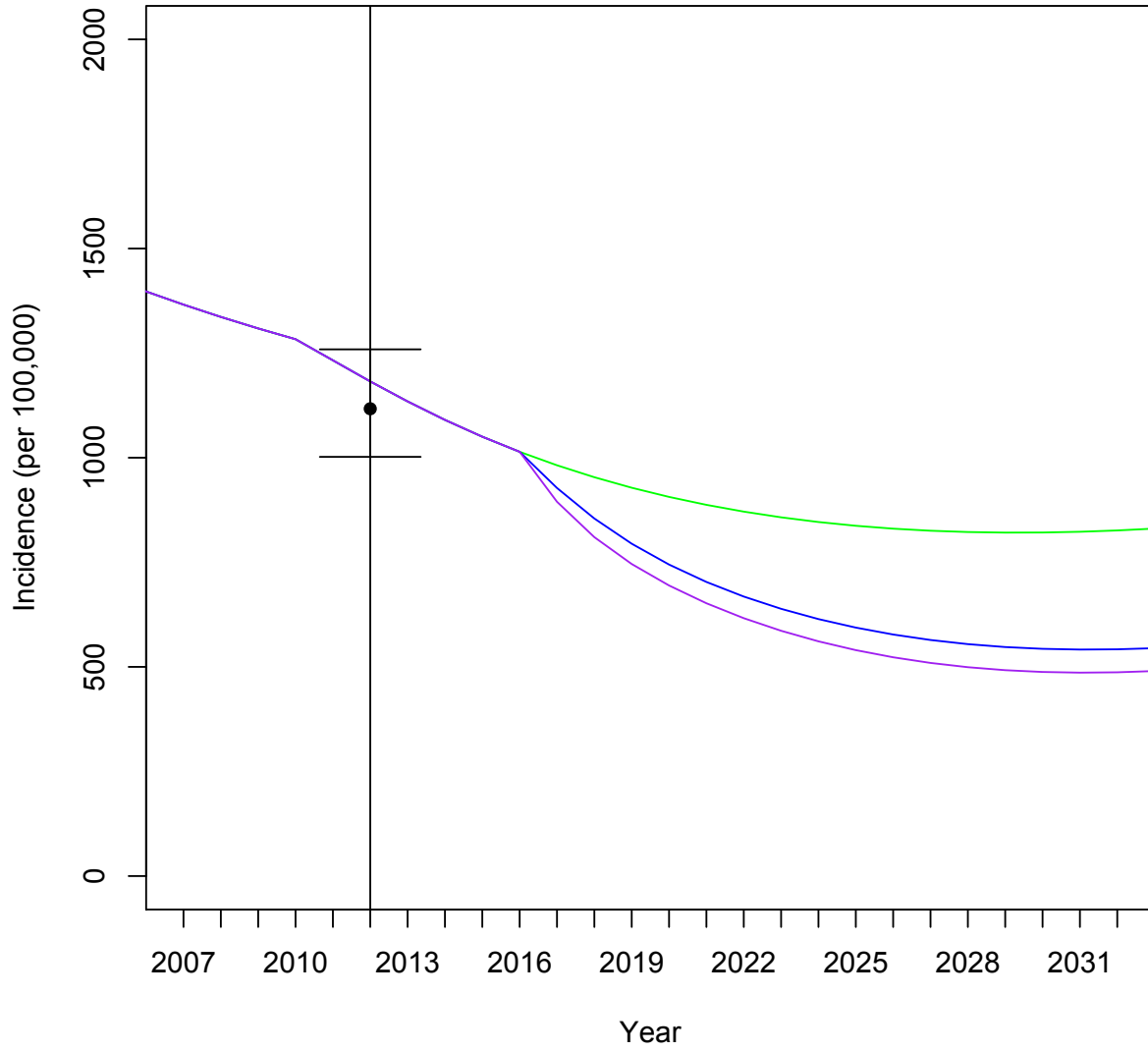
TB Activation in HIV+ of 0.5
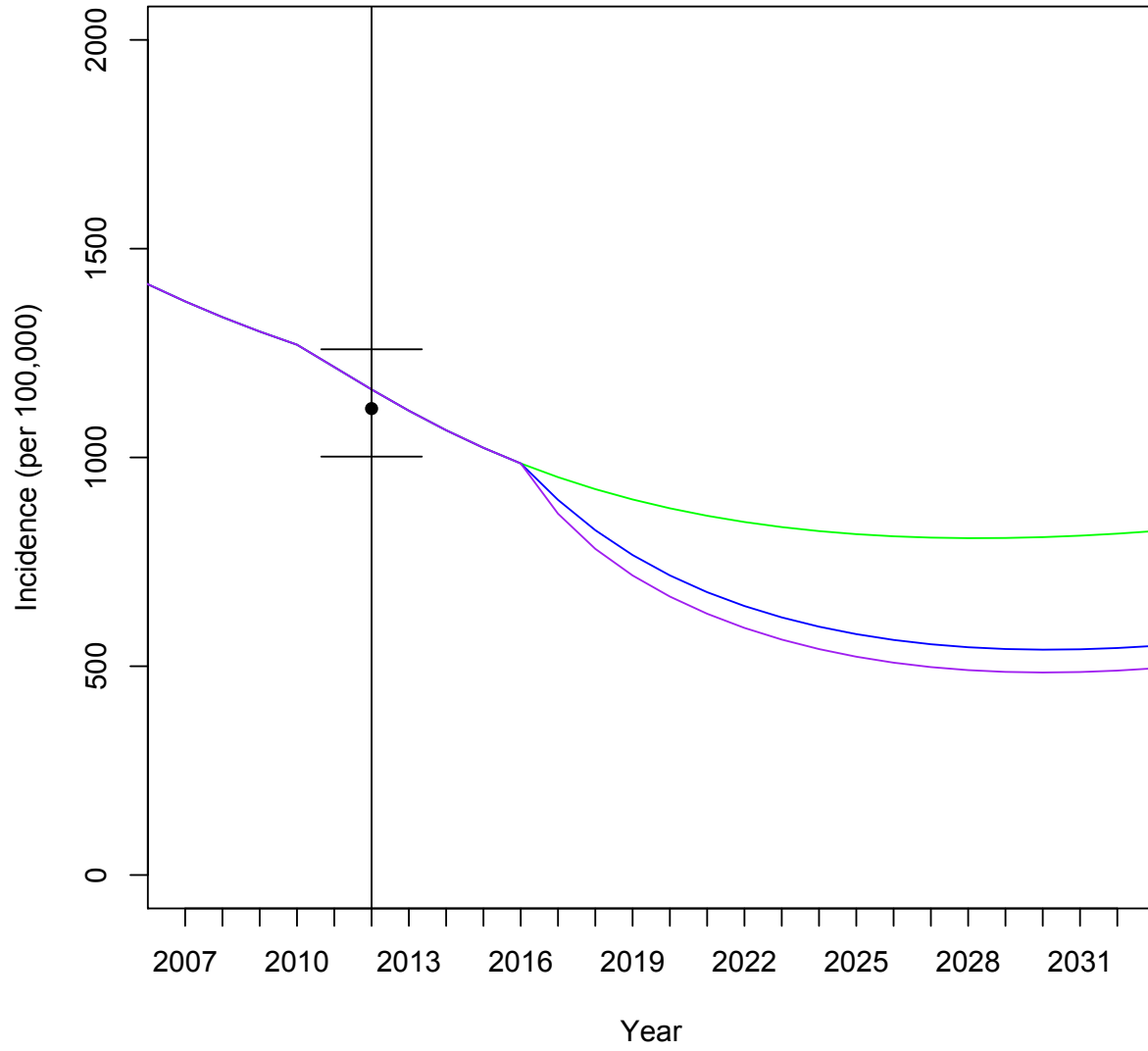
TB Activation in HIV+ of 0.9
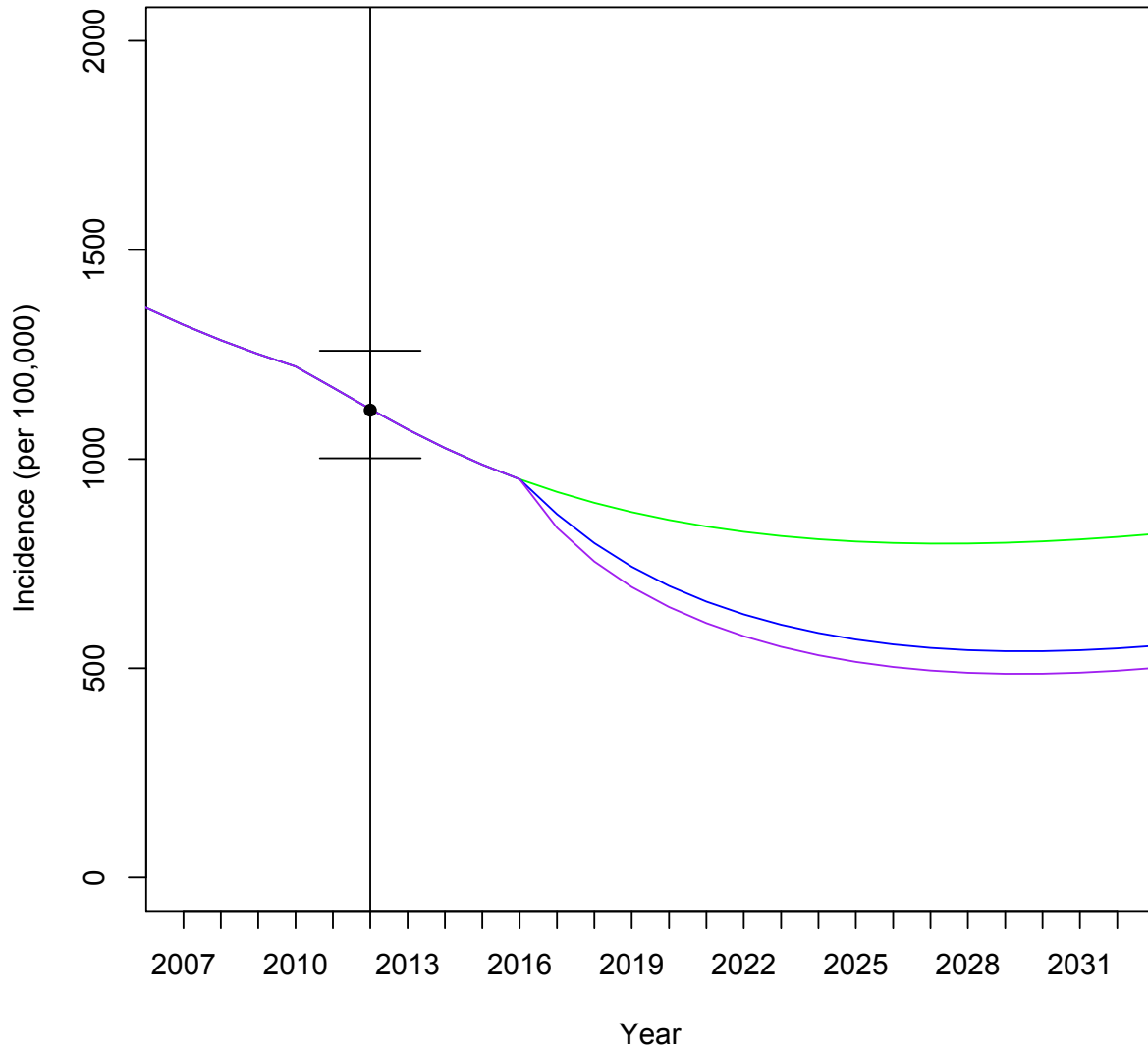
Figure S5: Sensitivity analysis of the rate at individuals who have recovered from TB cease IPT treatment

IPT Discontinuation Rate of 0.05
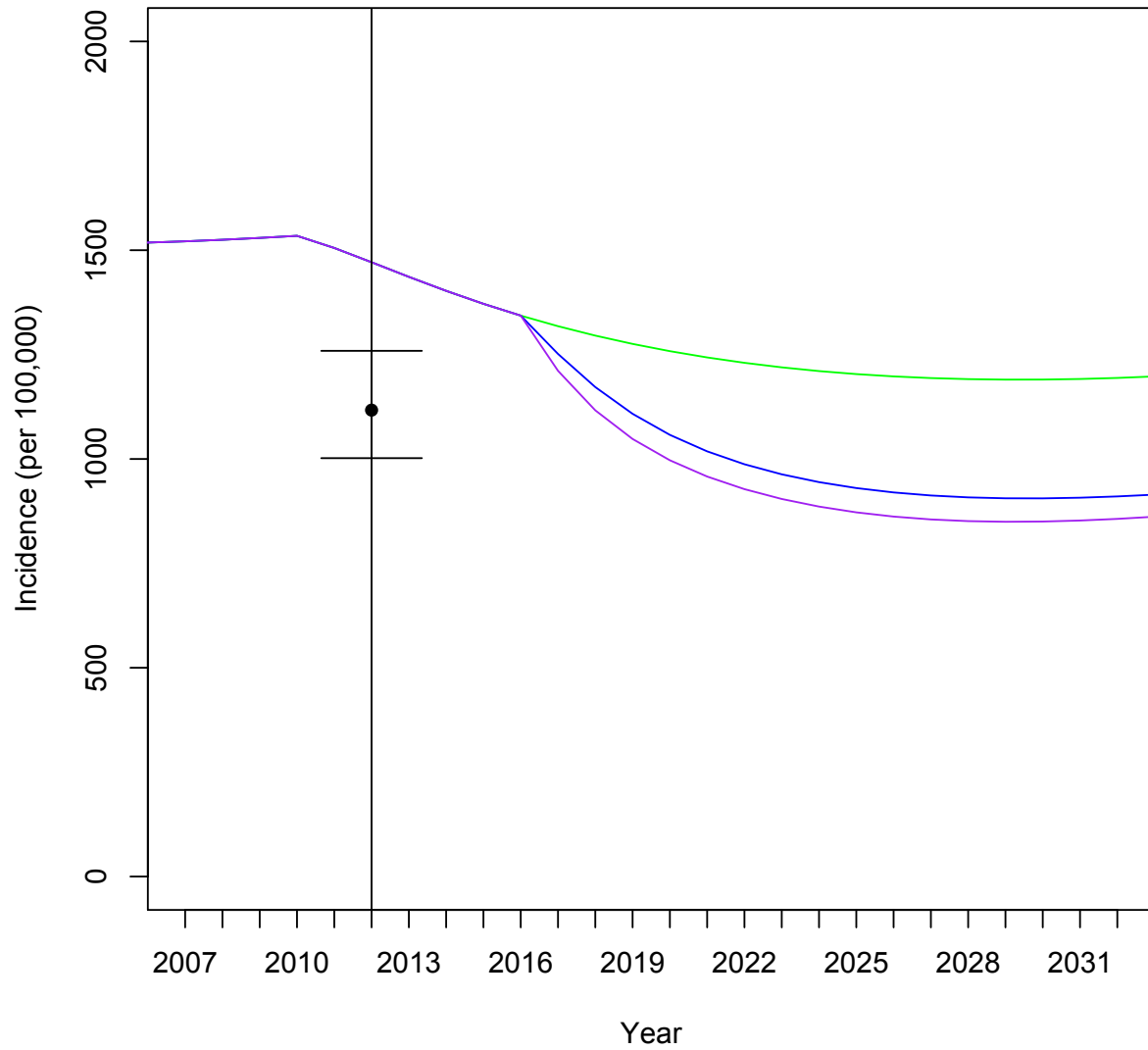
IPT Discontinuation Rate of 0.1
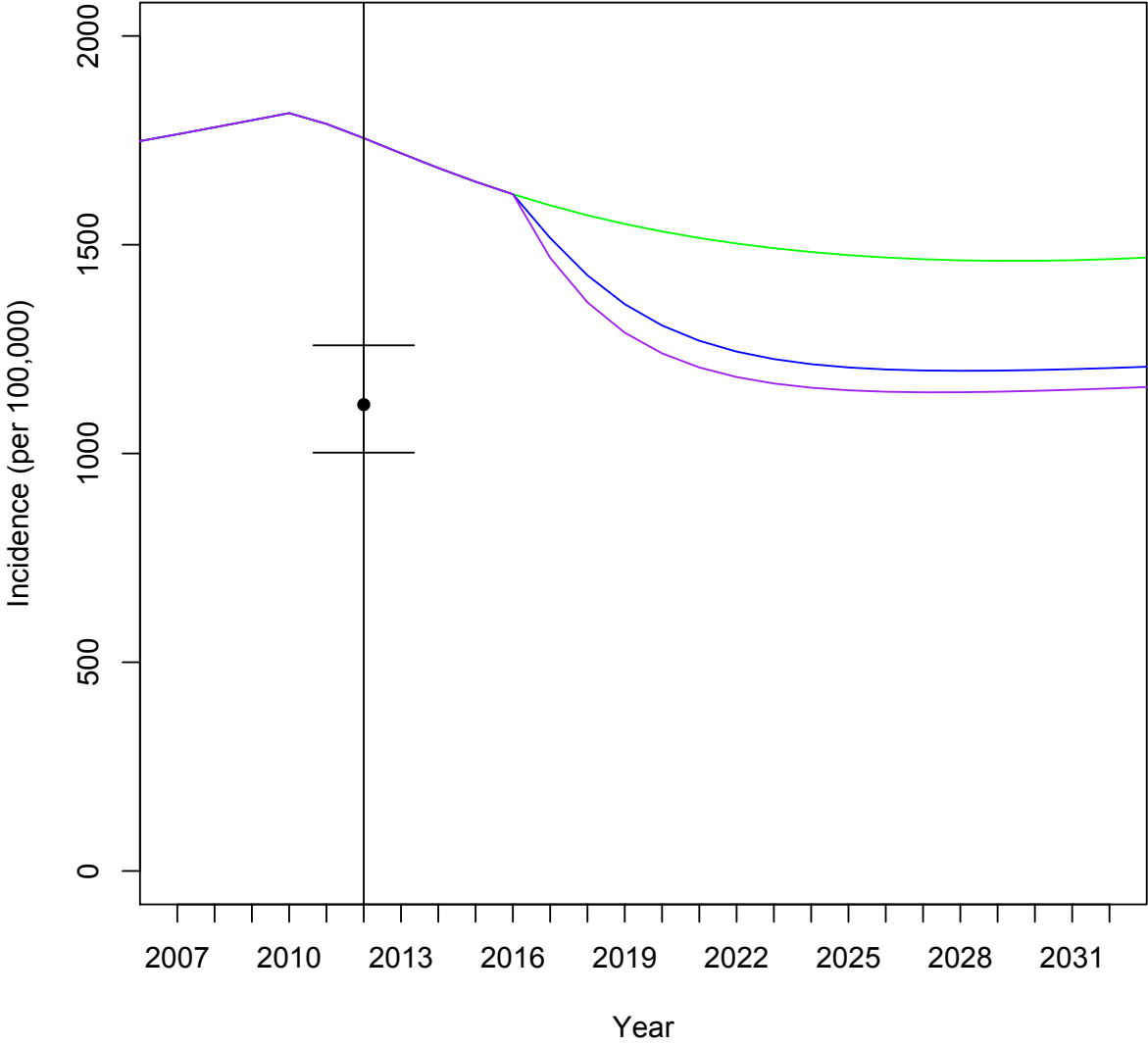
IPT Discontinuation Rate of 0.15
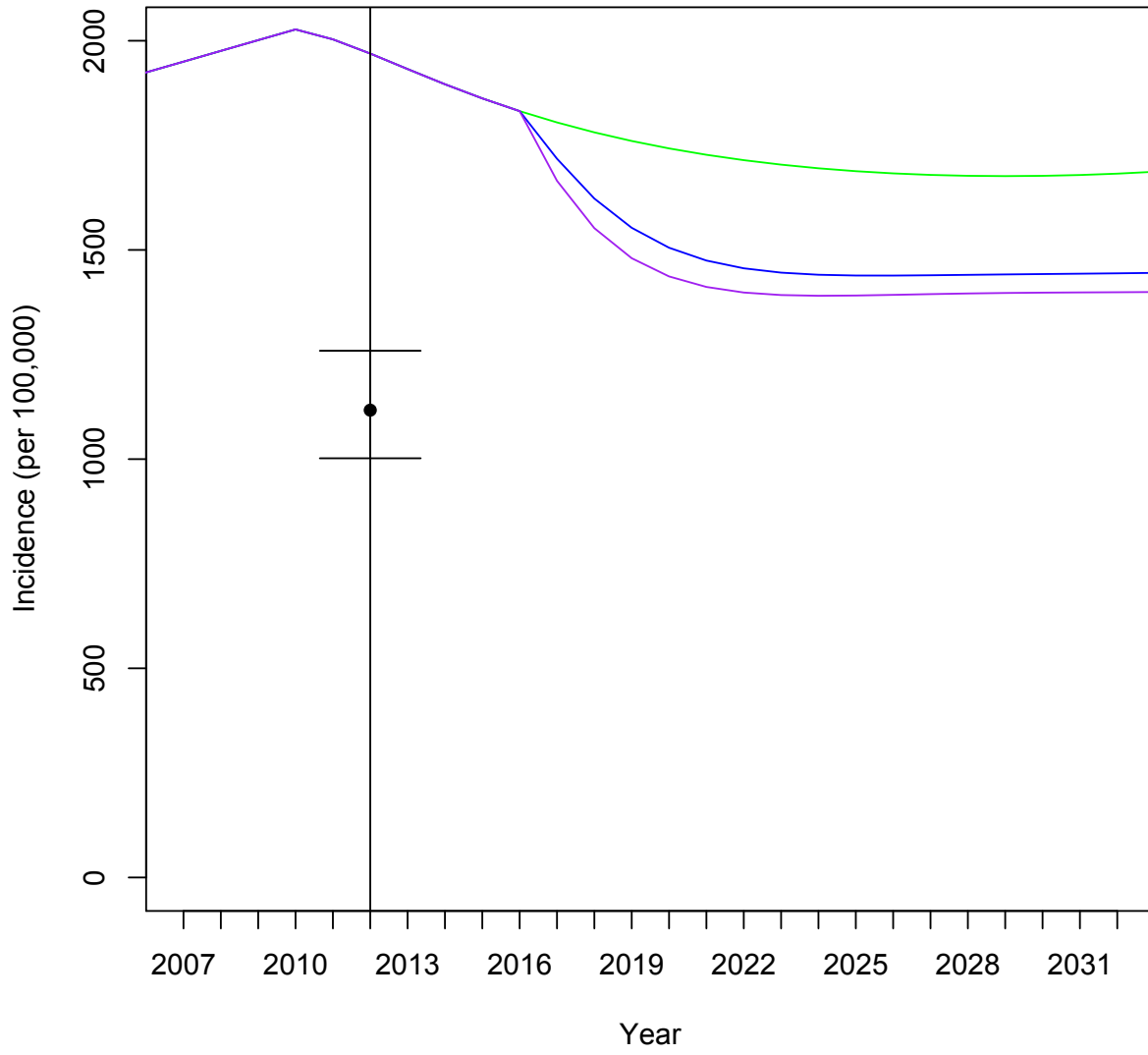
Figure S6: Effects of IPT on TB disease indicators

TB Incidence in Adolescents
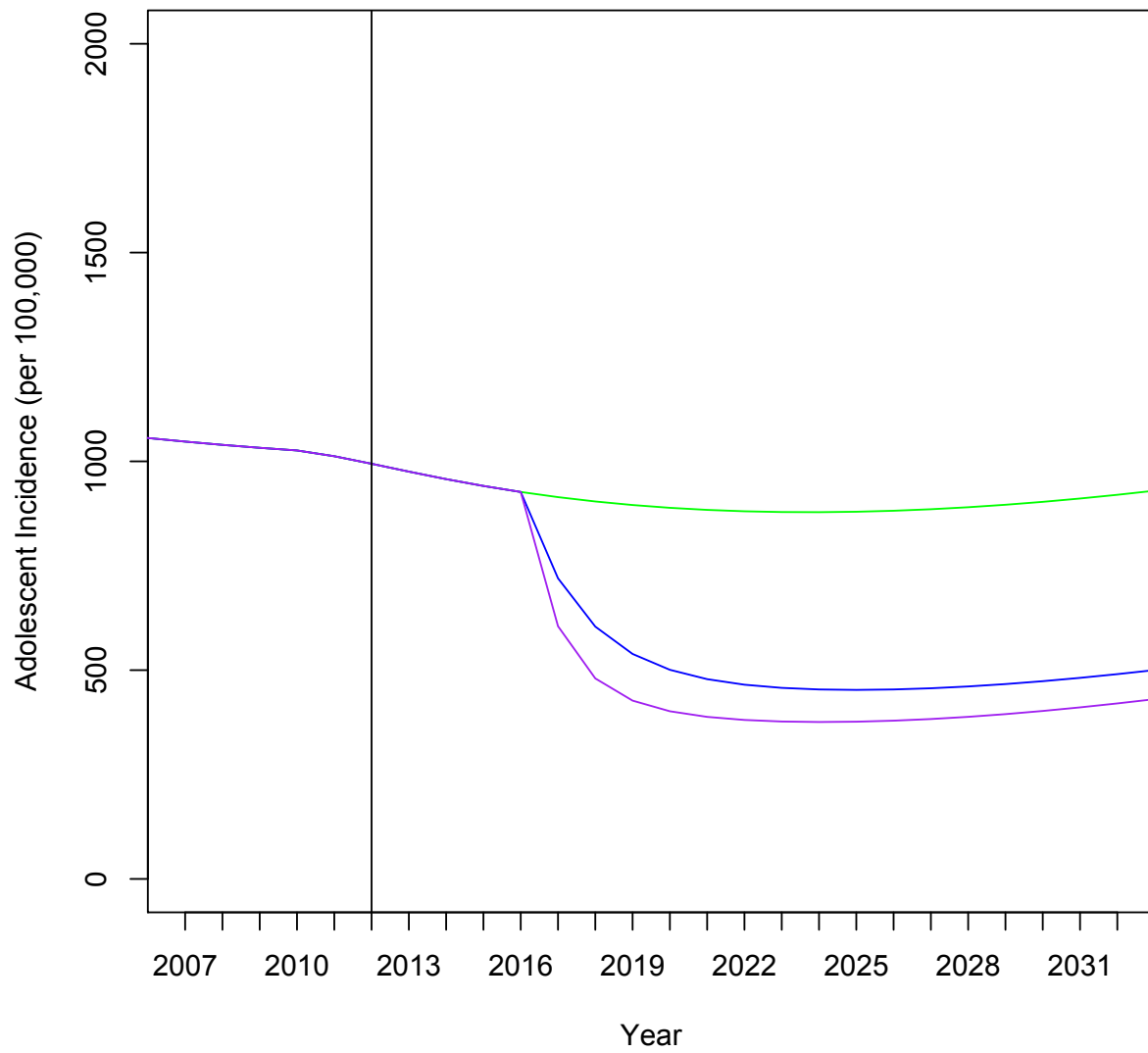
TB Incidence in Adults
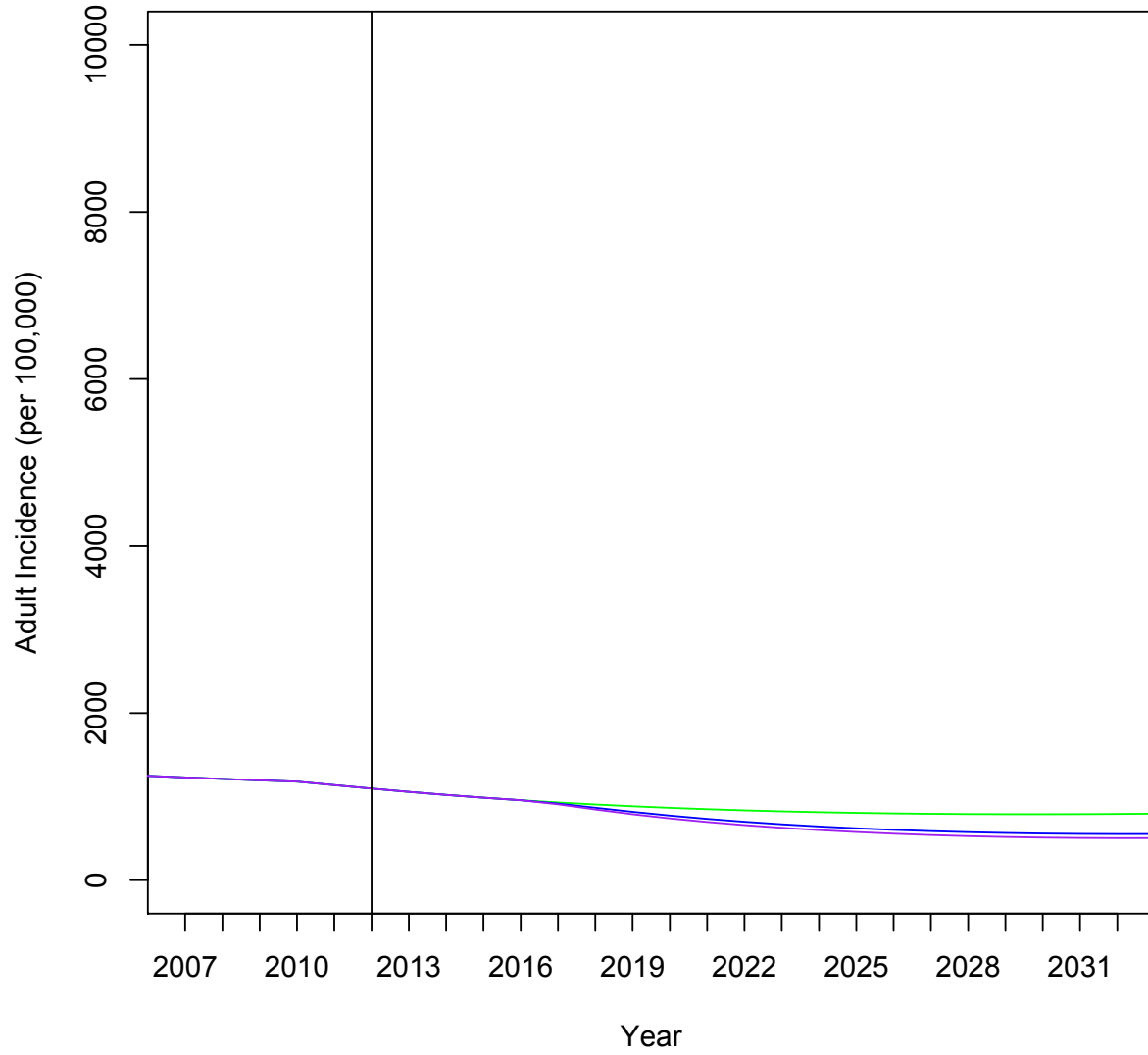
TB Prevalence in Adolescents
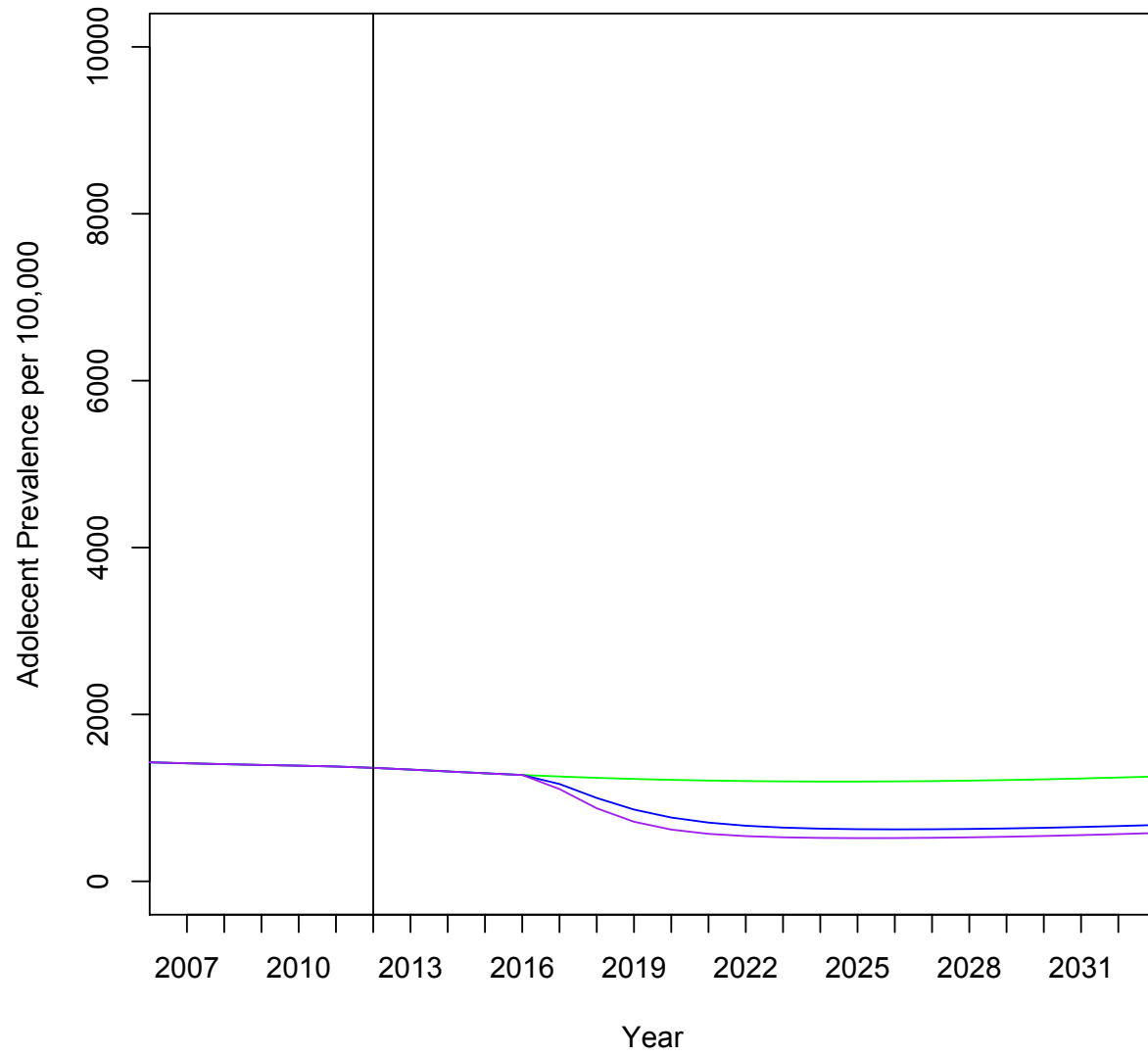
TB Prevalence in Adults
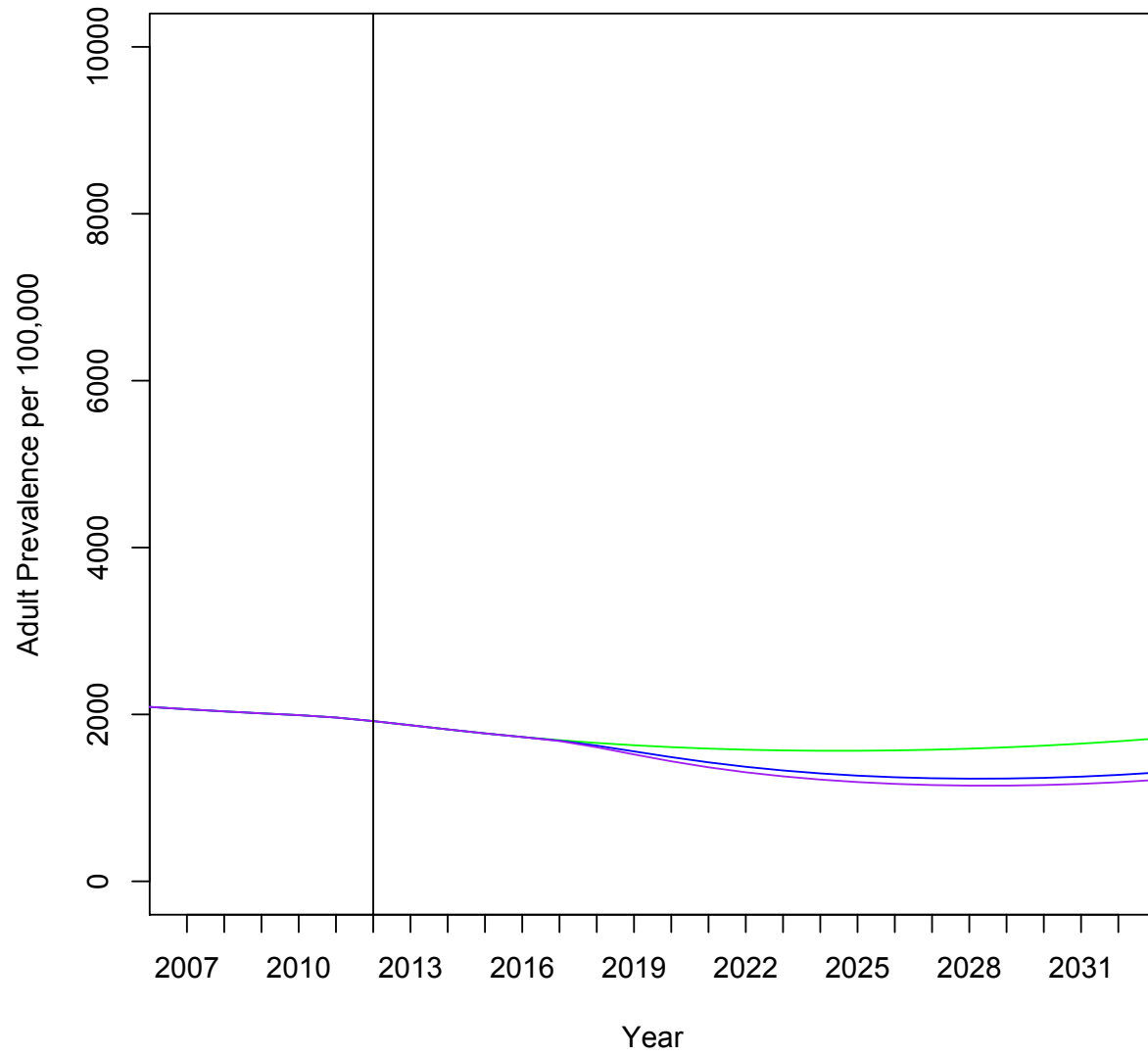
TB Mortality in Adolescents
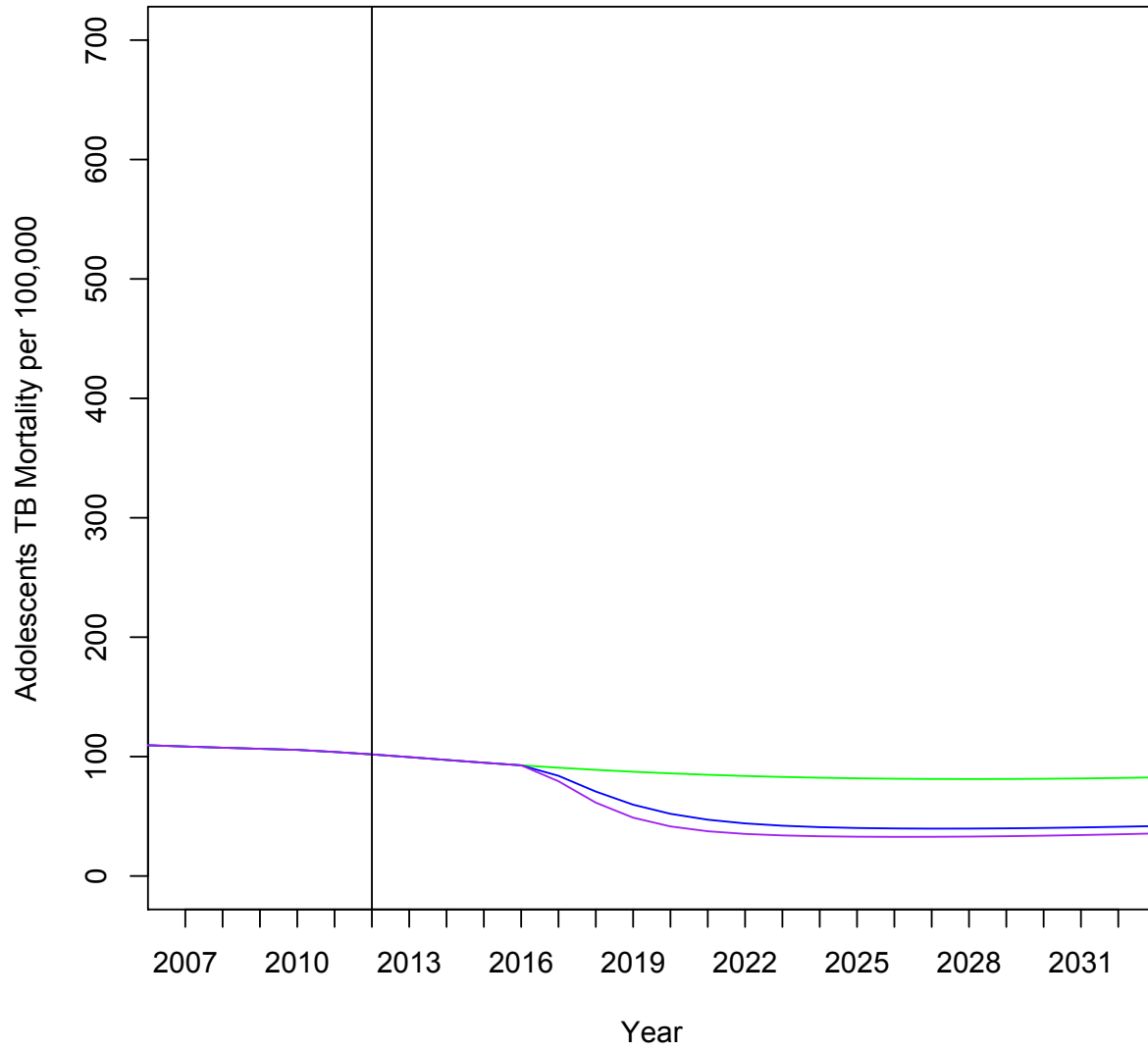
TB Mortality in Adults
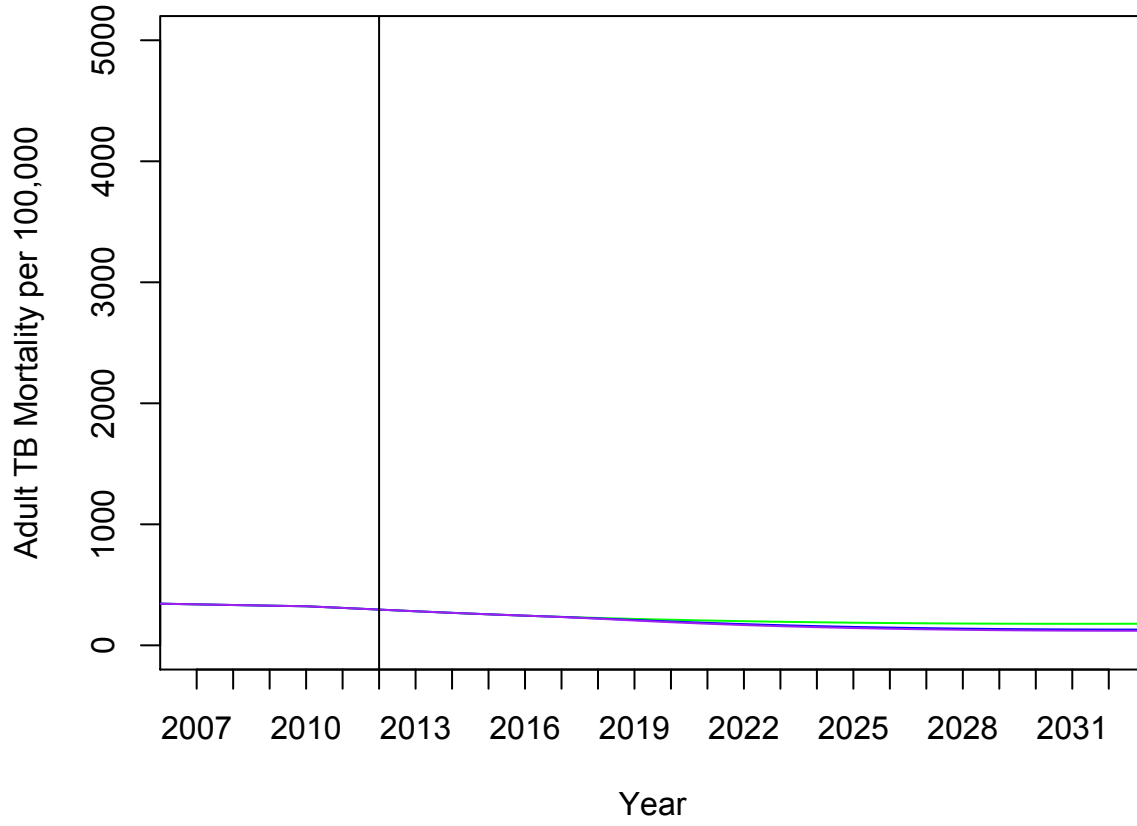
HIV Prevalence
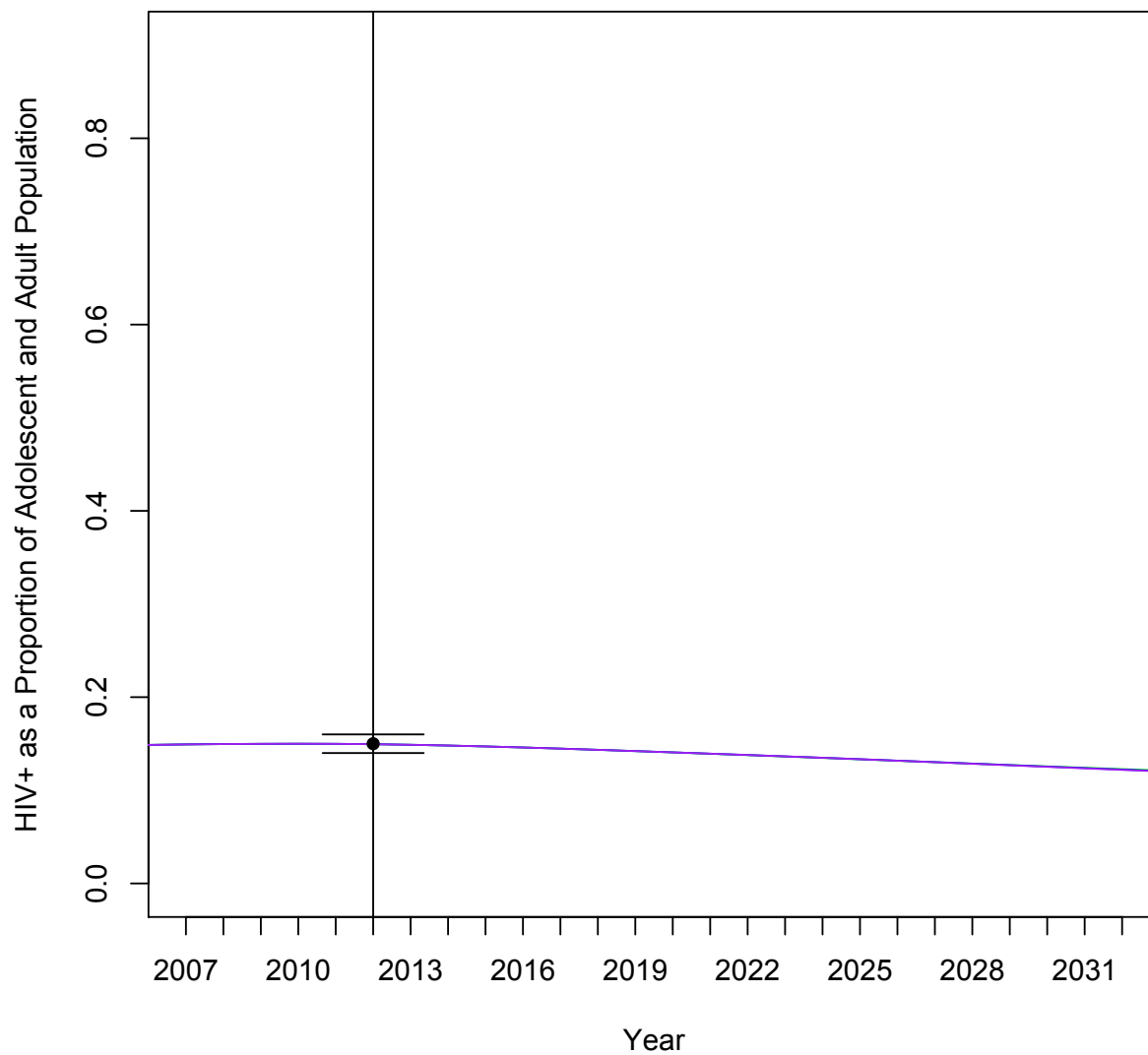
Proportion of HIV-Positive Individuals on ART
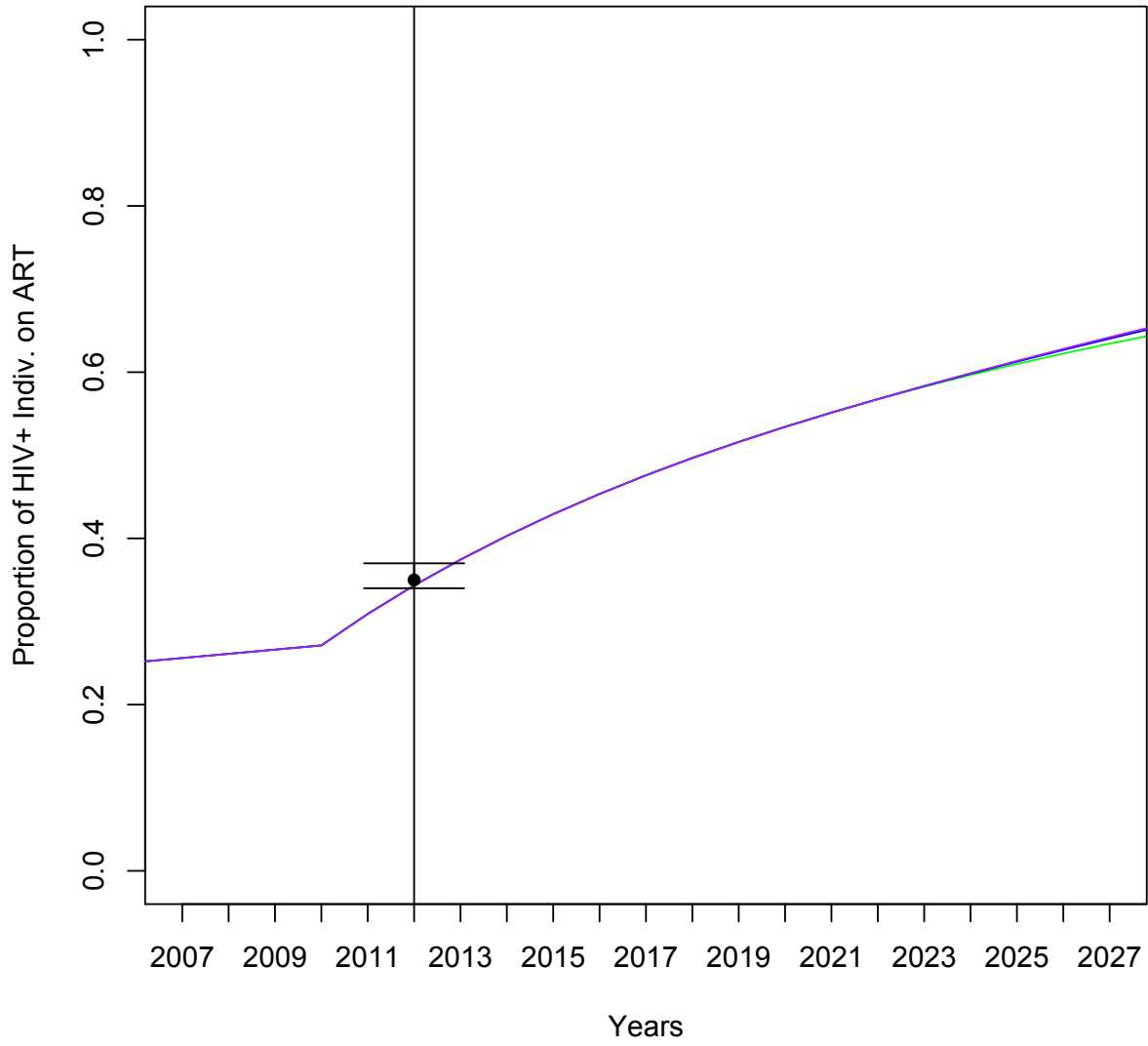
Proportion of TB Mortality in HIV-Positive Individuals
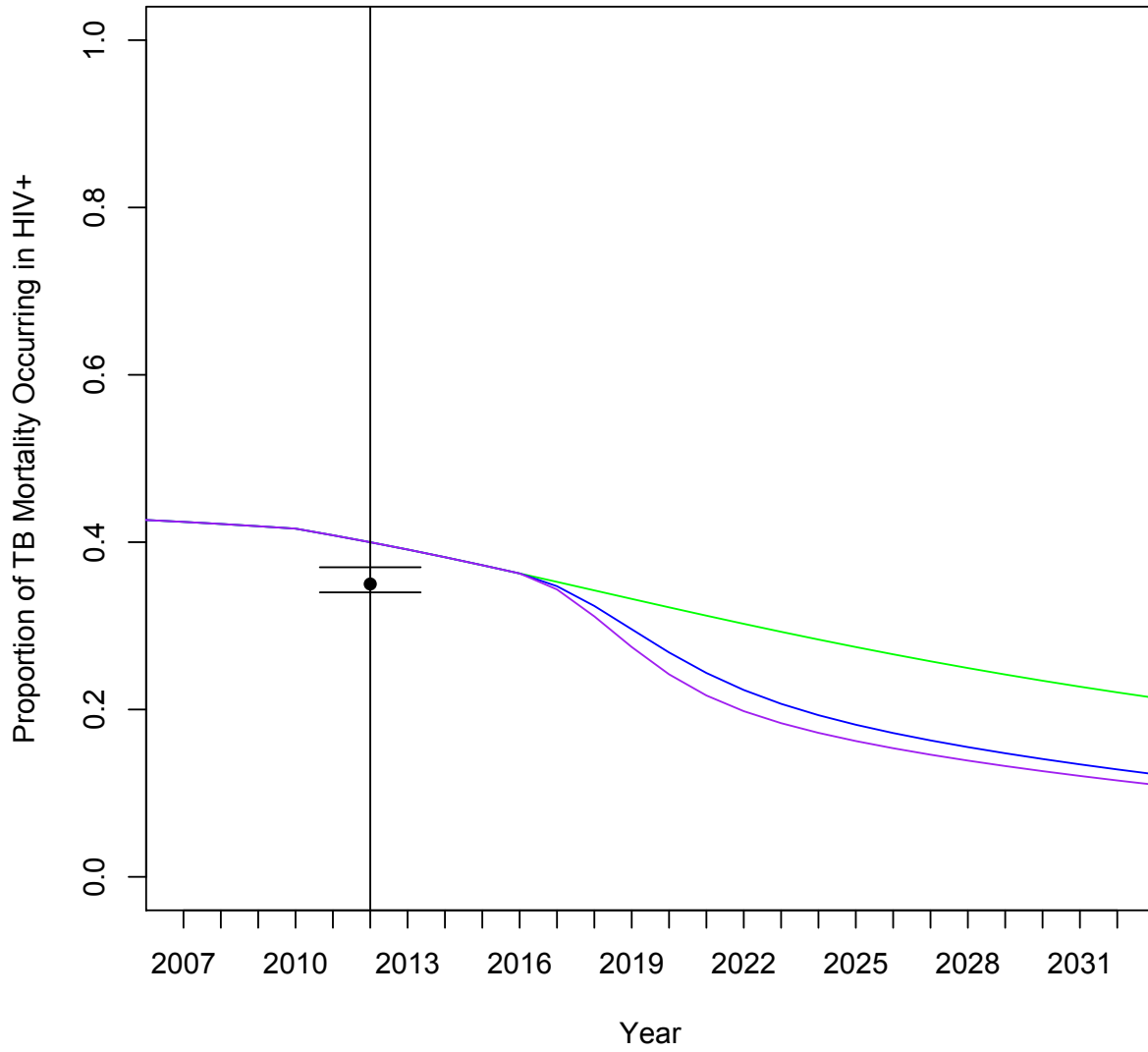
Figure S7: Sensitivity analysis to the rate of HIV infection

HIV Incidence of 0.0000000001
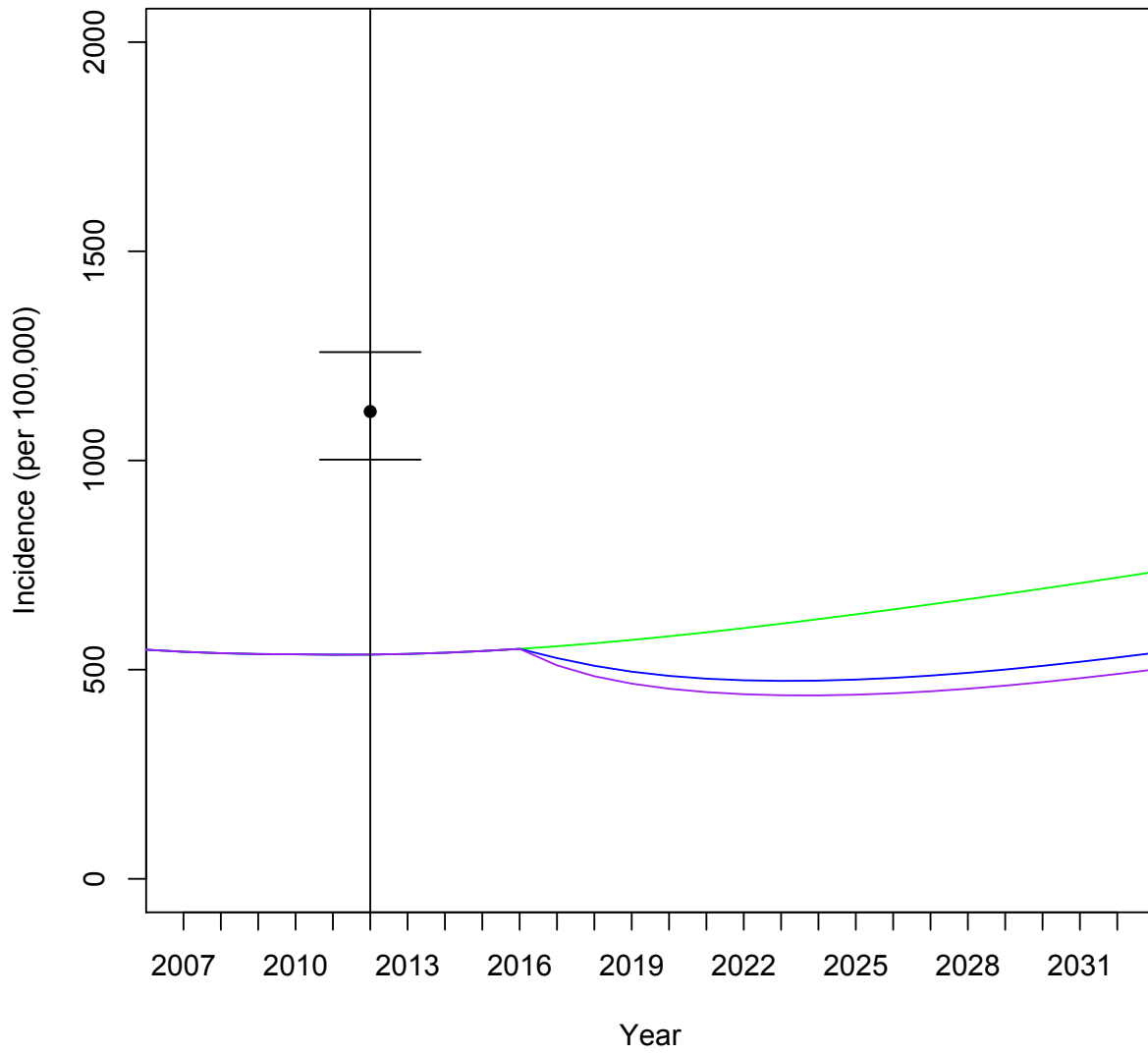
HIV Incidence of 0.000000001
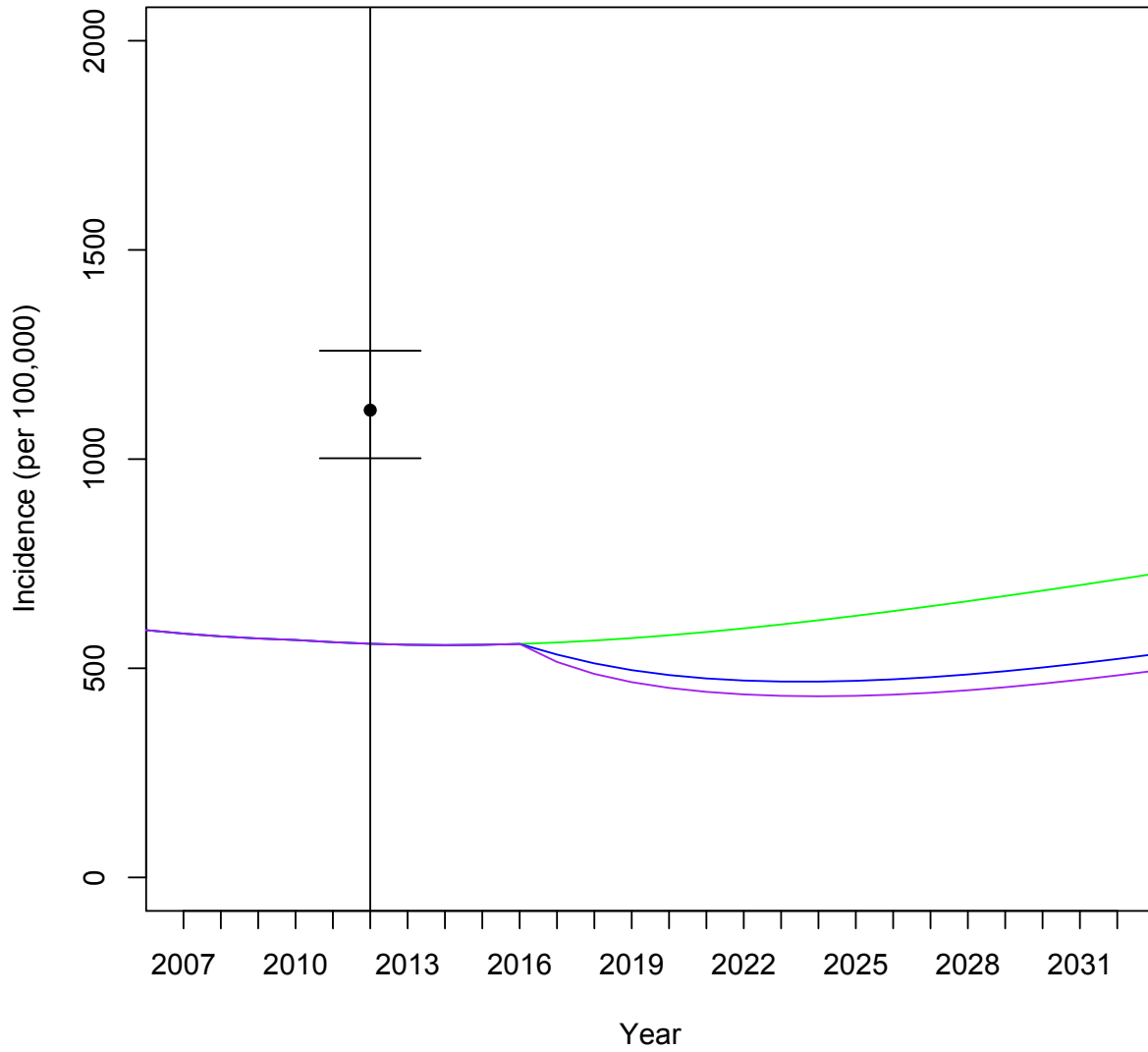
HIV Incidence of 0.000000002
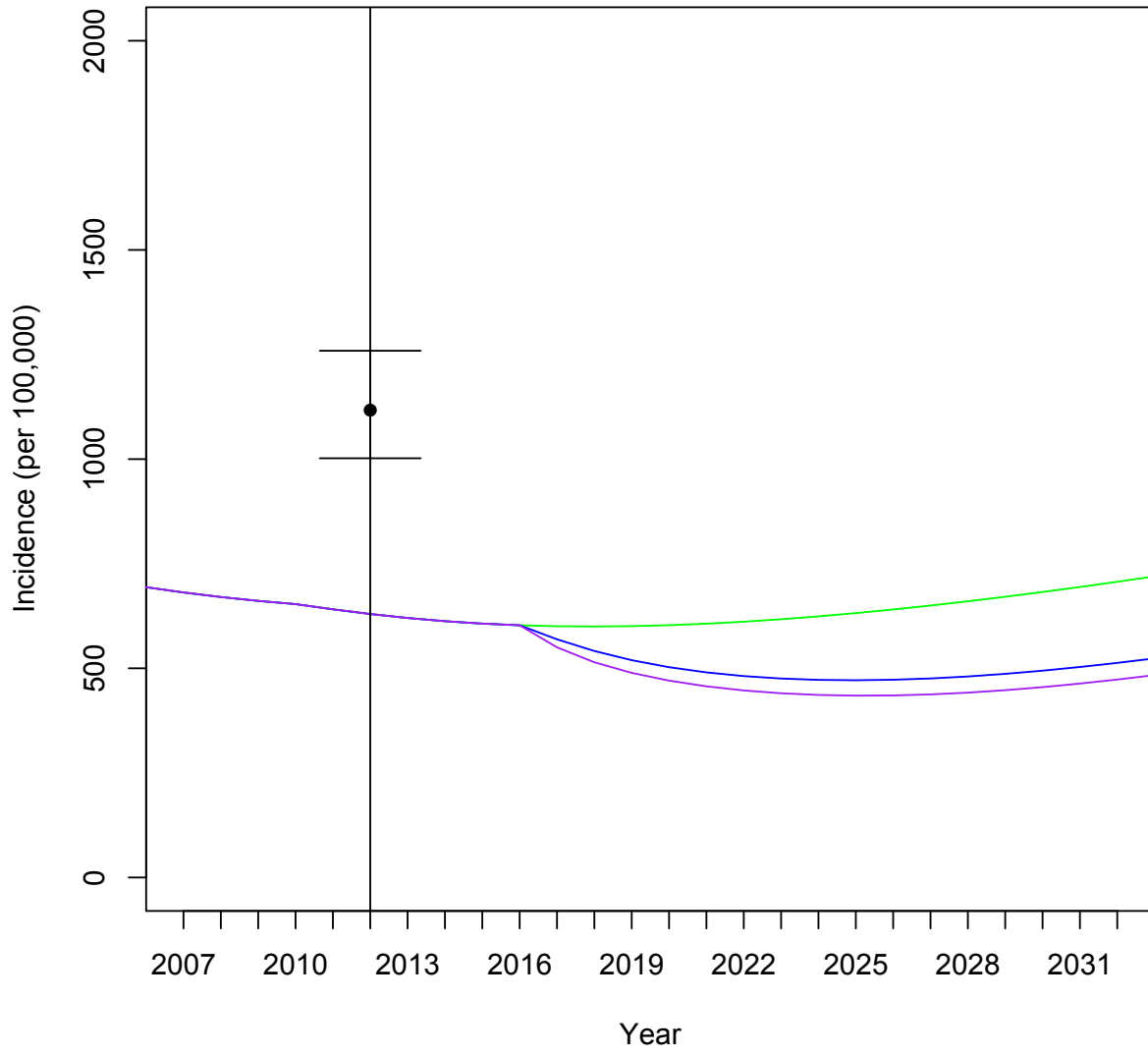
HIV Incidence of 0.000000003
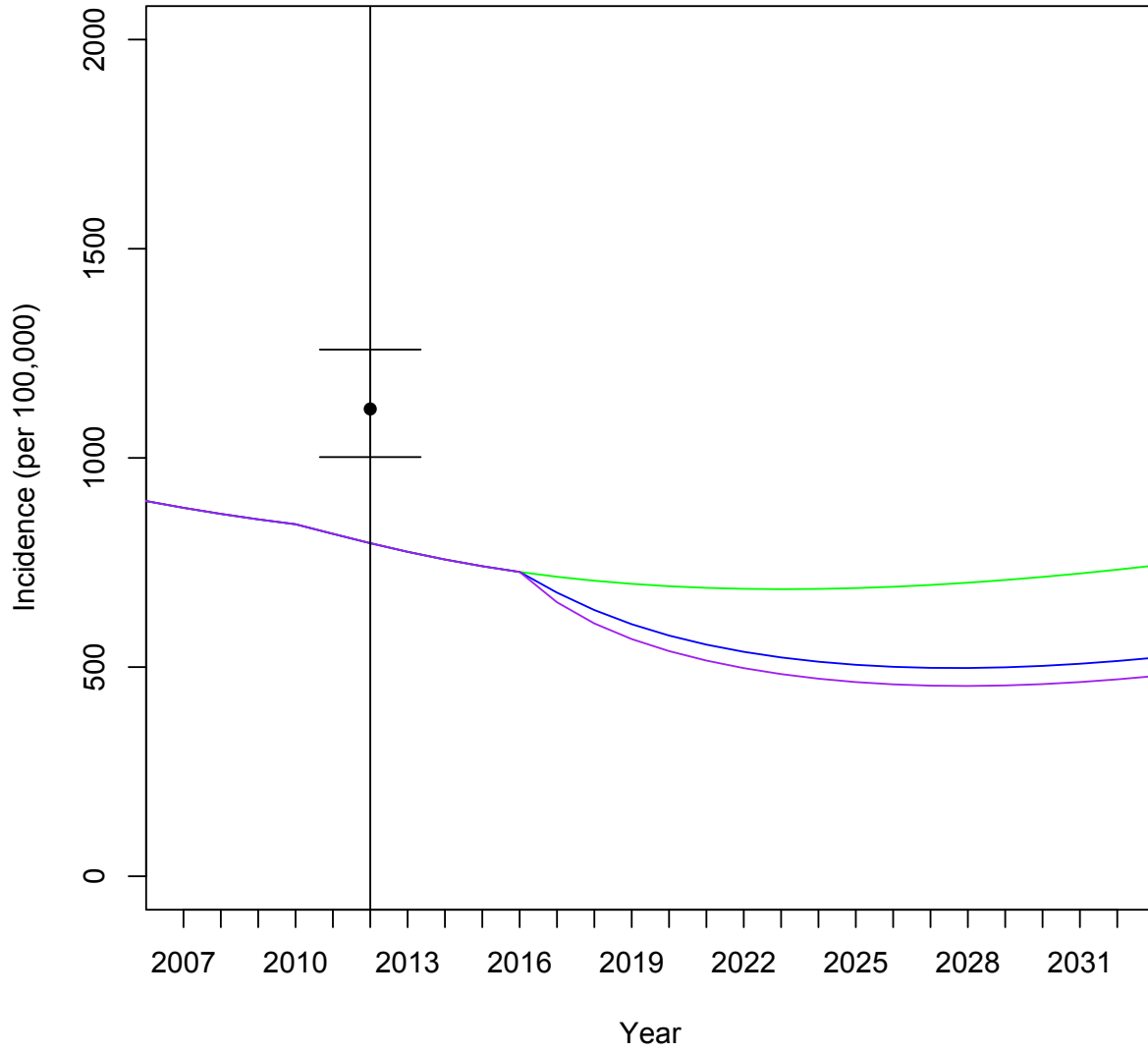
HIV Incidence of 0.000000004
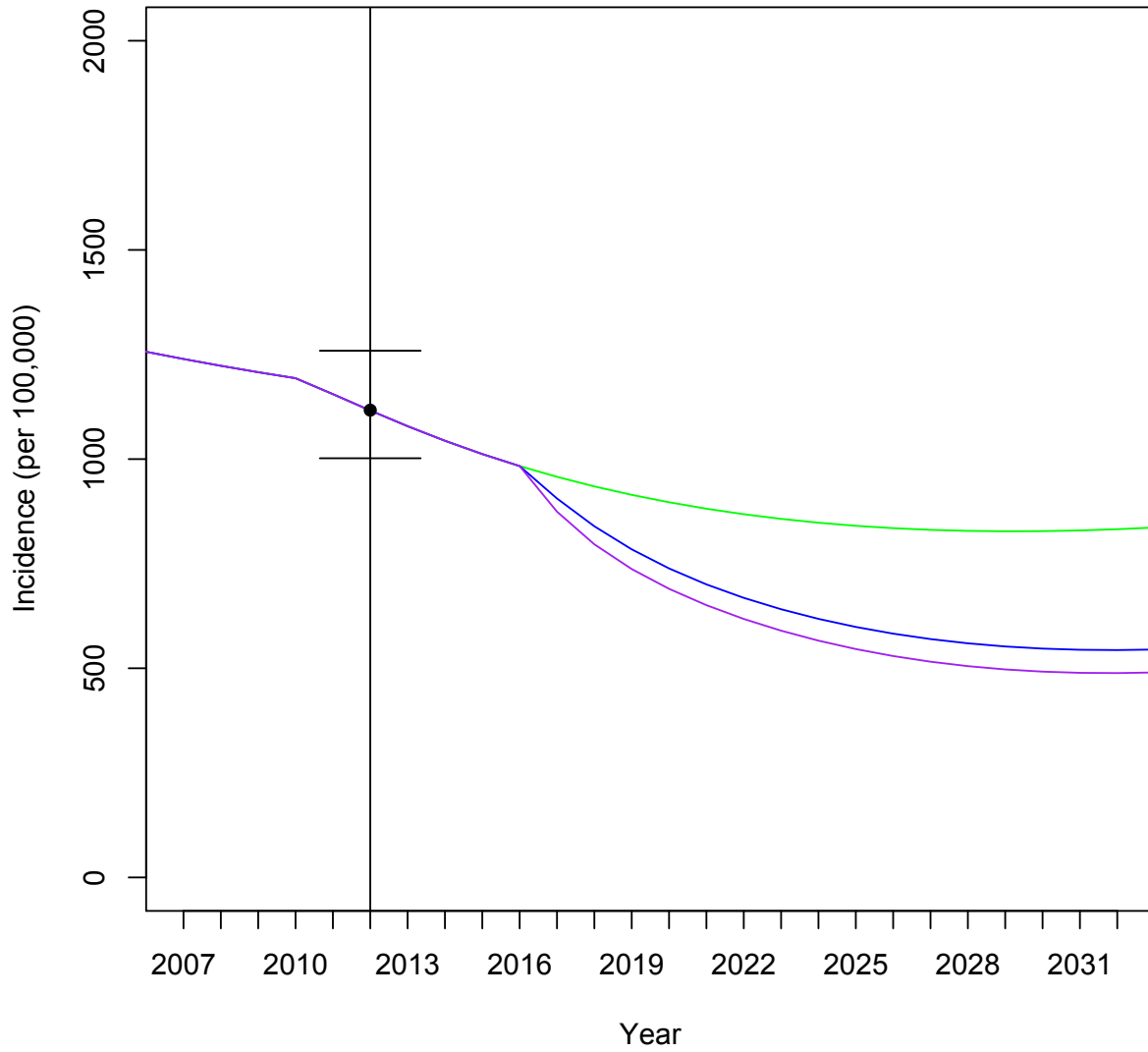
HIV Incidence of 0.000000005
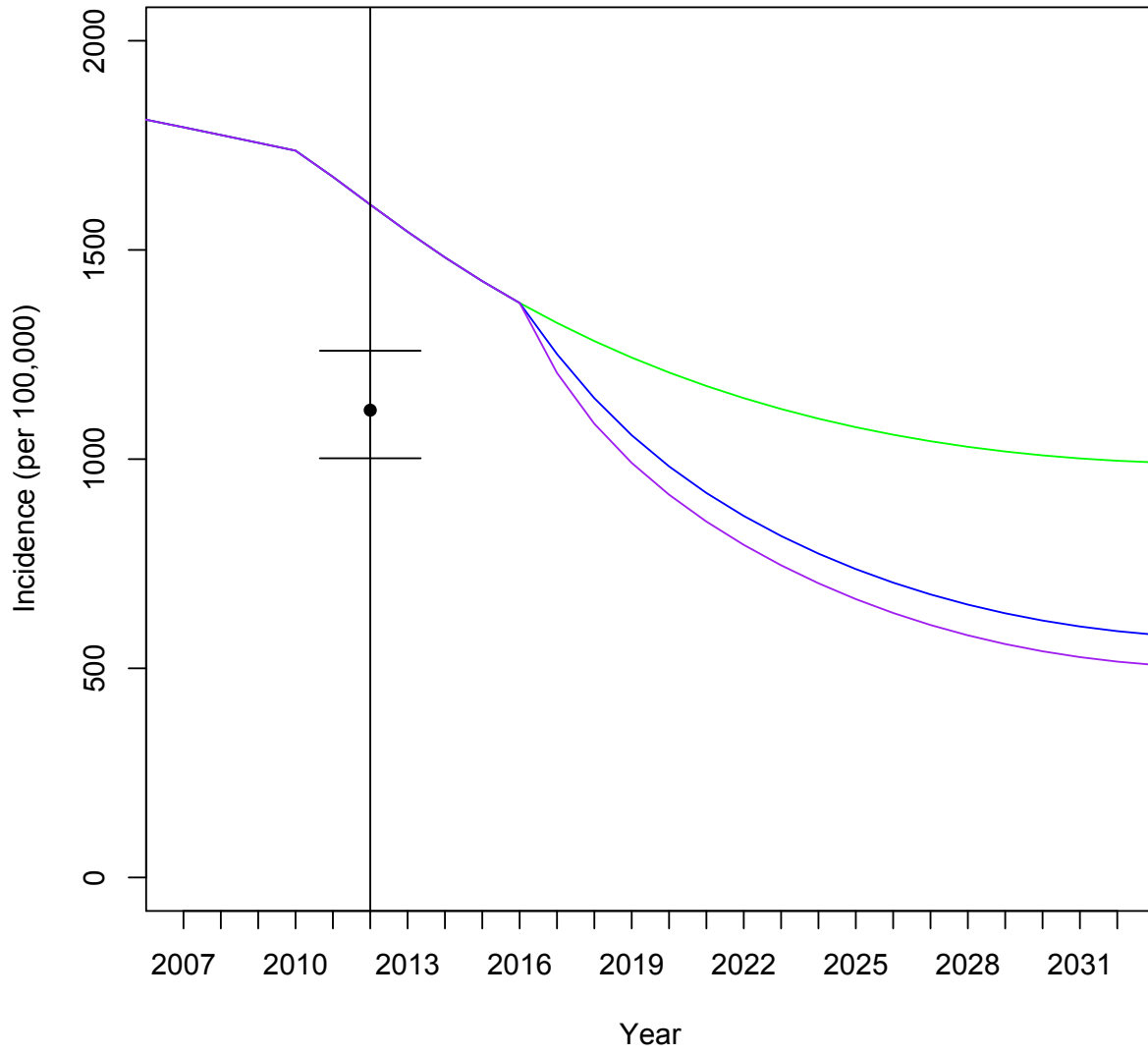
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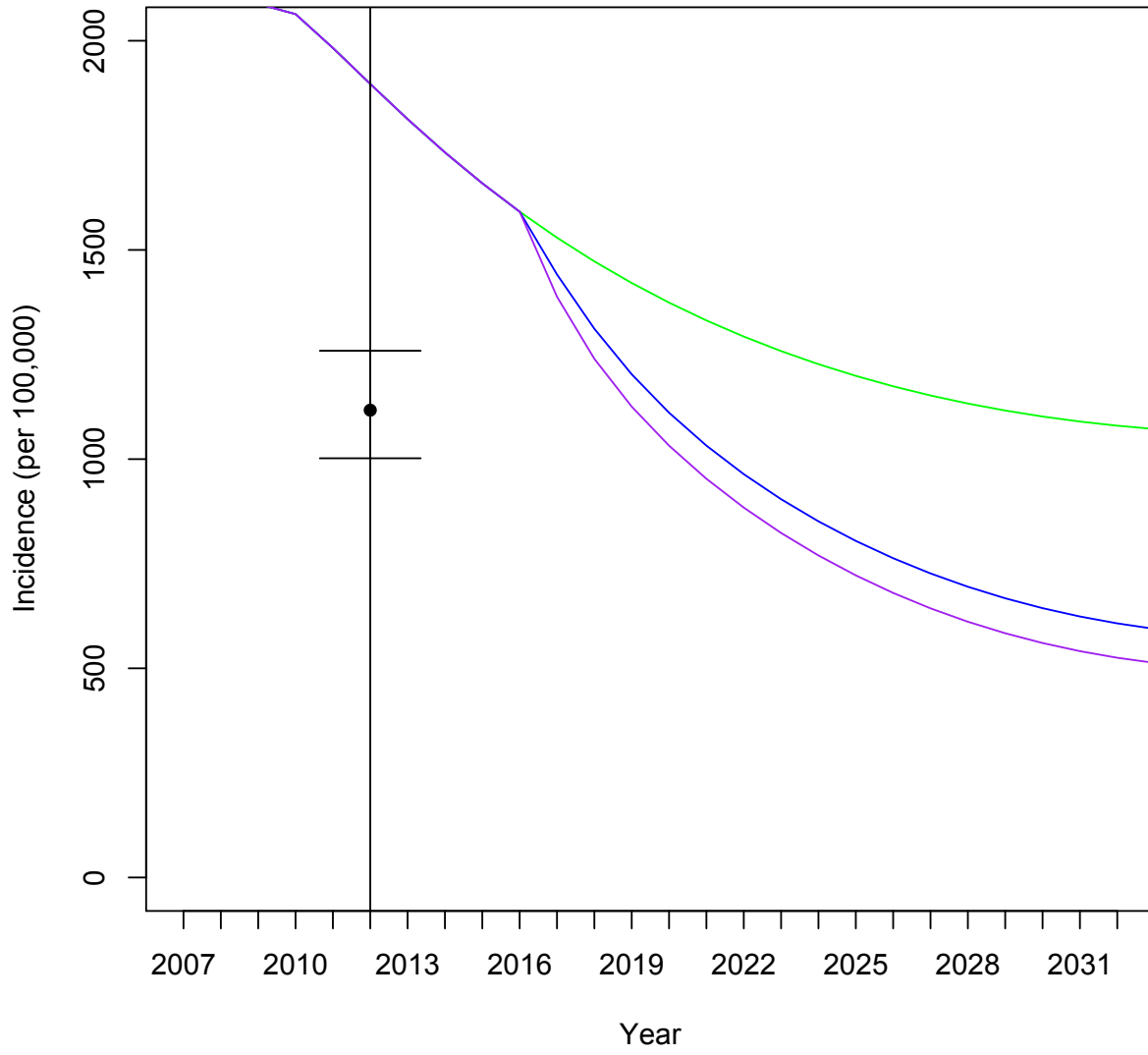
Figure S8: Sensitivity analysis to birthrate

Birthrate of 0.001
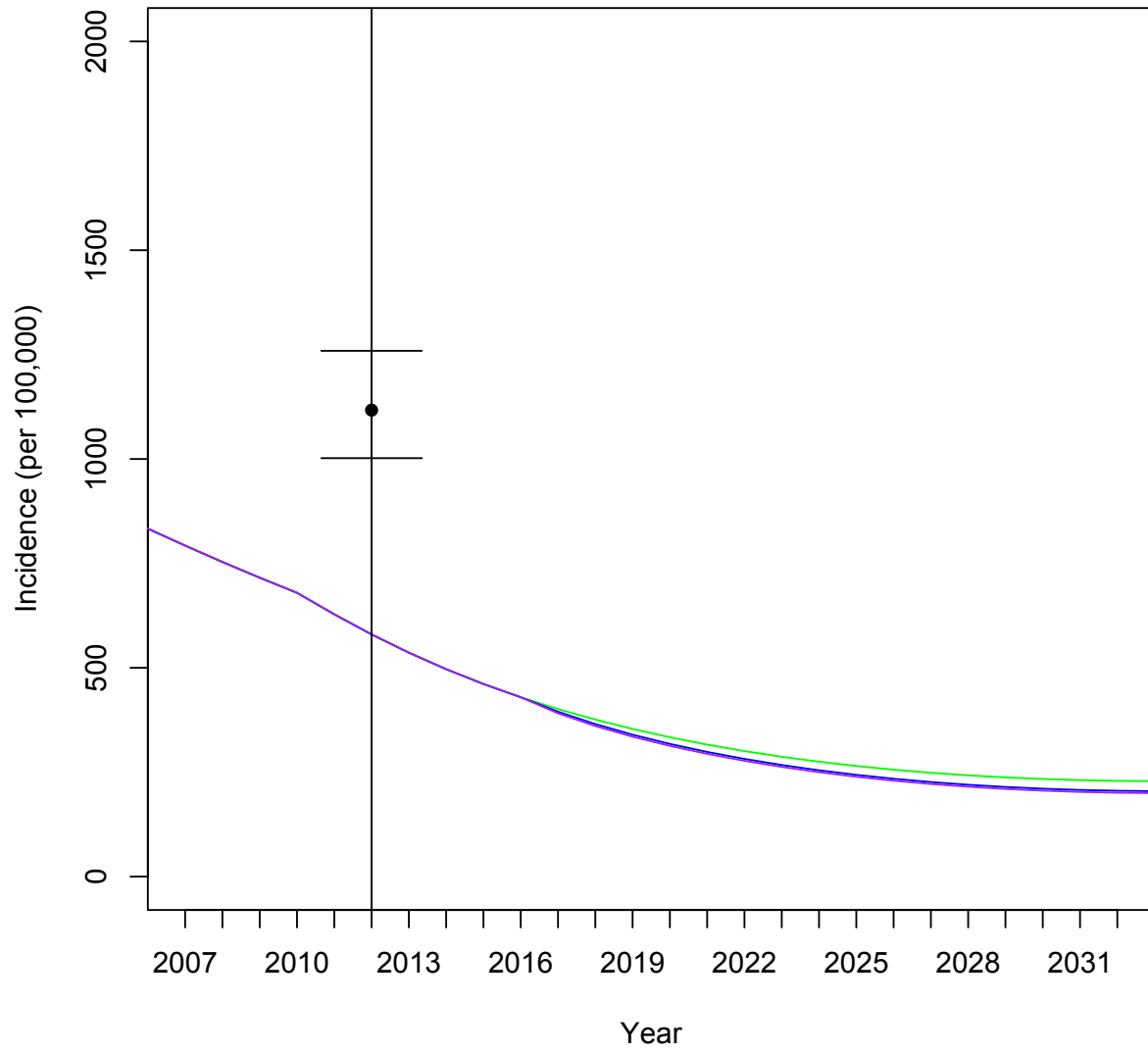
Birthrate of 0.01
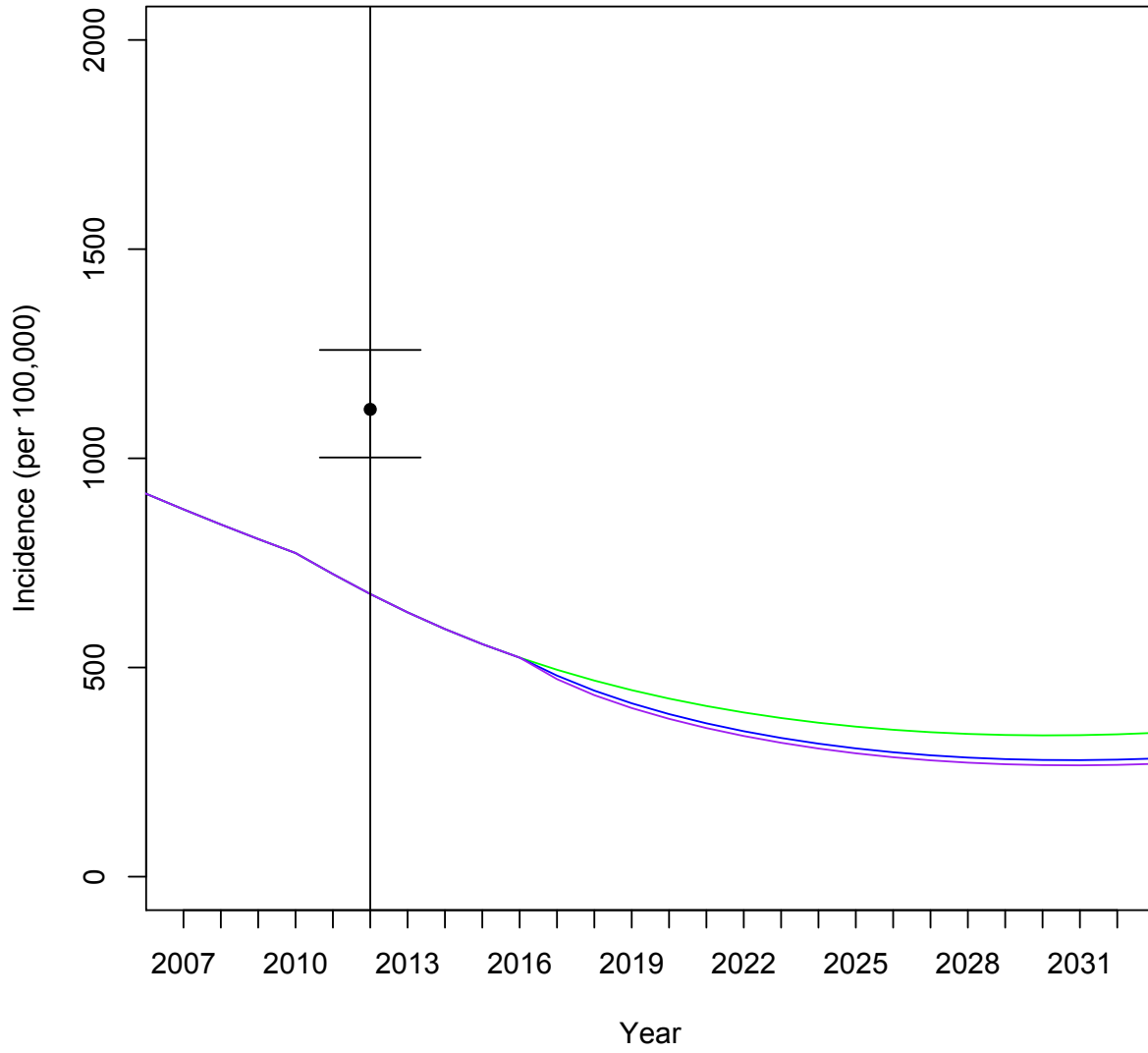
Birthrate of 0.02
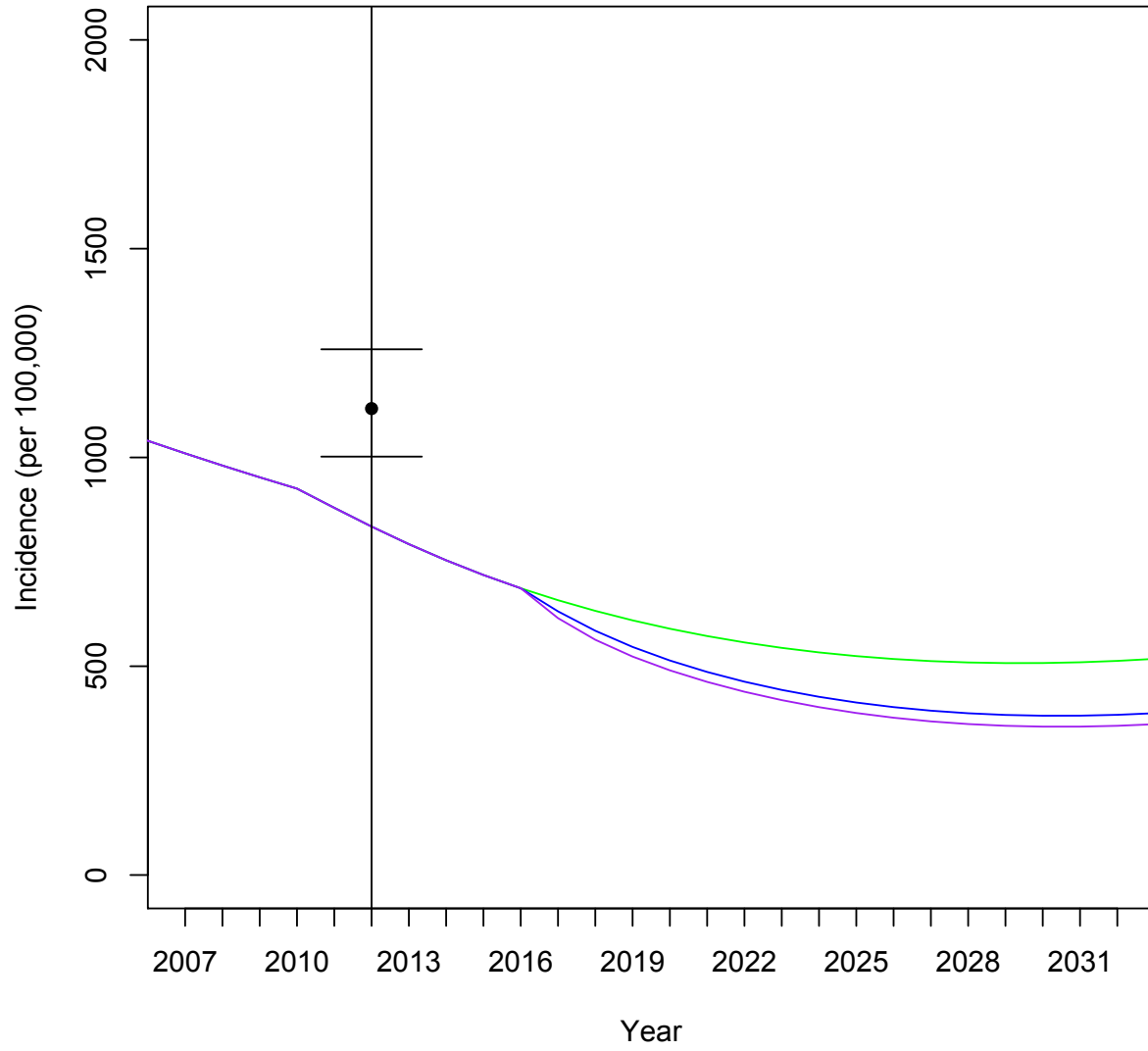
Birthrate of 0.04
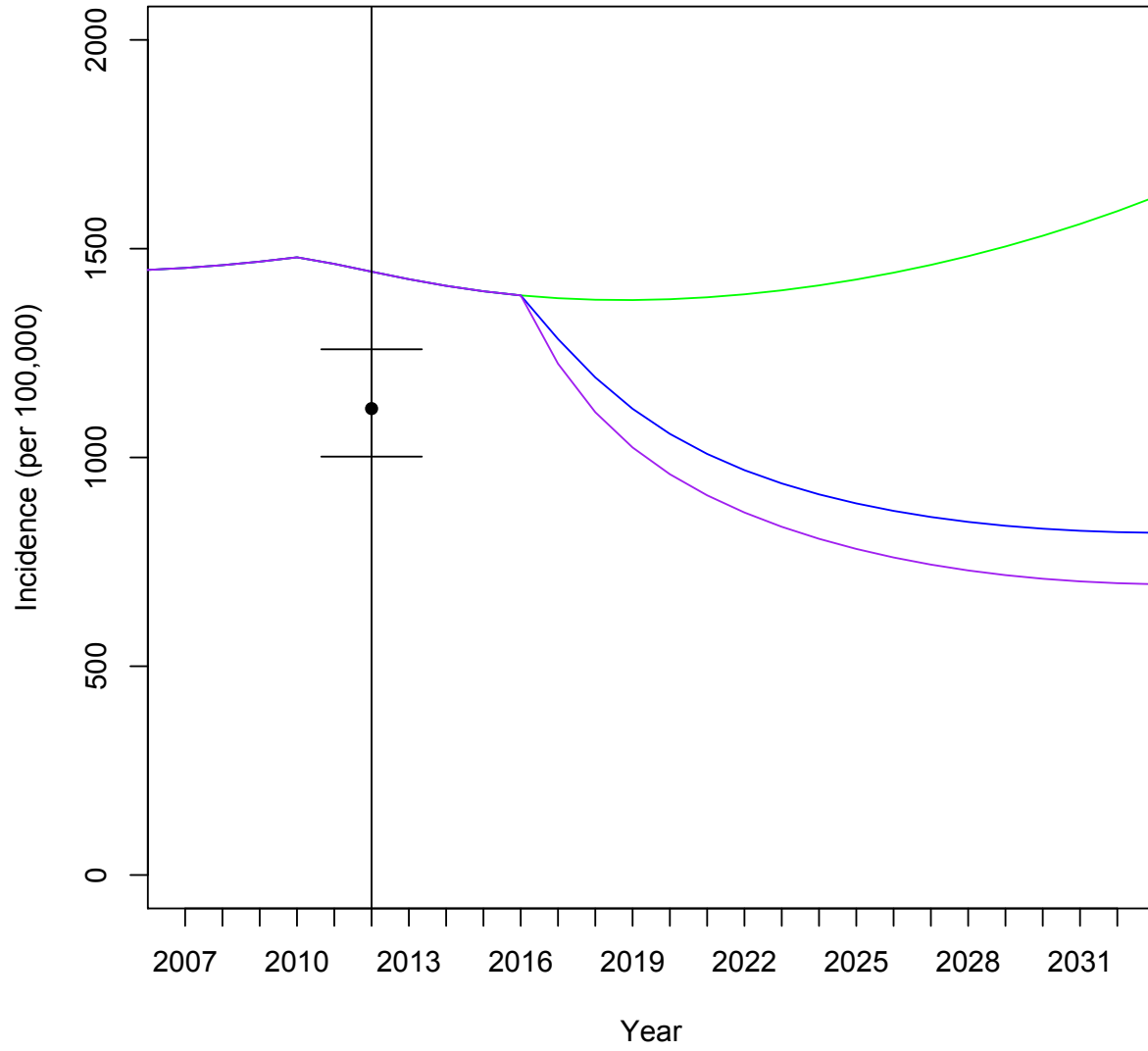
Figure S9: Effects of expanding IPT to adults

TB Incidence as a Result of Adolescent and Adult IPT
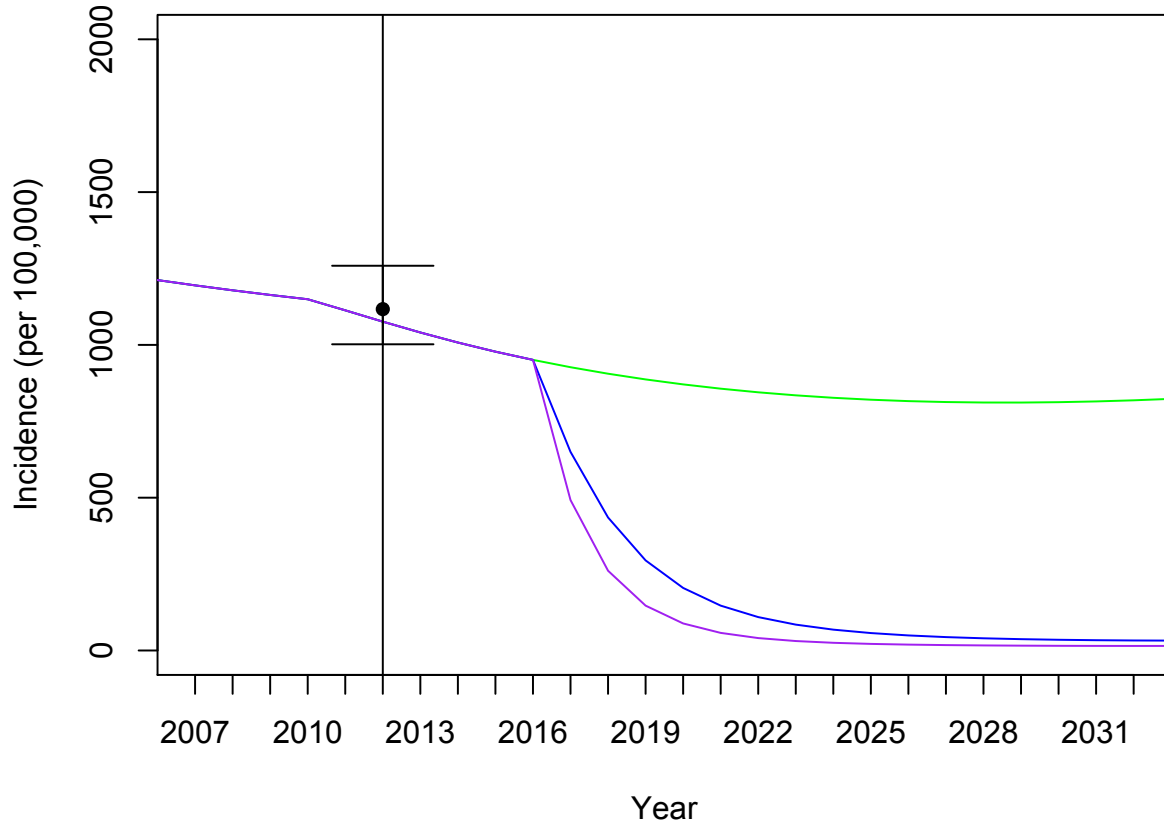
Figure S10: Effects of general adolescent IPT on TB incidence in HIV-positive population
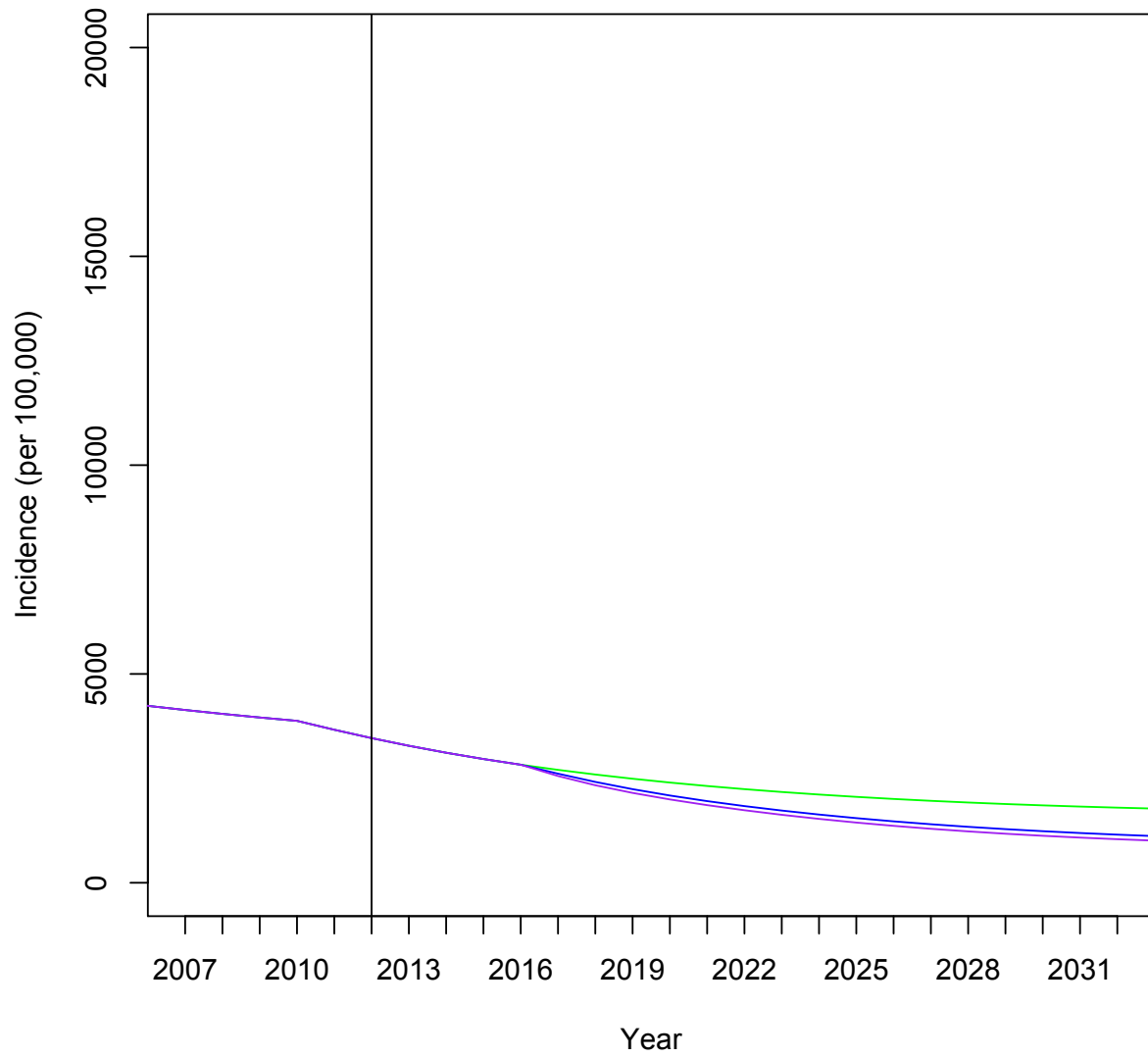
Figure S11: Effects of general adolescent IPT on TB incidence in HIV-positive adult population
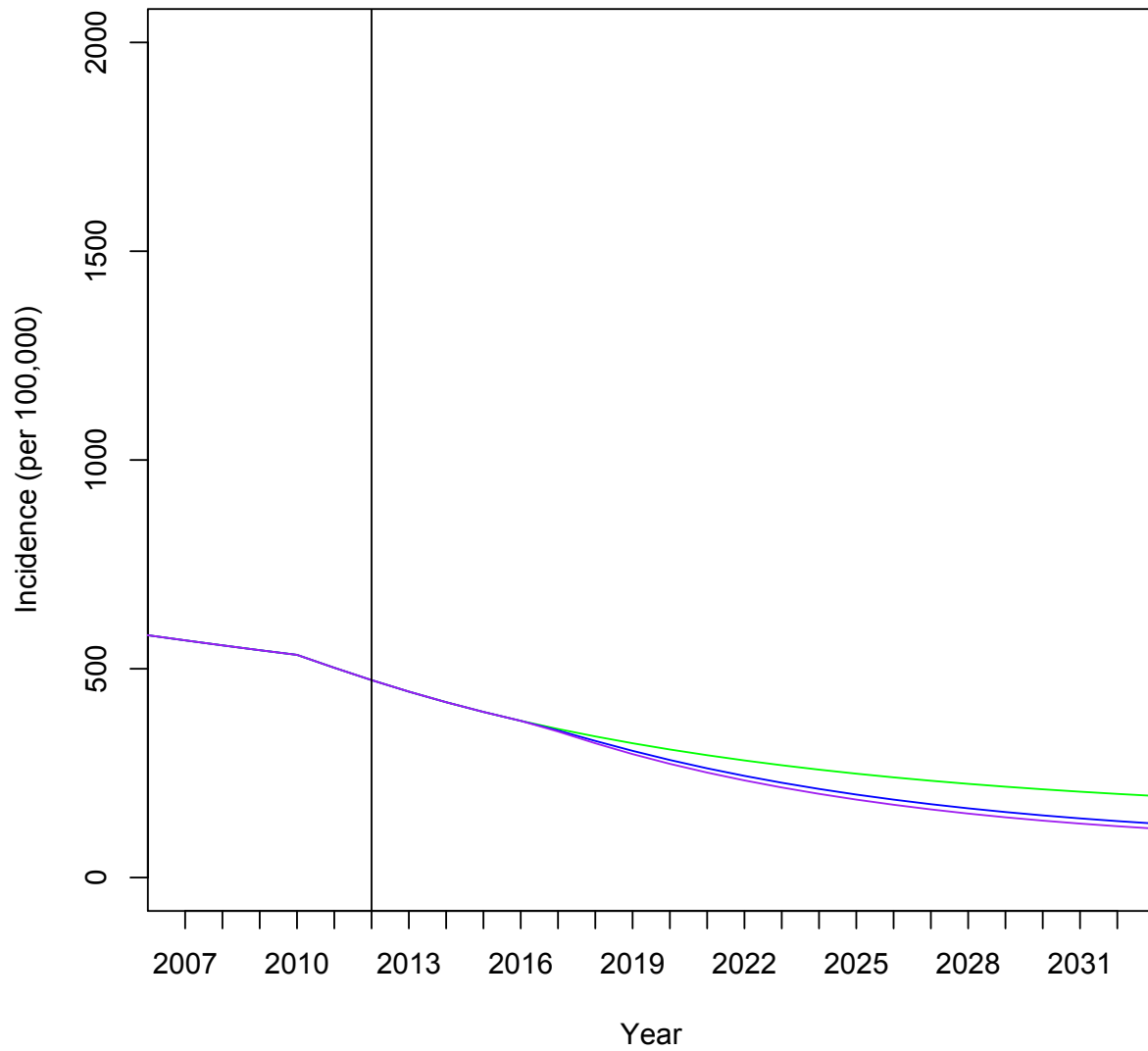
Figure S12: Conditions under which TB incidence could increase as a result of expanded IPT
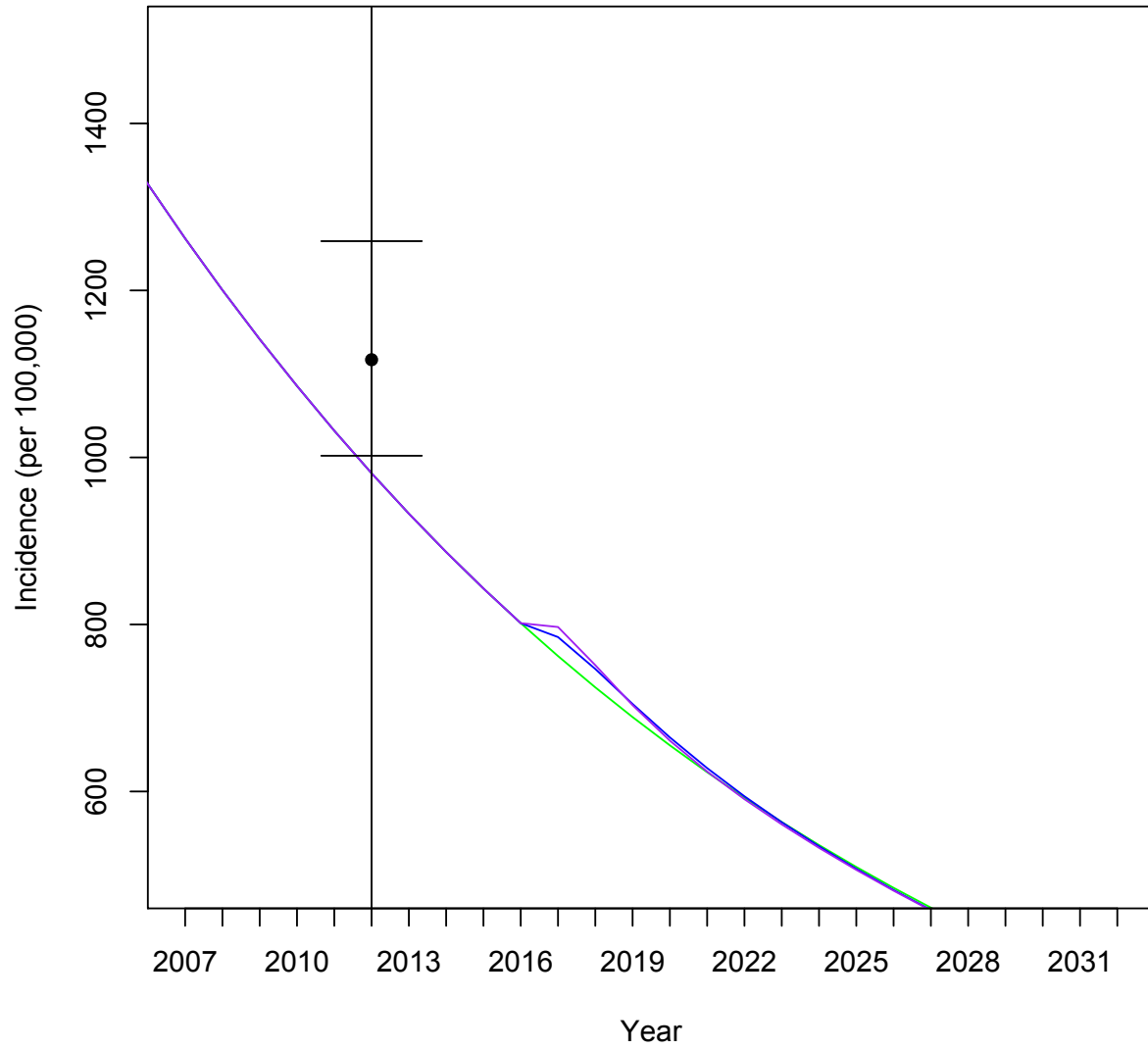
Figure S13: Impact of IPT access by demographic group
This figure compares the impact on general population TB incidence of IPT access in A)
adolescents only, B) adults only, C) both adolescents and adults

A) Population level TB incidence in response to IPT access in adolescents only
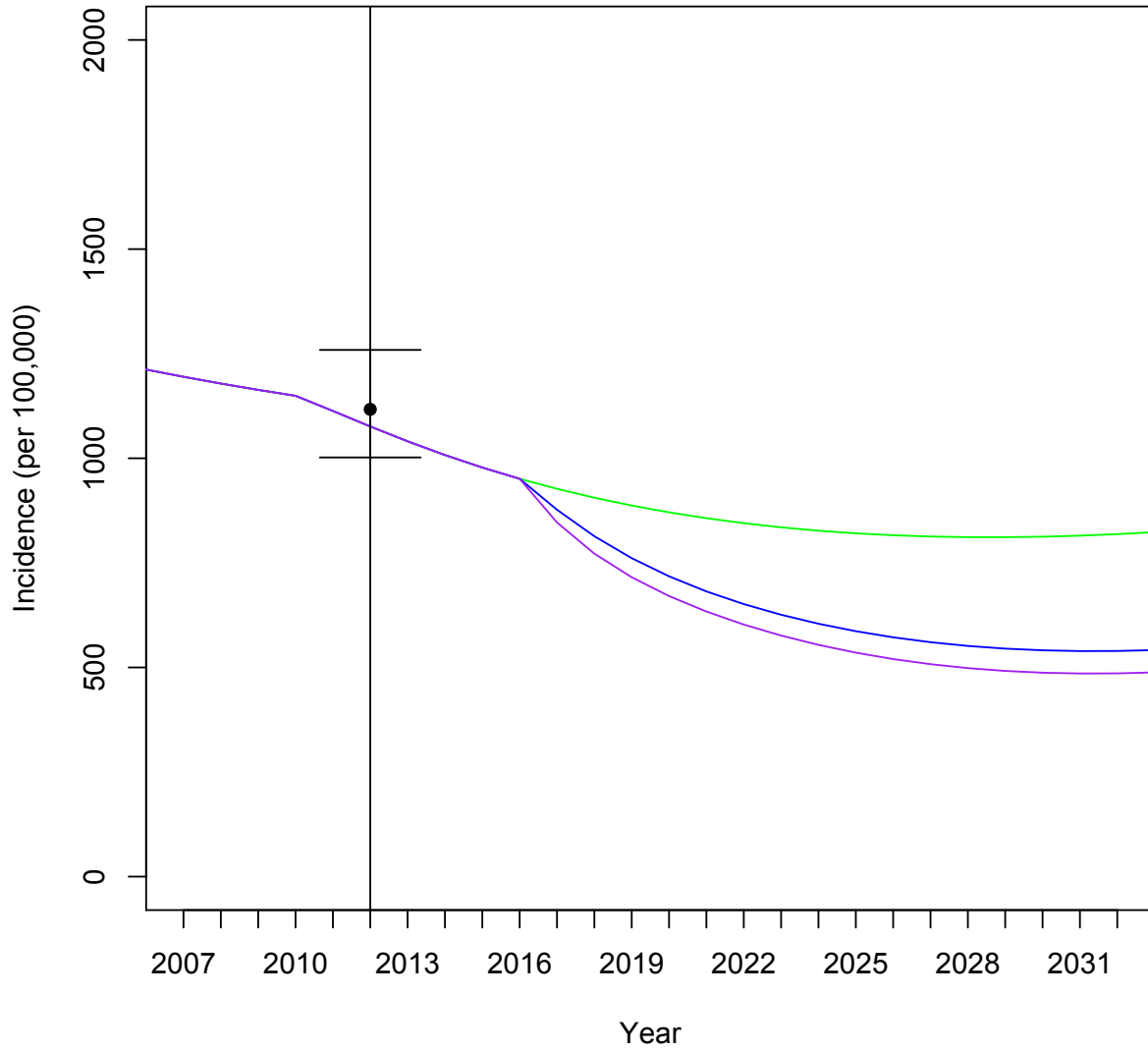
B) Population level TB incidence in response to IPT access in adults only
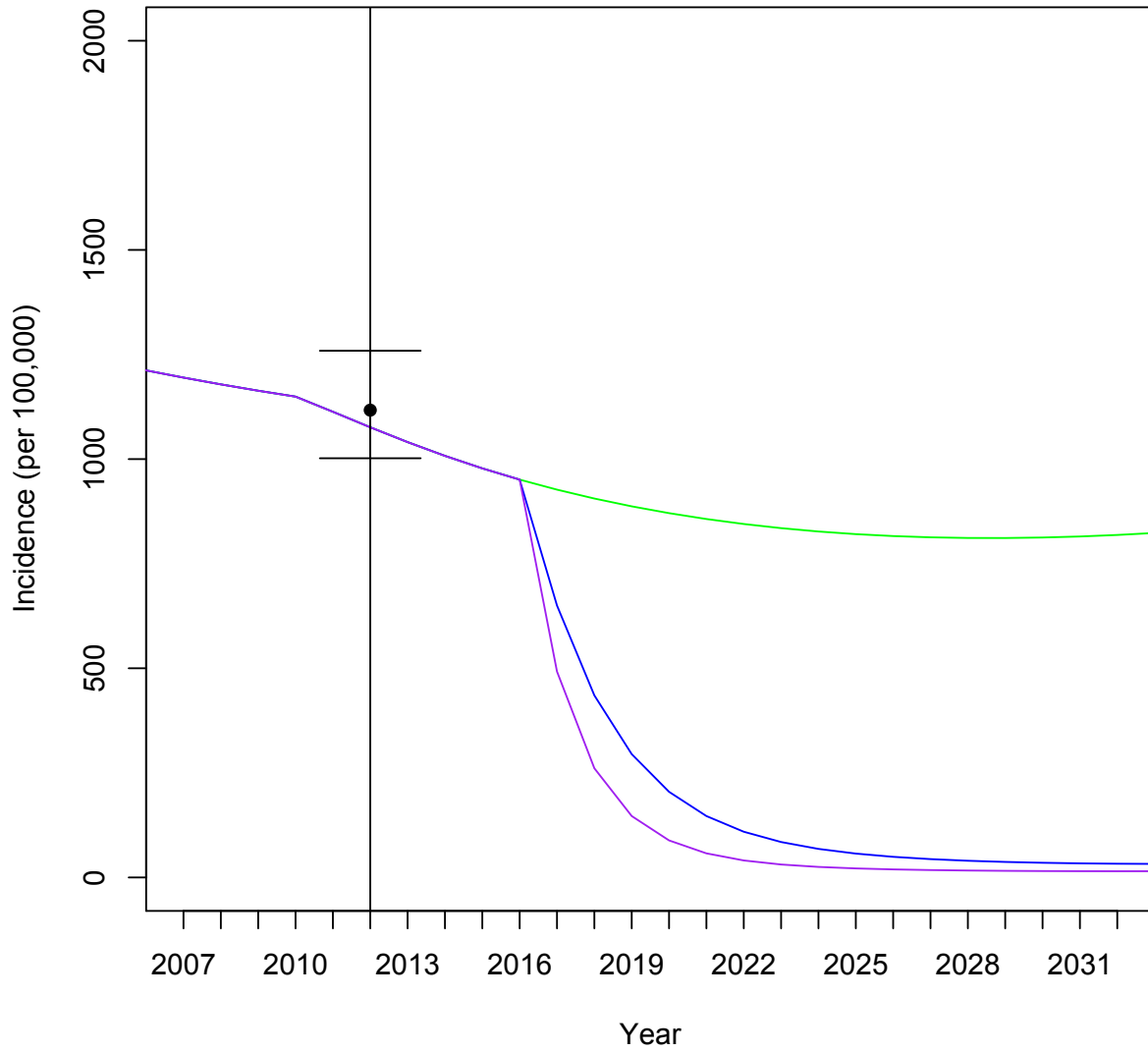
C) Population level TB incidence in response to IPT access in both adolescents and adults

Figure S14: Multiple-way sensitivity analyses

This figure shows two multi-way sensitivity analyses between key variables, A) TB activation in HIV-positive individuals versus rate of stopping IPT, and B) TB transmission rate versus stopping IPT

A) TB activation in HIV-positive individuals versus rate of stopping IPT

Rate of stopping IPT

TB Activation
Rate in HIV+
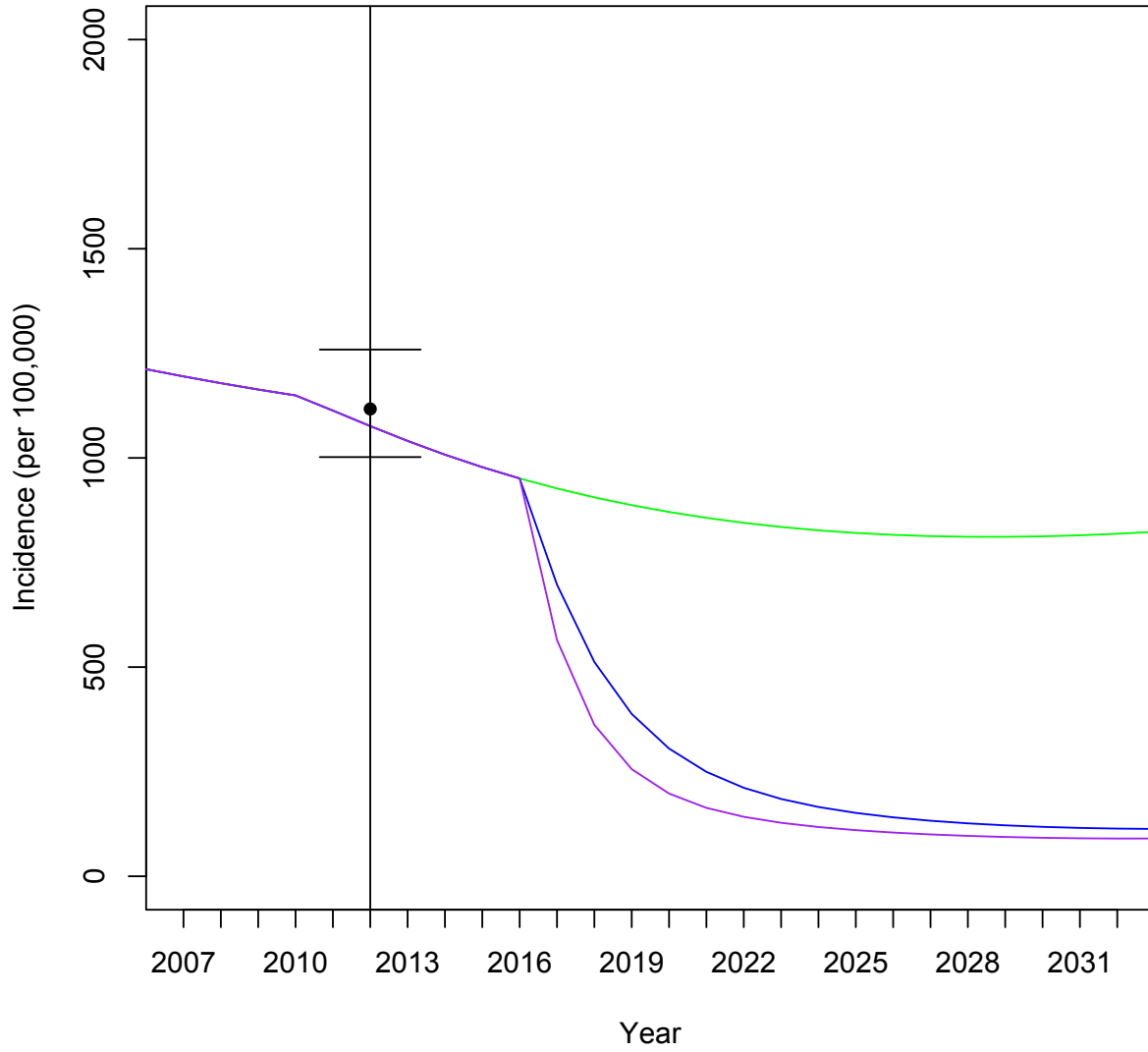
B) TB transmission rate versus stopping IPT

Rate of Stopping IPT

TB Transmission
Rate

0.05   0.1   0.15

0.0000001
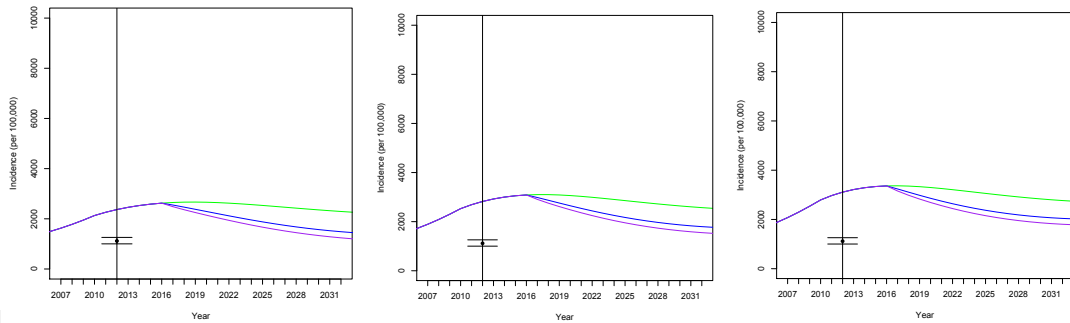
0.0000002
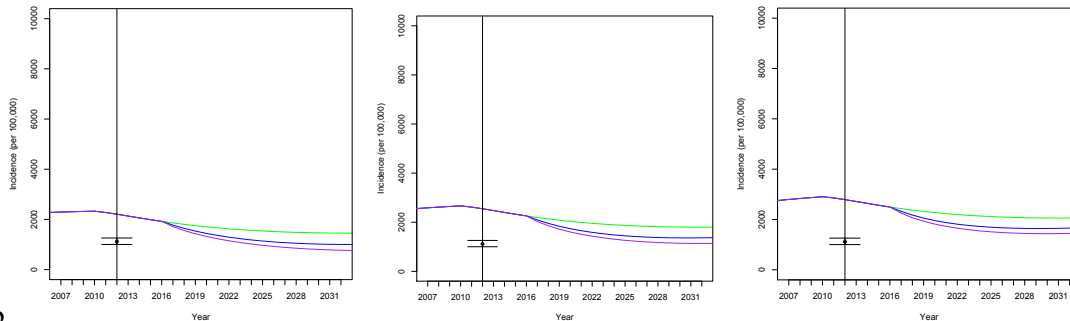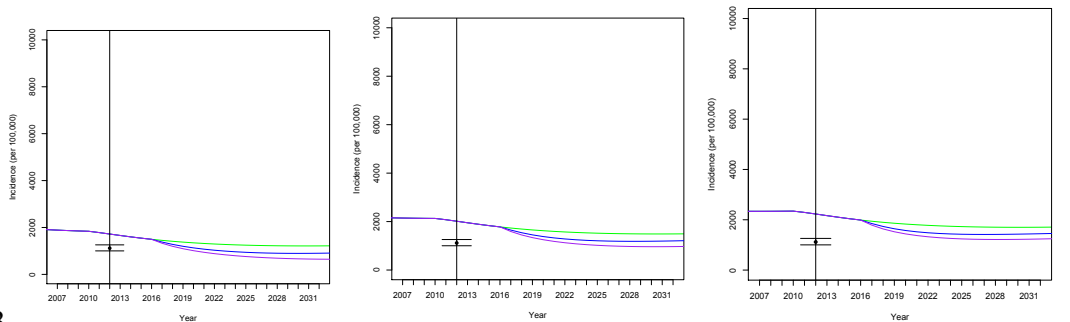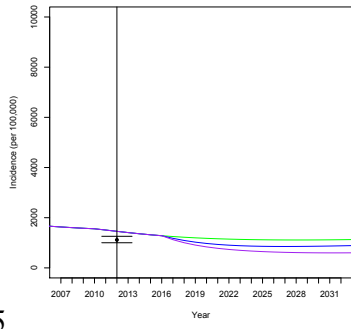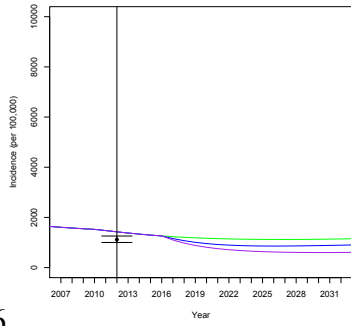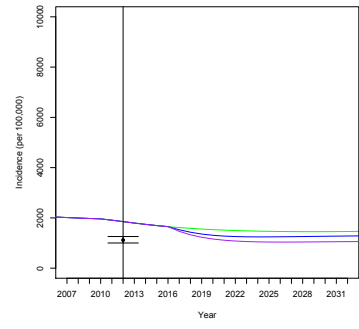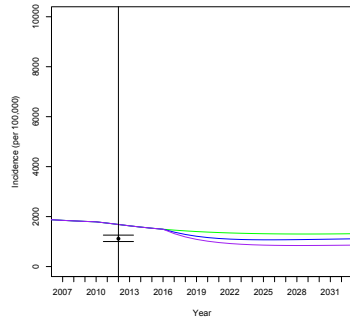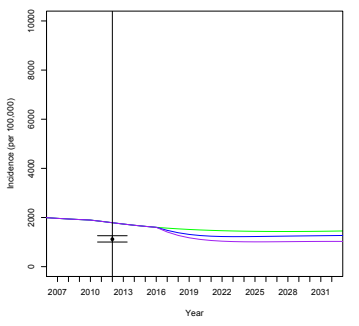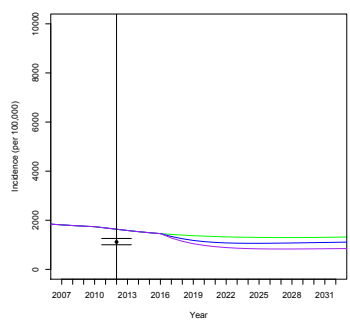
0.0000003

0.0000005

0.0000006

Figure S15: Effects of IPT in adolescents on various demographic groups
This figure shows the impact of IPT access in adolescents on TB incidence in A) adolescents, and B) adults

A) Impact of IPT access in adolescents on TB incidence in adolescents
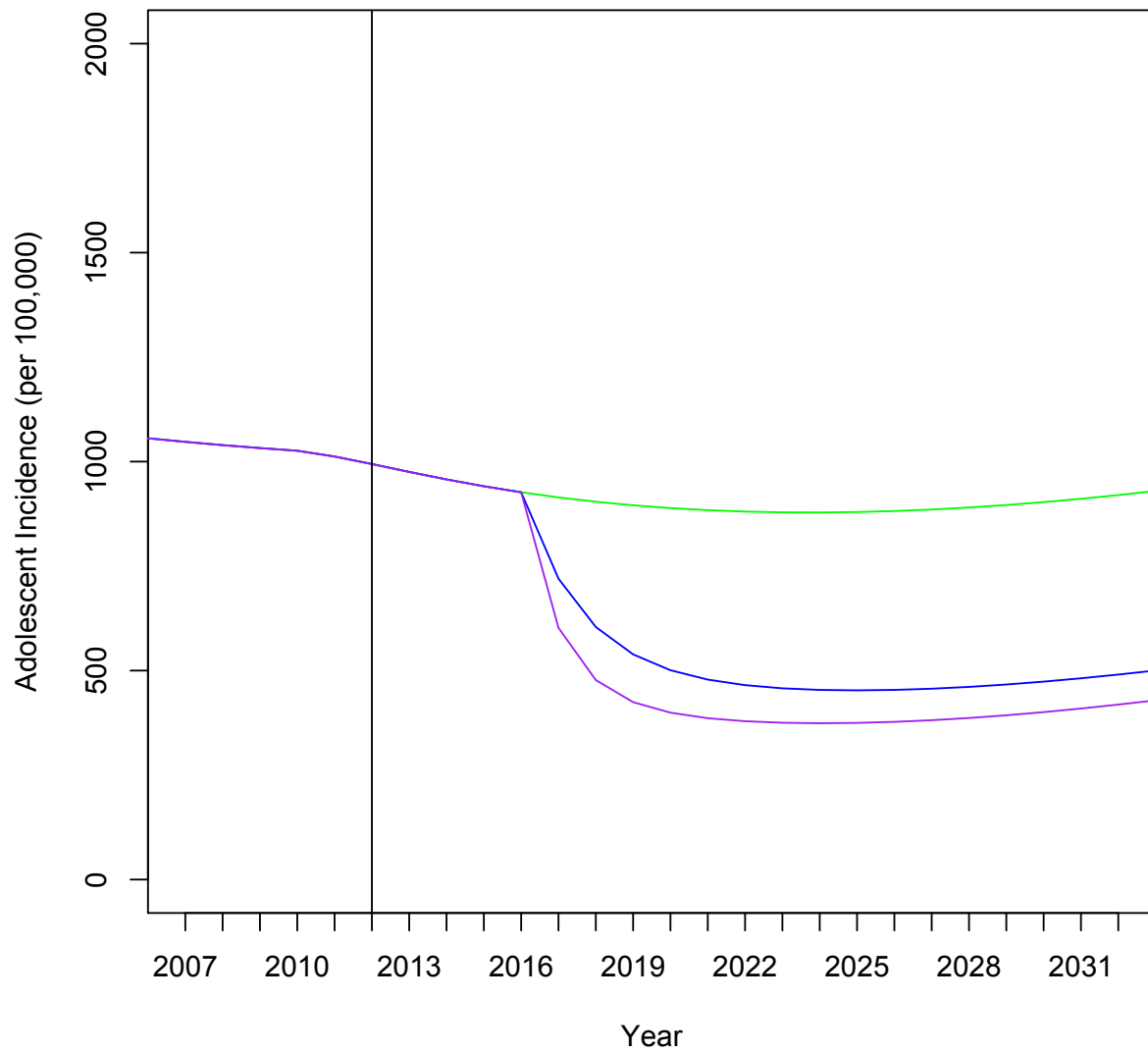
B) Impact of IPT access in adolescents on TB incidence in adults
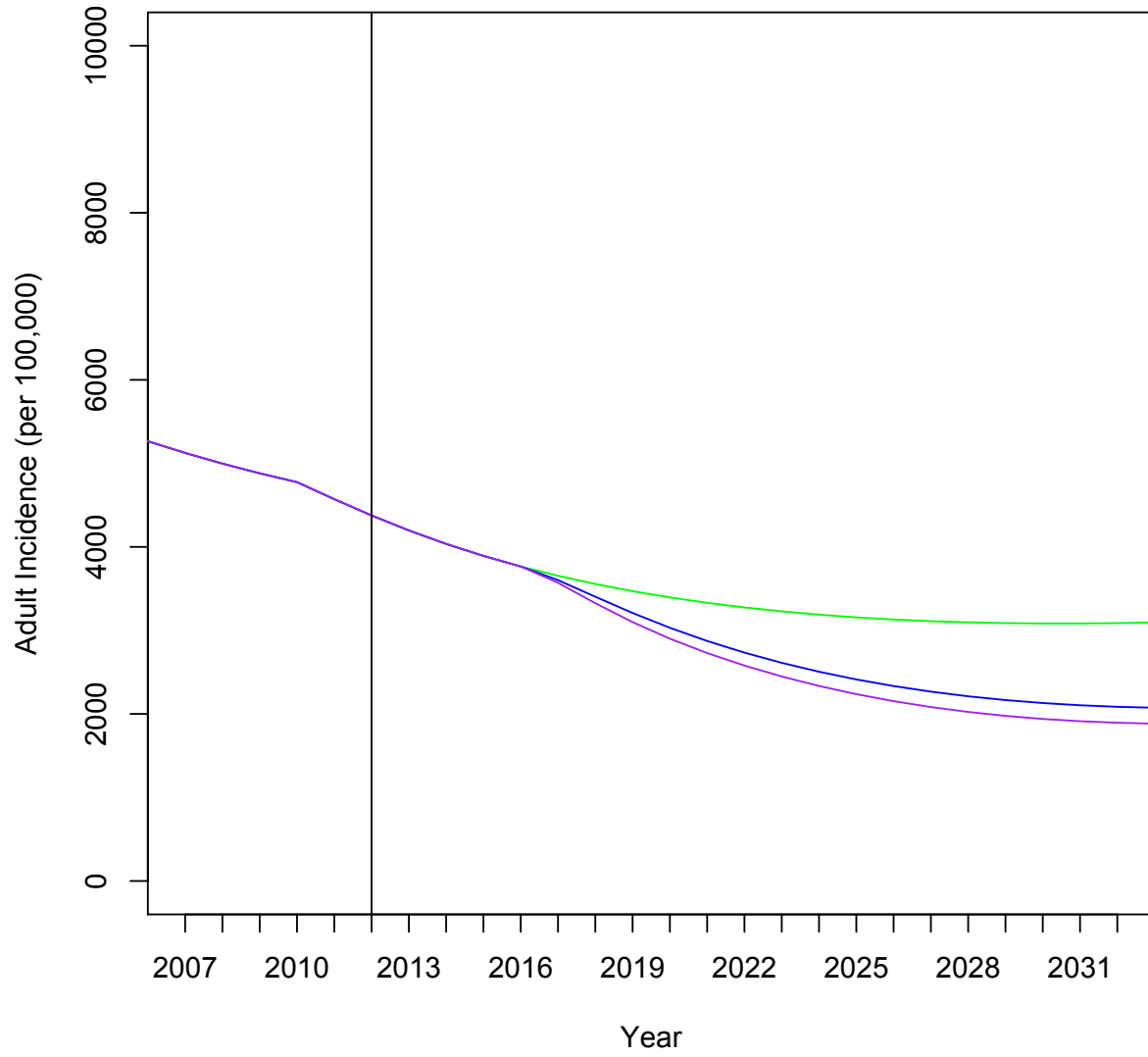
Equations

Children

$dS_C = \nu*(S_C + I_C + S_{AdoHIVn} + S_{AdoPreART} + S_{AdoARTelig} + S_{AdoOnARTCD4low} + S_{AdoOnARTCD4high} + L_{nottreatedAdoHIVn} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{nottreatedAdoOnARTCD4low} + L_{nottreatedAdoOnARTCD4high} + L_{IPTAdoHIVn} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + L_{IPTAdoOnARTCD4low} + L_{IPTAdoOnARTCD4high} + I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{nottreatedAdoOnARTCD4low} + I_{nottreatedAdoOnARTCD4high} + I_{CTAdoHIVn} + I_{CTAdoPreART} + I_{CTAdoARTelig} + I_{CTAdoOnARTCD4low} + I_{CTAdoOnARTCD4high} + S_{AdlHIVn} + S_{AdlPreART} + S_{AdlARTelig} + S_{AdlOnARTCD4low} + S_{AdlOnARTCD4high} + L_{nottreatedAdlHIVn} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{nottreatedAdlOnARTCD4low} + L_{nottreatedAdlOnARTCD4high} + L_{IPTAdlHIVn} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + L_{IPTAdlOnARTCD4low} + L_{IPTAdlOnARTCD4high} + I_{nottreatedAdlHIVn} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{nottreatedAdlOnARTCD4low} + I_{nottreatedAdlOnARTCD4high} + I_{CTAdlHIVn} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{CTAdlOnARTCD4low} + I_{CTAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high} + R_{AdoHIVn} + R_{AdoPreART} + R_{AdoARTelig} + R_{AdoOnARTCD4low} + R_{AdoOnARTCD4high} + R_{AdlHIVn} + R_{AdlPreART} + R_{AdlARTelig} + R_{AdlOnARTCD4low} + R_{AdlOnARTCD4high}) - \beta_{TBchi} *$
$(S_C*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}))) - \delta* S_C - \mu_{chi}*S_C$

$dI_C = \beta_{TBchi} * (S_C * ((I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{nottreatedAdoOnARTCD4low} + I_{nottreatedAdoOnARTCD4high} + I_{nottreatedAdlHIVn} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{nottreatedAdlOnARTCD4low} + I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}))) - \delta*I_C - \mu_{ChiInf} *I_C$

Adolescents

$dS_{AdoHIVn} = \nu * S_{Child} - S_{AdoHIVn} * \beta_{HIVInfAdo} * ((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) - \beta_{TBado} * S_{AdoHIVn} * ((I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{nottreatedAdoOnARTCD4low} + I_{nottreatedAdoOnARTCD4high} + I_{nottreatedAdlHIVn} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{nottreatedAdlOnARTCD4low} + I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + \gamma_{CT} * I_{CTAdoHIVn} + \kappa_{IPT} * R_{AdoHIVn} - \mu_{ado} * S_{AdoHIVn} - \varepsilon * S_{AdoHIVn}$

$dS_{AdoPreART} = S_{AdoHIVn} * \beta_{HIVInfAdo} * ((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) - \zeta_{HIV} * S_{AdoPreART} - \beta_{TBado} * S_{AdoPreART} * ((I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{nottreatedAdoOnARTCD4low} + I_{nottreatedAdoOnARTCD4high} + I_{nottreatedAdlHIVn} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{nottreatedAdlOnARTCD4low} + I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + \gamma_{CT} * I_{CTAdoPreART} + \kappa_{IPT} * R_{AdoPreART} - \mu_{PreARTAdo} * S_{AdoPreART} - \varepsilon * S_{AdoPreART}$

$dS_{AdoARTelig} = \zeta_{HIV} * S_{AdoPreART} - \eta_{HIV} * S_{AdoARTelig} - \beta_{TBado} * S_{AdoARTelig} * ((I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{nottreatedAdoOnARTCD4low} + I_{nottreatedAdoOnARTCD4high} + I_{nottreatedAdlHIVn} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{nottreatedAdlOnARTCD4low} + I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) +$

$\gamma_{CT}*I_{CTAdoARTelig} + \kappa_{IPT}*R_{AdoARTelig} - \mu_{ARTeligAdo}*S_{AdoARTelig} - \epsilon*S_{AdoARTelig}$

$dS_{AdoOnARTCD4low} = \eta_{HIV}*S_{AdoARTelig} - \theta_{HIV}*S_{AdoOnARTCD4low} -$
$\beta_{TBado}*S_{AdoOnARTCD4low}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) +$
$\gamma_{CT}*I_{CTAdoOnARTCD4low} + \kappa_{IPT}*R_{AdoOnARTCD4low} - \mu_{ARTlowAdo}*S_{AdoOnARTCD4low} - \epsilon*S_{AdoOnARTCD4low}$

$dS_{AdoOnARTCD4high} = \theta_{HIV}*S_{AdoOnARTCD4low} -$
$\beta_{TBado}*S_{AdoOnARTCD4high}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) +$
$\gamma_{CT}*I_{CTAdoOnARTCD4high} + \iota_{IPT}*R_{AdoOnARTCD4high} - \mu_{ARThighAdo}*S_{AdoOnARTCD4high} - \epsilon*S_{AdoOnARTCD4high}$

$dR_{AdoHIVn} = -R_{AdoHIVn}*\beta_{HIVInfAdo}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) + \gamma_{IPT}*L_{IPTAdoHIVn} - \kappa_{IPT}*R_{AdoHIVn} - \epsilon*R_{AdoHIVn} - \mu_{ado}*R_{AdoHIVn}$

$dR_{AdoPreART} = R_{AdoHIVn}*\beta_{HIVInfAdo}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) - \zeta_{HIV}*R_{AdoPreART} + \gamma_{IPT}*L_{IPTAdoPreART} - \kappa_{IPT}*R_{AdoPreART} - \epsilon*R_{AdoPreART} - \mu_{PreARTAdo}*R_{AdoPreART}$

$dR_{AdoARTelig} = \zeta_{HIV}*R_{AdoPreART} -\eta_{HIV}*R_{AdoARTelig} + \gamma_{IPT}*L_{IPTAdoARTelig} - \kappa_{IPT}*R_{AdoARTelig} - \epsilon*R_{AdoARTelig} - \mu_{ARTeligAdo}*R_{AdoARTelig}$

$dR_{AdoOnARTCD4low} = \eta_{HIV}*R_{AdoARTelig} - \theta_{HIV}*R_{AdoOnARTCD4low} + \gamma_{IPT}*L_{IPTAdoOnARTCD4low} - \kappa_{IPT}*R_{AdoOnARTCD4low} - \epsilon*R_{AdoOnARTCD4low} - \mu_{ARTlowAdo}*R_{AdoOnARTCD4low}$

$dR_{AdoOnARTCD4high} = \theta_{HIV}*R_{AdoOnARTCD4low} + \gamma_{IPT}*L_{IPTAdoOnARTCD4high} - \kappa_{IPT}*R_{AdoOnARTCD4high} - \epsilon*R_{AdoOnARTCD4high} - \mu_{ARThighAdo}*R_{AdoOnARTCD4high}$

$dL_{nottreatedAdoHIVn} = \epsilon*I_{Child} - L_{nottreatedAdoHIVn}*\beta_{HIVInfAdo}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) - \xi_{AdoHIVn}*L_{nottreatedAdoHIVn} + \beta_{TBAdo}*\rho_{TBinfHIVn}*S_{AdoHIVn}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) - (a*(1-\rho_{TBInfHIVn})*\beta_{TBAdo}*L_{nottreatedAdoHIVn}*(I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) + (\tau_{AdoHIVn}*L_{nottreatedAdoHIVn})) + \psi*I_{CTAdoHIVn} - \mu_{Ado}*L_{nottreatedAdoHIVn} + \iota_{IPT}*L_{IPTAdoHIVn} - L_{nottreatedAdoHIVn}*\vartheta_{HIVn} - \epsilon*L_{nottreatedAdoHIVn} + \varphi*I_{nottreatedAdoHIVn}$

$dL_{nottreatedAdoPreART} = L_{nottreatedAdoHIVn}*\beta_{HIVInfAdo}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART}$

$+ I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) - \zeta_{HIV}*L_{nottreatedAdoPreART} +$

$\beta_{TBAdo}*\rho_{TBInfHIVp}*S_{AdoPreART}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnART}$
$_{CD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnART}$
$_{CD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} +$
$I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} +$
$I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high}$
$+ I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) -$
$\xi_{AdoHIVp}*L_{nottreatedAdoPreART} - (a*(1-$
$\rho_{TBInfHIVp})*\beta_{TBAdo}*L_{nottreatedAdoPreART}*(I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedA}$
$_{doOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAd}$
$_{lOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} +$
$I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} +$
$I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high}$
$+ I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) +$
$(\tau_{AdoHIVp}*L_{nottreatedAdoPreART})) + \psi*I_{CTAdoPreART}-\mu_{PreARTAdo}*L_{nottreatedAdoPreART} + \iota_{IPT}*L_{IPTAdoPreART} -$
$L_{nottreatedAdoPreART}*\vartheta_{HIVp} -\varepsilon *L_{nottreatedAdoPreART}+ \varphi*I_{nottreatedAdoPreART}$

$dL_{nottreatedAdoARTelig} = \zeta_{HIV}*L_{nottreatedAdoPreART} - \eta_{HIV}*L_{nottreatedAdoARTelig}$
$+\beta_{TBAdo}*\rho_{TBinfHIVp}*S_{AdoARTelig}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnA}$
$_{RTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnAR}$
$_{TCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} +$
$I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} +$
$I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high}$
$+ I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) -$
$\xi_{AdoHIVp}*L_{nottreatedAdoARTelig} - (a*(1-$
$\rho_{TBinfHIVp})*\beta_{TBado}*L_{nottreatedAdoARTelig}*(I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedA}$
$_{doOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAd}$
$_{lOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} +$
$I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} +$
$I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high}$
$+ I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) +$
$(\tau_{AdoHIVp}*L_{nottreatedAdoARTelig})) + \psi*I_{CTAdoARTelig} + \iota_{IPT}*L_{IPTAdoARTelig} -L_{nottreatedAdoARTelig}*\vartheta_{HIVp} -$
$\mu_{ARTeligAdo}*L_{nottreatedAdoARTelig} - \varepsilon*L_{nottreatedAdoARTelig}$

$dL_{nottreatedAdoOnARTCD4low} = \eta_{HIV}*L_{nottreatedAdoARTelig} - \theta_{HIV}*L_{nottreatedAdoOnARTCD4low}$
$+natcure*I_{nottreatedAdoARTelig}+\beta_{TBado}*$
$\rho_{TBinfHIVART}*S_{AdoOnARTCD4low}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnAR}$
$_{TCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnART}$

$\text{CD4low}+I_{\text{nottreatedAdlOnARTCD4high}} + I_{\text{FNAdoHIVn}} + I_{\text{FNAdoPreART}} + I_{\text{FNAdoARTelig}} + I_{\text{FNAdoOnARTCD4low}} + I_{\text{FNAdoOnARTCD4high}} + I_{\text{IPTAdoHIVn}} + I_{\text{IPTAdoPreART}} + I_{\text{IPTAdoARTelig}} + I_{\text{IPTAdoOnARTCD4low}} + I_{\text{IPTAdoOnARTCD4high}} + I_{\text{FNAdlHIVn}} + I_{\text{FNAdlPreART}} + I_{\text{FNAdlARTelig}} + I_{\text{FNAdlOnARTCD4low}} + I_{\text{FNAdlOnARTCD4high}} + I_{\text{IPTAdlHIVn}} + I_{\text{IPTAdlPreART}} + I_{\text{IPTAdlARTelig}} + I_{\text{IPTAdlOnARTCD4low}} + I_{\text{IPTAdlOnARTCD4high}})) -$
$\xi_{\text{AdoHIVp}}*L_{\text{nottreatedAdoOnARTCD4low}}$ -
$(a*\beta_{\text{TBado}}*\rho_{\text{TBinfHIVART}}*L_{\text{nottreatedAdoOnARTCD4low}}*(I_{\text{nottreatedAdoHIVn}}+I_{\text{nottreatedAdoPreART}}+I_{\text{nottreatedAdoARTelig}}$
$+I_{\text{nottreatedAdoOnARTCD4low}}+I_{\text{nottreatedAdoOnARTCD4high}}+I_{\text{nottreatedAdlHIVn}}+I_{\text{nottreatedAdlPreART}}+I_{\text{nottreatedAdlARTelig}}$
$+I_{\text{nottreatedAdlOnARTCD4low}}+I_{\text{nottreatedAdlOnARTCD4high}} + I_{\text{FNAdoHIVn}} + I_{\text{FNAdoPreART}} + I_{\text{FNAdoARTelig}} +$
$I_{\text{FNAdoOnARTCD4low}} + I_{\text{FNAdoOnARTCD4high}} + I_{\text{IPTAdoHIVn}} + I_{\text{IPTAdoPreART}} + I_{\text{IPTAdoARTelig}} +$
$I_{\text{IPTAdoOnARTCD4low}} + I_{\text{IPTAdoOnARTCD4high}} + I_{\text{FNAdlHIVn}} + I_{\text{FNAdlPreART}} + I_{\text{FNAdlARTelig}} + I_{\text{FNAdlOnARTCD4low}}$
$+ I_{\text{FNAdlOnARTCD4high}} + I_{\text{IPTAdlHIVn}} + I_{\text{IPTAdlPreART}} + I_{\text{IPTAdlARTelig}} + I_{\text{IPTAdlOnARTCD4low}} +$
$I_{\text{IPTAdlOnARTCD4high}}) - (\tau_{\text{ART}}*L_{\text{nottreatedAdoOnARTCD4low}})) + \psi*I_{\text{CTAdoOnARTCD4low}} -$
$\mu_{\text{ARTlowAdo}}*L_{\text{nottreatedAdoOnARTCD4low}} + \iota*L_{\text{IPTAdoOnARTCD4low}} - L_{\text{nottreatedAdoOnARTCD4low}}*\vartheta_{\text{ART}} -$
$\varepsilon*L_{\text{nottreatedAdoOnARTCD4low}}+ \varphi*I_{\text{nottreatedAdoOnARTCD4low}}$

$dL_{\text{nottreatedAdoOnARTCD4high}} = \theta_{\text{HIV}}*L_{\text{nottreatedAdoOnARTCD4low}} +$
$\beta_{\text{TBado}}*\rho_{\text{TBinfHIVART}}*S_{\text{AdoOnARTCD4high}}*((I_{\text{nottreatedAdoHIVn}}+I_{\text{nottreatedAdoPreART}}+I_{\text{nottreatedAdoARTelig}}+I_{\text{nottreated}}$
$_{\text{AdoOnARTCD4low}}+I_{\text{nottreatedAdoOnARTCD4high}}+I_{\text{nottreatedAdlHIVn}}+I_{\text{nottreatedAdlPreART}}+I_{\text{nottreatedAdlARTelig}}+I_{\text{nottreatedA}}$
$_{\text{dlOnARTCD4low}}+I_{\text{nottreatedAdlOnARTCD4high}} + I_{\text{FNAdoHIVn}} + I_{\text{FNAdoPreART}} + I_{\text{FNAdoARTelig}} + I_{\text{FNAdoOnARTCD4low}}$
$+ I_{\text{FNAdoOnARTCD4high}} + I_{\text{IPTAdoHIVn}} + I_{\text{IPTAdoPreART}} + I_{\text{IPTAdoARTelig}} + I_{\text{IPTAdoOnARTCD4low}} +$
$I_{\text{IPTAdoOnARTCD4high}} + I_{\text{FNAdlHIVn}} + I_{\text{FNAdlPreART}} + I_{\text{FNAdlARTelig}} + I_{\text{FNAdlOnARTCD4low}} + I_{\text{FNAdlOnARTCD4high}}$
$+ I_{\text{IPTAdlHIVn}} + I_{\text{IPTAdlPreART}} + I_{\text{IPTAdlARTelig}} + I_{\text{IPTAdlOnARTCD4low}} + I_{\text{IPTAdlOnARTCD4high}})) -$
$\xi_{\text{AdoHIVp}}*L_{\text{nottreatedAdoOnARTCD4high}}$ -
$(a*\beta_{\text{TBado}}*\rho_{\text{TBinfHIVART}}*L_{\text{nottreatedAdoOnARTCD4high}}*(I_{\text{nottreatedAdoHIVn}}+I_{\text{nottreatedAdoPreART}}+I_{\text{nottreatedAdoARTeli}}$
$_{\text{g}}+I_{\text{nottreatedAdoOnARTCD4low}}+I_{\text{nottreatedAdoOnARTCD4high}}+I_{\text{nottreatedAdlHIVn}}+I_{\text{nottreatedAdlPreART}}+I_{\text{nottreatedAdlARTelig}}$
$+I_{\text{nottreatedAdlOnARTCD4low}}+I_{\text{nottreatedAdlOnARTCD4high}} + I_{\text{FNAdoHIVn}} + I_{\text{FNAdoPreART}} + I_{\text{FNAdoARTelig}} +$
$I_{\text{FNAdoOnARTCD4low}} + I_{\text{FNAdoOnARTCD4high}} + I_{\text{IPTAdoHIVn}} + I_{\text{IPTAdoPreART}} + I_{\text{IPTAdoARTelig}} +$
$I_{\text{IPTAdoOnARTCD4low}} + I_{\text{IPTAdoOnARTCD4high}} + I_{\text{FNAdlHIVn}} + I_{\text{FNAdlPreART}} + I_{\text{FNAdlARTelig}} + I_{\text{FNAdlOnARTCD4low}}$
$+ I_{\text{FNAdlOnARTCD4high}} + I_{\text{IPTAdlHIVn}} + I_{\text{IPTAdlPreART}} + I_{\text{IPTAdlARTelig}} + I_{\text{IPTAdlOnARTCD4low}} +$
$I_{\text{IPTAdlOnARTCD4high}}) + (\tau_{\text{AdoHIVART}}*L_{\text{nottreatedAdoOnARTCD4high}})) + \psi*I_{\text{CTAdoOnARTCD4high}}-$
$\mu_{\text{ARThighAdo}}*L_{\text{nottreatedAdoOnARTCD4high}} + \iota_{\text{IPT}}*L_{\text{IPTAdoOnARTCD4high}} -L_{\text{nottreatedAdoOnARTCD4high}}*\vartheta_{\text{ART}} - \varepsilon$
$*L_{\text{nottreatedAdoOnARTCD4high}}+ \varphi*I_{\text{nottreatedAdoOnARTCD4high}}$

$dL_{\text{IPTAdoHIVn}} = -L_{\text{IPTAdoHIVn}}*\beta_{\text{HIVInfAdo}}*((S_{\text{AdoPreART}} + S_{\text{AdoARTelig}} + L_{\text{nottreatedAdoPreART}} +$
$L_{\text{nottreatedAdoARTelig}} + L_{\text{IPTAdoPreART}} + L_{\text{IPTAdoARTelig}} + I_{\text{InottreatedAdoPreART}} + I_{\text{nottreatedAdoARTelig}} +$
$I_{\text{CTAdoPreART}} + I_{\text{CTAdoARTelig}} + S_{\text{AdlPreART}} + S_{\text{AdlARTelig}} + L_{\text{nottreatedAdlPreART}} + L_{\text{nottreatedAdlARTelig}} +$
$L_{\text{IPTAdlPreART}} + L_{\text{IPTAdlARTelig}} + I_{\text{nottreatedAdlPreART}} + I_{\text{nottreatedAdlARTelig}} + I_{\text{CTAdlPreART}} + I_{\text{CTAdlARTelig}} +$
$I_{\text{FNAdoPreART}} + I_{\text{FNAdoARTelig}} + I_{\text{IPTAdoPreART}} + I_{\text{IPTAdoARTelig}} + I_{\text{FNAdlPreART}} + I_{\text{FNAdlARTelig}} + I_{\text{IPTAdlPreART}}$

$+ I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) + \xi_{AdoHIVn}*L_{nottreatedAdoHIVn} - \gamma_{IPT}*L_{IPTAdoHIVn} - ((\lambda_{HIVn}*L_{IPTAdoHIVn})) - \iota_{IPT}*L_{IPTAdoHIVn} - \mu_{Ado}*L_{IPTAdoHIVn} - \varepsilon*L_{IPTAdoHIVn}$

$dL_{IPTAdoPreART} = L_{IPTAdoHIVn}*\beta_{HIVInfAdo}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) - \zeta_{HIV}*L_{IPTAdoPreART} + \xi_{AdoHIVp}*L_{nottreatedAdoPreART} - \gamma_{IPT}*L_{IPTAdoPreART} - ((\lambda_{HIVp}*L_{IPTAdoPreART})) - \iota_{IPT}*L_{IPTAdoPreART} - \mu_{PreARTAdo}*L_{IPTAdoPreART} - \varepsilon*L_{IPTAdoPreART}$

$dL_{IPTAdoARTelig} = \zeta_{HIV}*L_{IPTAdoPreART} - \eta_{HIV}*L_{IPTAdoARTelig} + \xi_{AdoHIVp}*L_{nottreatedAdoARTelig} - \gamma_{IPT}*L_{IPTAdoARTelig} - ((\lambda_{HIVp}*L_{IPTAdoARTelig})) - \iota_{IPT}*L_{IPTAdoARTelig} - \mu_{ARTeligAdo}*L_{IPTAdoARTelig} - \varepsilon*L_{IPTAdoARTelig}$

$dL_{IPTAdoOnARTCD4low} = \eta_{HIV}*L_{IPTAdoARTelig} - \theta_{HIV}*L_{nottreatedAdoOnARTCD4low} + \xi_{AdoHIVp}*L_{nottreatedAdoOnARTCD4low} - \gamma_{IPT}*L_{IPTAdoOnARTCD4low} - ((\lambda_{HIVp}*L_{IPTAdoOnARTCD4low})) - \iota_{IPT}*L_{IPTAdoOnARTCD4low} - \mu_{ARTlowAdo}*L_{IPTAdoOnARTCD4low} - \varepsilon*L_{IPTAdoOnARTCD4low}$

$dL_{IPTAdoOnARTCD4high} = \theta_{HIV}*L_{nottreatedAdoOnARTCD4low} + \xi_{AdoHIVp}*L_{nottreatedAdoOnARTCD4high} - \theta_{IPT}*L_{IPTAdoOnARTCD4high} - ((\lambda_{HIVp}*L_{IPTAdoOnARTCD4high})) - \iota_{IPT}*L_{IPTAdoOnARTCD4high} - \mu_{ARThighAdo}*L_{IPTAdoOnARTCD4high} - \varepsilon*L_{IPTAdoOnARTCD4high}$

$dI_{nottreatedAdoHIVn} = -I_{nottreatedAdoHIVn}*\beta_{HIVInfado}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) + \beta_{TBado}*(1-\rho_{TBinfHIVn})*S_{AdoHIVn}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} +$

$I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + (a*(1-\rho_{TBinfHIVn})*\beta_{TBado}*L_{nottreatedAdoHIVn}*(I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) + (\tau_{AdoHIVn}*L_{nottreatedAdoHIVn})) - \chi_{Ado}*I_{nottreatedAdoHIVn} + \sigma*I_{CTAdoHIVn} - \mu_{TBAdo}*I_{nottreatedAdoHIVn} - \varepsilon*I_{nottreatedAdoHIVn} - \varphi*I_{nottreatedAdoHIVn}$

$dI_{nottreatedAdoPreART} = I_{nottreatedAdoHIVn}*\beta_{HIVInfAdo}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) - \zeta_{HIV}*I_{nottreatedAdoPreART} + \beta_{TBado}*(1 - \rho_{TBinfHIVp}) * S_{AdoPreART} * ((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + (a*(1-\rho_{TBinfHIVp})*\beta_{TBado}*L_{nottreatedAdoPreART}*(I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) + (\tau_{AdoHIVp}*L_{nottreatedAdoPreART})) - \chi_{Ado}*I_{nottreatedAdoPreART} + \sigma*I_{CTAdoPreART} - \mu_{TBPreARTAdo}*I_{nottreatedAdoPreART} - \varepsilon*I_{nottreatedAdoPreART} - \varphi*I_{nottreatedAdoPreART}$

$dI_{nottreatedAdoARTelig} = \zeta_{HIV}*I_{nottreatedAdoPreART} - \eta_{HIV}*I_{nottreatedAdoARTelig} + \beta_{TBAdo}*(1-\rho_{TBinfHIVp})*S_{AdoARTelig}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} +$

$I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + (a*(1-\rho_{TBInfHIVp})*\beta_{TBAdo}*L_{nottreatedAdoARTelig}*(I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) + (\tau_{AdoHIVp}*L_{nottreatedAdoARTelig})) + \sigma*I_{CTAdoARTelig} - \chi_{Ado}*I_{nottreatedAdoARTelig} - \mu_{TBARTeligAdo}*I_{nottreatedAdoARTelig} - \varepsilon*I_{nottreatedAdoARTelig} - \varphi*I_{nottreatedAdoARTelig}$

$dI_{nottreatedAdoOnARTCD4low} = \eta_{HIV}*I_{nottreatedAdoARTelig} - \theta_{HIV}*I_{nottreatedAdoOnARTCD4low} + \beta_{TBAdo}*(1-\rho_{TBinfHIVART})*S_{AdoOnARTCD4low}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + (a*\beta_{TBAdo}*\rho_{TBinfHIVART}*L_{nottreatedAdoOnARTCD4low}*(I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) + (\tau_{AdoHIVART}*L_{nottreatedAdoOnARTCD4low})) - \chi_{Ado}*I_{nottreatedAdoOnARTCD4low} + \sigma*I_{CTAdoOnARTCD4low} - \mu_{TBARTlowAdo}*I_{nottreatedAdoOnARTCD4low} - \varepsilon*I_{nottreatedAdoOnARTCD4low} - \varphi*I_{nottreatedAdoOnARTCD4low}$

$dI_{nottreatedAdoOnARTCD4high} = \theta_{HIV}*I_{nottreatedAdoOnARTCD4low} + \beta_{TBado}*(1-\rho_{TBinfHIVART})*S_{AdoOnARTCD4high}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + (a*\beta_{TBado}*\rho_{TBinfHIVART}*L_{nottreatedAdoOnARTCD4high}*(I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} +$

$I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) + (\tau_{AdoHIVART}*L_{nottreatedAdoOnARTCD4high})) - \chi_{Ado}*I_{nottreatedAdoOnARTCD4high} + \sigma*I_{CTAdoOnARTCD4high} - \mu_{TBARThighAdo}* I_{nottreatedAdoOnARTCD4high} - \varepsilon*I_{nottreatedAdoOnARTCD4high} - \varphi*I_{nottreatedAdoOnARTCD4high}$

$d_{ICTAdoHIVn} = -I_{CTAdoHIVn}*\beta_{HIVinfado}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) + \chi_{Ado}*I_{nottreatedAdoHIVn} - \gamma_{CT}*I_{CTAdoHIVn} - \psi*I_{CTAdoHIVn} - \sigma*I_{CTAdoHIVn} + \phi*I_{IPTAdoHIVn} - \mu_{TBAdo}*I_{CTAdoHIVn} - \varepsilon *I_{CTAdoHIVn}$

$dI_{CTAdoPreART} = I_{CTAdoHIVn}*\beta_{HIVInfAdo}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) - \zeta_{HIV}*I_{CTAdoPreART} + \chi_{Ado}*I_{nottreatedAdoPreART} - \gamma_{CT}*I_{CTAdoPreART} - \sigma*I_{CTAdoPreART} - \psi*I_{CTAdoPreART} + \phi*I_{IPTAdoPreART} - \mu_{TBPreARTAdo}*I_{CTAdoPreART} - \varepsilon*I_{CTAdoPreART}$

$dI_{CTAdoARTelig} = \zeta_{HIV}*I_{CTAdoPreART} - \eta_{HIV}*I_{CTAdoARTelig} + \chi_{Ado}*I_{nottreatedAdoARTelig} - \gamma_{CT}*I_{CTAdoARTelig} - \psi*I_{CTAdoARTelig} + \phi*I_{IPTAdoARTelig} - \mu_{TBARTeligAdo}*I_{CTAdoARTelig} - \varepsilon*I_{CTAdoARTelig}$

$dI_{CTAdoOnARTCD4low} = \eta_{HIV}*I_{CTAdoARTelig} - \theta_{HIV}*I_{CTAdoOnARTCD4low} + \chi_{Ado}*I_{nottreatedAdoOnARTCD4low} - \gamma_{CT}*I_{CTAdoOnARTCD4low} - \psi*I_{CTAdoOnARTCD4low} + \phi*I_{IPTAdoOnARTCD4low} - \sigma*I_{CTAdoARTelig} - \mu_{TBARTlowAdo}*I_{CTAdoOnARTCD4low} - \varepsilon *I_{CTAdoOnARTCD4low}$

$dI_{CTAdoOnARTCD4high} = \theta_{HIV}*I_{CTAdoOnARTCD4low} + \chi_{Ado}*I_{nottreatedAdoOnARTCD4high} - \gamma_{CT}*I_{CTAdoOnARTCD4high} - \psi*I_{CTAdoOnARTCD4high} + \phi*I_{IPTAdoOnARTCD4high} - \sigma*I_{CTAdoOnARTCD4high} - \mu_{TBARThighAdo}*I_{CTAdoOnARTCD4high} - \varepsilon *I_{CTAdoOnARTCD4high}$

$dI_{FNAdoHIVn}$ = -$I_{FNAdoHIVn}$* $\beta_{HIVInfAdo}$*(($S_{AdoPreART}$ + $S_{AdoARTelig}$ + $L_{nottreatedAdoPreART}$ + $L_{nottreatedAdoARTelig}$ + $L_{IPTAdoPreART}$ + $L_{IPTAdoARTelig}$ + $I_{InottreatedAdoPreART}$ + $I_{nottreatedAdoARTelig}$ + $I_{CTAdoPreART}$ + $I_{CTAdoARTelig}$ + $S_{AdlPreART}$ + $S_{AdlARTelig}$ + $L_{nottreatedAdlPreART}$ + $L_{nottreatedAdlARTelig}$ + $L_{IPTAdlPreART}$ + $L_{IPTAdlARTelig}$ + $I_{nottreatedAdlPreART}$ + $I_{nottreatedAdlARTelig}$ + $I_{CTAdlPreART}$ + $I_{CTAdlARTelig}$ + $I_{FNAdoPreART}$ + $I_{FNAdoARTelig}$ + $I_{IPTAdoPreART}$ + $I_{IPTAdoARTelig}$ + $I_{FNAdlPreART}$ + $I_{FNAdlARTelig}$ + $I_{IPTAdlPreART}$ + $I_{IPTAdlARTelig}$ + $R_{AdoPreART}$ + $R_{AdoARTelig}$)) +$L_{nottreatedAdoHIVn}$*ϑ_{HIVn} −$I_{FNAdoHIVn}$*ξ_{Ado} −ε *$I_{FNAdoHIVn}$ − μ_{TBAdo}*$I_{FNAdoHIVn}$

$dI_{FNAdoPreART}$ = $I_{FNAdoHIVn}$*$\beta_{HIVinfado}$*(($S_{AdoPreART}$ + $S_{AdoARTelig}$ + $L_{nottreatedAdoPreART}$ + $L_{nottreatedAdoARTelig}$ + $L_{IPTAdoPreART}$ + $L_{IPTAdoARTelig}$ + $I_{InottreatedAdoPreART}$ + $I_{nottreatedAdoARTelig}$ + $I_{CTAdoPreART}$ + $I_{CTAdoARTelig}$ + $S_{AdlPreART}$ + $S_{AdlARTelig}$ + $L_{nottreatedAdlPreART}$ + $L_{nottreatedAdlARTelig}$ + $L_{IPTAdlPreART}$ + $L_{IPTAdlARTelig}$ + $I_{nottreatedAdlPreART}$ + $I_{nottreatedAdlARTelig}$ + $I_{CTAdlPreART}$ + $I_{CTAdlARTelig}$ + $I_{FNAdoPreART}$ + $I_{FNAdoARTelig}$ + $I_{IPTAdoPreART}$ + $I_{IPTAdoARTelig}$ + $I_{FNAdlPreART}$ + $I_{FNAdlARTelig}$ + $I_{IPTAdlPreART}$ + $I_{IPTAdlARTelig}$ + $R_{AdoPreART}$ + $R_{AdoARTelig}$)) − ζ_{HIV}* $I_{FNAdoPreART}$ + $L_{nottreatedAdoPreART}$*ϑ_{HIVp} − $I_{FNAdoPreART}$*ξ_{Ado} -ε*$I_{FNAdoPreART}$ − $\mu_{TBPreARTAdo}$*$I_{FNAdoPreART}$

$dI_{FNAdoARTelig}$ = $L_{nottreatedAdoARTelig}$*ϑ_{HIVp} −$I_{FNAdoARTelig}$*ξ_{Ado} + ζ_{HIV}*$I_{FNAdoPreART}$ −θ_{HIV}*$I_{FNAdoARTelig}$ -ε *$I_{FNAdoARTelig}$ − $\mu_{TBARTeligAdo}$*$I_{FNAdoARTelig}$

$dI_{FNAdoOnARTCD4low}$ = $L_{nottreatedAdoOnARTCD4low}$*$\vartheta_{ART}$ − $I_{FNAdoOnARTCD4low}$*ξ_{Ado} + η_{HIV}*$I_{FNAdoARTelig}$ − θ_{HIV}*$I_{FNAdoOnARTCD4low}$ - ε*$I_{FNAdoOnARTCD4low}$ − $\mu_{TBARTlowAdo}$* $I_{FNAdoOnARTCD4low}$

$d_{IFNAdoOnARTCD4high}$ = $L_{nottreatedAdoOnARTCD4high}$*$\vartheta_{ART}$ −$I_{FNAdoOnARTCD4high}$*ξ_{Ado} + θ_{HIV}*$I_{FNAdoOnARTCD4low}$ -ε*$I_{FNAdoOnARTCD4high}$ − $\mu_{TBARThighAdo}$*$I_{FNAdoOnARTCD4high}$

$dI_{IPTAdoHIVn}$ = -$I_{IPTAdoHIVn}$*$\beta_{HIVInfAdo}$*(($S_{AdoPreART}$ + $S_{AdoARTelig}$ + $L_{nottreatedAdoPreART}$ + $L_{nottreatedAdoARTelig}$ + $L_{IPTAdoPreART}$ + $L_{IPTAdoARTelig}$ + $I_{InottreatedAdoPreART}$ + $I_{nottreatedAdoARTelig}$ + $I_{CTAdoPreART}$ + $I_{CTAdoARTelig}$ + $S_{AdlPreART}$ + $S_{AdlARTelig}$ + $L_{nottreatedAdlPreART}$ + $L_{nottreatedAdlARTelig}$ + $L_{IPTAdlPreART}$ + $L_{IPTAdlARTelig}$ + $I_{nottreatedAdlPreART}$ + $I_{nottreatedAdlARTelig}$ + $I_{CTAdlPreART}$ + $I_{CTAdlARTelig}$ + $I_{FNAdoPreART}$ + $I_{FNAdoARTelig}$ + $I_{IPTAdoPreART}$ + $I_{IPTAdoARTelig}$ + $I_{FNAdlPreART}$ + $I_{FNAdlARTelig}$ + $I_{IPTAdlPreART}$ + $I_{IPTAdlARTelig}$ + $R_{AdoPreART}$ + $R_{AdoARTelig}$)) + $I_{FNAdoHIVn}$*ξ_{Ado} + ((λ_{HIVn}*$L_{IPTAdoHIVn}$)) - ϕ*$I_{IPTAdoHIVn}$ -ε *$I_{IPTAdoHIVn}$ − μ_{TBAdo}*$I_{IPTAdoHIVn}$

$$dI_{IPTAdoPreART} = I_{IPTAdoHIVn}*\beta_{HIVInfAdo}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) - \zeta_{HIV}*I_{IPTAdoPreART} + I_{FNAdoPreART}*\xi_{Ado} + ((\lambda_{HIVp}*L_{IPTAdoPreART})) - \phi*I_{IPTAdoPreART} - \varepsilon * I_{IPTAdoPreART} - \mu_{TBPreARTAdo}*I_{IPTAdoPreART}$$

$$dI_{IPTAdoARTelig} = \zeta_{HIV}*I_{IPTAdoPreART} - \eta_{HIV}*I_{IPTAdoARTelig} + I_{FNAdoARTelig}*\xi_{Ado} + ((\lambda_{HIVp}*L_{IPTAdoARTelig})) - \phi*I_{IPTAdoARTelig} - \varepsilon*I_{IPTAdoARTelig} - \mu_{TBARTeligAdo}*I_{IPTAdoARTelig}$$

$$dI_{IPTAdoOnARTCD4low} = \eta_{HIV}*I_{IPTAdoARTelig} - \theta_{HIV}* I_{IPTAdoOnARTCD4low} + I_{FNAdoOnARTCD4low}*\xi_{Ado} + ((\lambda_{HIVp}*L_{IPTAdoOnARTCD4low})) - \phi*I_{IPTAdoOnARTCD4low} - \varepsilon*I_{IPTAdoOnARTCD4low} - \mu_{TBARTlowAdo}*I_{IPTAdoOnARTCD4low}$$

$$dI_{IPTAdoOnARTCD4high} = \theta_{HIV}*I_{IPTAdoOnARTCD4low} + I_{FNAdoOnARTCD4high}*\xi_{Ado} + ((\lambda_{HIVp}*L_{IPTAdoOnARTCD4high})) - \phi*I_{IPTAdoOnARTCD4high} - \varepsilon*I_{IPTAdoOnARTCD4high} - \mu_{TBARThighAdo}*I_{IPTAdoOnARTCD4high}$$

Adults

$$dS_{AdlHIVn} = \varepsilon*S_{AdoHIVn} - S_{AdlHIVn}*\beta_{HIVInfAdl}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) -$$
$$\beta_{TBadl}*S_{AdlHIVn}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + \gamma_{CT}*I_{CTAdlHIVn} + \kappa_{IPT}*R_{AdlHIVn} - \mu_{nat}*S_{AdlHIVn}$$

$dS_{AdlPreART} = \varepsilon * S_{AdoPreART} + S_{AdlHIVn} * \beta_{HIVInfAdl} * ((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) - progHIV * S_{AdlPreART} - \beta_{TBAdl} * S_{AdlPreART} * ((I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{nottreatedAdoOnARTCD4low} + I_{nottreatedAdoOnARTCD4high} + I_{nottreatedAdlHIVn} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{nottreatedAdlOnARTCD4low} + I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + \gamma_{CT} * I_{CTAdlPreART} + \kappa_{IPT} * R_{AdlPreART} - \mu_{PreARTAdl} * S_{AdlPreART}$

$dS_{AdlARTelig} = \varepsilon * S_{AdoARTelig} + \zeta_{HIV} * S_{AdlPreART} - \eta_{HIV} * S_{AdlARTelig} - \beta_{TBAdl} * S_{AdlARTelig} * ((I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{nottreatedAdoOnARTCD4low} + I_{nottreatedAdoOnARTCD4high} + I_{nottreatedAdlHIVn} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{nottreatedAdlOnARTCD4low} + I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + \gamma_{CT} * I_{CTAdlARTelig} + \kappa_{IPT} * R_{AdlARTelig} - \mu_{ARTeligAdl} * S_{AdlARTelig}$

$dS_{AdlOnARTCD4low} = \varepsilon * S_{AdoOnARTCD4low} + \eta_{HIV} * S_{AdlARTelig} - \theta_{HIV} * S_{AdlOnARTCD4low} - \beta_{TBAdl} * S_{AdlOnARTCD4low} * ((I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{nottreatedAdoOnARTCD4low} + I_{nottreatedAdoOnARTCD4high} + I_{nottreatedAdlHIVn} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{nottreatedAdlOnARTCD4low} + I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + \Upsilon_{CT} * I_{CTAdlOnARTCD4low} + \kappa_{IPT} * R_{AdlOnARTCD4low} - \mu_{ARTlowAdl} * S_{AdlOnARTCD4low}$

$dS_{AdlOnARTCD4high} = \varepsilon * S_{AdoOnARTCD4high} + \theta_{HIV} * S_{AdlOnARTCD4low} - \beta_{TBAdl} * S_{AdlOnARTCD4high} * ((I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{nottreatedAdoOnARTCD4low} + I_{nottreatedAdoOnARTCD4high} + I_{nottreatedAdlHIVn} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{nottreatedAdlOnARTCD4low}$

$+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + \gamma_{CT}*I_{CTAdlOnARTCD4high} + \kappa_{IPT}*R_{AdlOnARTCD4high} - \mu_{ARThighAdl}*S_{AdlOnARTCD4high}$

$dR_{AdlHIVn} = -R_{AdlHIVn}*\beta_{HIVInfAdl}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) -$
$\beta_{TBadl}*R_{AdlHIVn}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + \gamma_{IPT}*L_{IPTAdlHIVn} - \kappa_{IPT}*R_{AdlHIVn} + \varepsilon*R_{AdoHIVn} - \mu_{nat}*R_{AdlHIVn}$

$dR_{AdlPreART} = R_{AdlHIVn}*\beta_{HIVinfadl}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) -$
$\beta_{TBadl}*R_{AdlPreART}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) - \zeta_{HIV}*R_{AdlPreART} + \gamma_{IPT}*L_{IPTAdlPreART} - \kappa_{IPT}*R_{AdlPreART} + \varepsilon*R_{AdoPreART} - \mu_{PreARTAdl}*R_{AdlPreART}$

$dR_{AdlARTelig} = \zeta_{HIV}*R_{AdlPreART} - \eta_{HIV}*R_{AdlARTelig} + \gamma_{IPT}*L_{IPTAdlARTelig} - \kappa_{IPT}*R_{AdlARTelig} + \varepsilon*R_{AdoARTelig} - \mu_{ARTeligAdl}*R_{AdlARTelig}$

$dR_{AdlOnARTCD4low} = \eta_{HIV}*R_{AdlARTelig} - \theta_{HIV}*R_{AdlOnARTCD4low} + \gamma_{IPT}*L_{IPTAdlOnARTCD4low} - \kappa_{IPT}*R_{AdlOnARTCD4low} + agingtwo*R_{AdoOnARTCD4low} - \eta_{ARTlowAdl}*R_{AdlOnARTCD4low}$

$dR_{AdlOnARTCD4high} = \theta_{HIV}*R_{AdlOnARTCD4low} + \gamma_{IPT}*L_{IPTAdlOnARTCD4high} - \kappa_{IPT}*R_{AdlOnARTCD4high} + \varepsilon*R_{AdoOnARTCD4high} - \mu_{ARThighAdl}*R_{AdlOnARTCD4high}$

$dL_{nottreatedAdlHIVn} = \varepsilon*L_{nottreatedAdoHIVn} - L_{nottreatedAdlHIVn}*\beta_{HIVinfadl}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig}))$
$+ \beta_{TBadl}*\rho_{TBinfHIVn}*S_{AdlHIVn}*((I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{nottreatedAdoOnARTCD4low} + I_{nottreatedAdoOnARTCD4high} + I_{nottreatedAdlHIVn} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{nottreatedAdlOnARTCD4low} + I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) - \xi_{AdlHIVn}*L_{nottreatedAdlHIVn} - (a*(1-\rho_{TBinfHIVn})*\beta_{TBadl}*L_{nottreatedAdlHIVn}*(I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{nottreatedAdoOnARTCD4low} + I_{nottreatedAdoOnARTCD4high} + I_{nottreatedAdlHIVn} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{nottreatedAdlOnARTCD4low} + I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) + (\tau_{AdlHIVn}*L_{nottreatedAdlHIVn})) + \psi*I_{CTAdlHIVn} - \mu_{nat}*L_{nottreatedAdlHIVn} + \varphi*I_{nottreatedAdlHIVn} + u_{IPT}*L_{IPTAdlHIVn} - L_{nottreatedAdlHIVn}*\vartheta_{HIVn}$

$dL_{nottreatedAdlPreART} = \varepsilon*L_{nottreatedAdoPreART} + L_{nottreatedAdlHIVn}*\beta_{HIVinfadl}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) - \zeta_{HIV}*L_{nottreatedAdlPreART} + \beta_{TBadl}*\rho_{TBinfHIVp}*S_{AdlPreART}*((I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdo}$

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ARTelig+$I_{nottreatedAdoOnARTCD4low}$+$I_{nottreatedAdoOnARTCD4high}$+$I_{nottreatedAdlHIVn}$+$I_{nottreatedAdlPreART}$+$I_{nottreatedAdlARTelig}$+$I_{nottreatedAdlOnARTCD4low}$+$I_{nottreatedAdlOnARTCD4high}$ + $I_{FNAdoHIVn}$ + $I_{FNAdoPreART}$ + $I_{FNAdoARTelig}$ + $I_{FNAdoOnARTCD4low}$ + $I_{FNAdoOnARTCD4high}$ + $I_{IPTAdoHIVn}$ + $I_{IPTAdoPreART}$ + $I_{IPTAdoARTelig}$ + $I_{IPTAdoOnARTCD4low}$ + $I_{IPTAdoOnARTCD4high}$ + $I_{FNAdlHIVn}$ + $I_{FNAdlPreART}$ + $I_{FNAdlARTelig}$ + $I_{FNAdlOnARTCD4low}$ + $I_{FNAdlOnARTCD4high}$ + $I_{IPTAdlHIVn}$ + $I_{IPTAdlPreART}$ + $I_{IPTAdlARTelig}$ + $I_{IPTAdlOnARTCD4low}$ + $I_{IPTAdlOnARTCD4high}$)) - $\xi_{AdoHIVp}$*$L_{nottreatedAdlPreART}$ - (a*(1-$\rho_{TBinfHIVp}$)*β_{TBadl}*$L_{nottreatedAdlPreART}$*($I_{nottreatedAdoHIVn}$+$I_{nottreatedAdoPreART}$+$I_{nottreatedAdoARTelig}$+$I_{nottreatedAdoOnARTCD4low}$+$I_{nottreatedAdoOnARTCD4high}$+$I_{nottreatedAdlHIVn}$+$I_{nottreatedAdlPreART}$+$I_{nottreatedAdlARTelig}$+$I_{nottreatedAdlOnARTCD4low}$+$I_{nottreatedAdlOnARTCD4high}$ + $I_{FNAdoHIVn}$ + $I_{FNAdoPreART}$ + $I_{FNAdoARTelig}$ + $I_{FNAdoOnARTCD4low}$ + $I_{FNAdoOnARTCD4high}$ + $I_{IPTAdoHIVn}$ + $I_{IPTAdoPreART}$ + $I_{IPTAdoARTelig}$ + $I_{IPTAdoOnARTCD4low}$ + $I_{IPTAdoOnARTCD4high}$ + $I_{FNAdlHIVn}$ + $I_{FNAdlPreART}$ + $I_{FNAdlARTelig}$ + $I_{FNAdlOnARTCD4low}$ + $I_{FNAdlOnARTCD4high}$ + $I_{IPTAdlHIVn}$ + $I_{IPTAdlPreART}$ + $I_{IPTAdlARTelig}$ + $I_{IPTAdlOnARTCD4low}$ + $I_{IPTAdlOnARTCD4high}$) + ($\tau_{AdlHIVp}$*$L_{nottreatedAdlPreART}$)) + ψ*$I_{CTAdlPreART}$-$\mu_{PreARTAdl}$*$L_{nottreatedAdlPreART}$+ φ*$I_{nottreatedAdlPreART}$ + ι_{IPT}*$L_{IPTAdlPreART}$ − $L_{nottreatedAdlPreART}$*ϑ_{HIVp}

$dL_{nottreatedAdlARTelig}$ = ε*$L_{nottreatedAdoARTelig}$+ζ_{HIV}*$L_{nottreatedAdlPreART}$ − η_{HIV}*$L_{nottreatedAdlARTelig}$ +β_{TBadl}*$\rho_{TBinfHIVp}$*$S_{AdlARTelig}$*(($I_{nottreatedAdoHIVn}$+$I_{nottreatedAdoPreART}$+$I_{nottreatedAdoARTelig}$+$I_{nottreatedAdoOnARTCD4low}$+$I_{nottreatedAdoOnARTCD4high}$+$I_{nottreatedAdlHIVn}$+$I_{nottreatedAdlPreART}$+$I_{nottreatedAdlARTelig}$+$I_{nottreatedAdlOnARTCD4low}$+$I_{nottreatedAdlOnARTCD4high}$ + $I_{FNAdoHIVn}$ + $I_{FNAdoPreART}$ + $I_{FNAdoARTelig}$ + $I_{FNAdoOnARTCD4low}$ + $I_{FNAdoOnARTCD4high}$ + $I_{IPTAdoHIVn}$ + $I_{IPTAdoPreART}$ + $I_{IPTAdoARTelig}$ + $I_{IPTAdoOnARTCD4low}$ + $I_{IPTAdoOnARTCD4high}$ + $I_{FNAdlHIVn}$ + $I_{FNAdlPreART}$ + $I_{FNAdlARTelig}$ + $I_{FNAdlOnARTCD4low}$ + $I_{FNAdlOnARTCD4high}$ + $I_{IPTAdlHIVn}$ + $I_{IPTAdlPreART}$ + $I_{IPTAdlARTelig}$ + $I_{IPTAdlOnARTCD4low}$ + $I_{IPTAdlOnARTCD4high}$)) - ξ_{Adl}*$L_{nottreatedAdlARTelig}$ -(a*(1-$\rho_{TBinfHIVp}$)*β_{TBadl}*$L_{nottreatedAdlARTelig}$*($I_{nottreatedAdoHIVn}$+$I_{nottreatedAdoPreART}$+$I_{nottreatedAdoARTelig}$+$I_{nottreatedAdoOnARTCD4low}$+$I_{nottreatedAdoOnARTCD4high}$+$I_{nottreatedAdlHIVn}$+$I_{nottreatedAdlPreART}$+$I_{nottreatedAdlARTelig}$+$I_{nottreatedAdlOnARTCD4low}$+$I_{nottreatedAdlOnARTCD4high}$ + $I_{FNAdoHIVn}$ + $I_{FNAdoPreART}$ + $I_{FNAdoARTelig}$ + $I_{FNAdoOnARTCD4low}$ + $I_{FNAdoOnARTCD4high}$ + $I_{IPTAdoHIVn}$ + $I_{IPTAdoPreART}$ + $I_{IPTAdoARTelig}$ + $I_{IPTAdoOnARTCD4low}$ + $I_{IPTAdoOnARTCD4high}$ + $I_{FNAdlHIVn}$ + $I_{FNAdlPreART}$ + $I_{FNAdlARTelig}$ + $I_{FNAdlOnARTCD4low}$ + $I_{FNAdlOnARTCD4high}$ + $I_{IPTAdlHIVn}$ + $I_{IPTAdlPreART}$ + $I_{IPTAdlARTelig}$ + $I_{IPTAdlOnARTCD4low}$ + $I_{IPTAdlOnARTCD4high}$) + ($\tau_{AdlHIVp}$*$L_{nottreatedAdlARTelig}$)) + ψ*$I_{CTAdlARTelig}$- $\mu_{ARTeligAdl}$*$L_{nottreatedAdlARTelig}$+ φ*$I_{nottreatedAdlARTelig}$ + ι_{IPT}*$L_{IPTAdlARTelig}$ − $L_{nottreatedAdlARTelig}$*ζ_{HIVp}

$dL_{nottreatedAdlOnARTCD4low}$ = ε*$L_{nottreatedAdoOnARTCD4low}$+$\eta_{HIV}$*$L_{nottreatedAdlARTelig}$ − θ_{HIV}*$L_{nottreatedAdoOnARTCD4low}$ +β_{TBadl}*$\rho_{infHIVART}$*$S_{AdlOnARTCD4low}$*(($I_{nottreatedAdoHIVn}$+$I_{nottreatedAdoPreART}$+$I_{nottreatedAdoARTelig}$+$I_{nottreatedAdoOnARTCD4low}$+$I_{nottreatedAdoOnARTCD4high}$+$I_{nottreatedAdlHIVn}$+$I_{nottreatedAdlPreART}$+$I_{nottreatedAdlARTelig}$+$I_{nottreatedAdlOnARTCD4low}$+$I_{nottreatedAdlOnARTCD4high}$ + $I_{FNAdoHIVn}$ + $I_{FNAdoPreART}$ + $I_{FNAdoARTelig}$ + $I_{FNAdoOnARTCD4low}$ + $I_{FNAdoOnARTCD4high}$ + $I_{IPTAdoHIVn}$ + $I_{IPTAdoPreART}$ + $I_{IPTAdoARTelig}$ + $I_{IPTAdoOnARTCD4low}$ +

$I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) - \xi_{Adl}*L_{nottreatedAdlOnARTCD4low} -(a*(1-\rho_{TBinfHIVART})*\beta_{TBadl}*L_{nottreatedAdlOnARTCD4low}*(I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) + (\tau_{AdlHIVART}*L_{nottreatedAdlOnARTCD4low})) + \psi*I_{CTAdlARTelig}-\mu_{ARTlowAdl}*L_{nottreatedAdlOnARTCD4low}+ \varphi*I_{nottreatedAdlOnARTCD4low} + \upsilon_{IPT}*L_{IPTAdlOnARTCD4low} - L_{nottreatedAdlOnARTCD4low}*\vartheta_{ART}$

$dL_{nottreatedAdlOnARTCD4high} = \varepsilon*L_{nottreatedAdoOnARTCD4high}+ \theta_{HIV}*L_{nottreatedAdoOnARTCD4low} + \beta_{TBadl}*\rho_{TBinfHIVART}*S_{AdlOnARTCD4high}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) - \xi_{Adl}*L_{nottreatedAdlOnARTCD4high} -(a*(1-\rho_{TBinfHIVART})*\beta_{TBadl}*L_{nottreatedAdlOnARTCD4high}*(I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) + (\tau_{AdlHIVART}*L_{nottreatedAdlOnARTCD4high})) + \psi*I_{CTAdlARTelig}-\mu_{ARThighAdl}*L_{nottreatedAdlOnARTCD4high}+ \varphi*I_{nottreatedAdlOnARTCD4high} + \upsilon_{IPT}*L_{IPTAdlOnARTCD4high} - L_{nottreatedAdlOnARTCD4high}*\vartheta_{ART}$

$dL_{IPTAdlHIVn} = \varepsilon*L_{IPTAdoHIVn} - L_{IPTAdlHIVn}*\beta_{HIVinfadl}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) +\xi_{Adl}*L_{nottreatedAdlHIVn} -((\lambda_{HIVn}*L_{IPTAdlHIVn})) - \gamma_{IPT}*L_{IPTAdlHIVn}- \mu_{nat}*L_{IPTAdlHIVn} - \upsilon_{IPT}*L_{IPTAdlHIVn}$

$dL_{IPTAdlPreART} = \epsilon*L_{IPTAdoPreART} + L_{IPTAdlHIVn}*\beta_{HIVInfAdl}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) -\zeta_{HIV}*L_{IPTAdlPreART} + \xi_{Adl}*L_{nottreatedAdlPreART} -((\lambda_{HIVp}*L_{IPTAdlPreART})) - \gamma_{IPT}*L_{IPTAdlPreART} - \mu_{PreARTAdl}*L_{IPTAdlPreART} - \iota_{IPT}*L_{IPTAdlPreART}$

$dL_{IPTAdlARTelig} = \epsilon*L_{IPTAdoARTelig} + \zeta_{HIV}*L_{IPTAdlPreART} - \eta_{HIV}*L_{IPTAdlARTelig} + \xi_{Adl}*L_{nottreatedAdlARTelig} - ((activIPTHIVp*L_{IPTAdlARTelig})) - \gamma_{IPT}*L_{IPTAdlARTelig} - \mu_{ARTeligAdl}*L_{IPTAdlARTelig} - \iota_{IPT}*L_{IPTAdlARTelig}$

$dL_{IPTAdlOnARTCD4low} = \epsilon*L_{IPTAdoOnARTCD4low} + \eta_{HIV}*L_{IPTAdlARTelig} - \theta_{HIV}*L_{IPTAdlOnARTCD4low} + \xi_{Adl}*L_{nottreatedAdlOnARTCD4low} - \lambda_{HIVp}*L_{IPTAdlOnARTCD4low} - \gamma_{IPT}*L_{IPTAdlOnARTCD4low} - \mu_{ARTlowAdl}*L_{IPTAdlOnARTCD4low} - \iota_{IPT}*L_{IPTAdlOnARTCD4low}$

$dL_{IPTAdlOnARTCD4high} = \epsilon*L_{IPTAdoOnARTCD4high} + \theta_{HIV}*L_{nottreatedAdlOnARTCD4low} + \xi_{Adl}*L_{nottreatedAdlOnARTCD4high} - \lambda_{HIVp}*L_{IPTAdlOnARTCD4high} - \gamma_{IPT}*L_{IPTAdlOnARTCD4high} - \mu_{ARThighAdl}*L_{IPTAdlOnARTCD4high} - \iota_{IPT}*L_{IPTAdlOnARTCD4high} - \kappa_{IPT}*L_{IPTAdlOnARTCD4high}$

$dI_{nottreatedAdlHIVn} = \epsilon*I_{nottreatedAdoHIVn} - I_{InottreatedAdlHIVn}*\beta_{HIVInfAdl}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) + \beta_{TBadl}*(1-\rho_{TBinfHIVn})*S_{AdlHIVn}*((I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{nottreatedAdoOnARTCD4low} + I_{nottreatedAdoOnARTCD4high} + I_{nottreatedAdlHIVn} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{nottreatedAdlOnARTCD4low} + I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) + (a*(1-\rho_{TBinfHIVn})*\beta_{TBadl}*L_{nottreatedAdlHIVn}*(I_{nottreatedAdoHIVn} + I_{nottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{nottreatedAdoOnARTCD4low} + I_{nottreatedAdoOnARTCD4high} + I_{nottreatedAdlHIVn} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{nottreatedAdlOn}$

$\text{ARTCD4low} + I_{\text{nottreatedAdlOnARTCD4high}} + I_{\text{FNAdoHIVn}} + I_{\text{FNAdoPreART}} + I_{\text{FNAdoARTelig}} + I_{\text{FNAdoOnARTCD4low}} + I_{\text{FNAdoOnARTCD4high}} + I_{\text{IPTAdoHIVn}} + I_{\text{IPTAdoPreART}} + I_{\text{IPTAdoARTelig}} + I_{\text{IPTAdoOnARTCD4low}} + I_{\text{IPTAdoOnARTCD4high}} + I_{\text{FNAdlHIVn}} + I_{\text{FNAdlPreART}} + I_{\text{FNAdlARTelig}} + I_{\text{FNAdlOnARTCD4low}} + I_{\text{FNAdlOnARTCD4high}} + I_{\text{IPTAdlHIVn}} + I_{\text{IPTAdlPreART}} + I_{\text{IPTAdlARTelig}} + I_{\text{IPTAdlOnARTCD4low}} + I_{\text{IPTAdlOnARTCD4high}}) + (\tau_{\text{AdlHIVn}} * L_{\text{nottreatedAdlHIVn}})) - \chi_{\text{Adl}} * I_{\text{nottreatedAdlHIVn}} + \sigma * I_{\text{CTAdlHIVn}} - \mu_{\text{TBAdl}} * I_{\text{nottreatedAdlHIVn}} - \varphi * I_{\text{nottreatedAdlHIVn}}$

$dI_{\text{nottreatedAdlPreART}} = \varepsilon * I_{\text{nottreatedAdoPreART}} + I_{\text{nottreatedAdlHIVn}} * \beta_{\text{HIVInfAdl}} * ((S_{\text{AdoPreART}} + S_{\text{AdoARTelig}} + L_{\text{nottreatedAdoPreART}} + L_{\text{nottreatedAdoARTelig}} + L_{\text{IPTAdoPreART}} + L_{\text{IPTAdoARTelig}} + I_{\text{InottreatedAdoPreART}} + I_{\text{nottreatedAdoARTelig}} + I_{\text{CTAdoPreART}} + I_{\text{CTAdoARTelig}} + S_{\text{AdlPreART}} + S_{\text{AdlARTelig}} + L_{\text{nottreatedAdlPreART}} + L_{\text{nottreatedAdlARTelig}} + L_{\text{IPTAdlPreART}} + L_{\text{IPTAdlARTelig}} + I_{\text{nottreatedAdlPreART}} + I_{\text{nottreatedAdlARTelig}} + I_{\text{CTAdlPreART}} + I_{\text{CTAdlARTelig}} + I_{\text{FNAdoPreART}} + I_{\text{FNAdoARTelig}} + I_{\text{IPTAdoPreART}} + I_{\text{IPTAdoARTelig}} + I_{\text{FNAdlPreART}} + I_{\text{FNAdlARTelig}} + I_{\text{IPTAdlPreART}} + I_{\text{IPTAdlARTelig}} + R_{\text{AdoPreART}} + R_{\text{AdoARTelig}})) - \zeta_{\text{HIV}} * I_{\text{nottreatedAdlPreART}} + \beta_{\text{TBadl}} * (1-$

$\rho_{\text{TBinfHIVp}}) * S_{\text{AdlPreART}} * ((I_{\text{nottreatedAdoHIVn}} + I_{\text{nottreatedAdoPreART}} + I_{\text{nottreatedAdoARTelig}} + I_{\text{nottreatedAdoOnARTCD4low}} + I_{\text{nottreatedAdoOnARTCD4high}} + I_{\text{nottreatedAdlHIVn}} + I_{\text{nottreatedAdlPreART}} + I_{\text{nottreatedAdlARTelig}} + I_{\text{nottreatedAdlOnARTCD4low}} + I_{\text{nottreatedAdlOnARTCD4high}} + I_{\text{FNAdoHIVn}} + I_{\text{FNAdoPreART}} + I_{\text{FNAdoARTelig}} + I_{\text{FNAdoOnARTCD4low}} + I_{\text{FNAdoOnARTCD4high}} + I_{\text{IPTAdoHIVn}} + I_{\text{IPTAdoPreART}} + I_{\text{IPTAdoARTelig}} + I_{\text{IPTAdoOnARTCD4low}} + I_{\text{IPTAdoOnARTCD4high}} + I_{\text{FNAdlHIVn}} + I_{\text{FNAdlPreART}} + I_{\text{FNAdlARTelig}} + I_{\text{FNAdlOnARTCD4low}} + I_{\text{FNAdlOnARTCD4high}} + I_{\text{IPTAdlHIVn}} + I_{\text{IPTAdlPreART}} + I_{\text{IPTAdlARTelig}} + I_{\text{IPTAdlOnARTCD4low}} + I_{\text{IPTAdlOnARTCD4high}})) + (a * (1-$

$\rho_{\text{TBinfHIVp}}) * \beta_{\text{TBadl}} * L_{\text{nottreatedAdlPreART}} * (I_{\text{nottreatedAdoHIVn}} + I_{\text{nottreatedAdoPreART}} + I_{\text{nottreatedAdoARTelig}} + I_{\text{nottreatedAdoOnARTCD4low}} + I_{\text{nottreatedAdoOnARTCD4high}} + I_{\text{nottreatedAdlHIVn}} + I_{\text{nottreatedAdlPreART}} + I_{\text{nottreatedAdlARTelig}} + I_{\text{nottreatedAdlOnARTCD4low}} + I_{\text{nottreatedAdlOnARTCD4high}} + I_{\text{FNAdoHIVn}} + I_{\text{FNAdoPreART}} + I_{\text{FNAdoARTelig}} + I_{\text{FNAdoOnARTCD4low}} + I_{\text{FNAdoOnARTCD4high}} + I_{\text{IPTAdoHIVn}} + I_{\text{IPTAdoPreART}} + I_{\text{IPTAdoARTelig}} + I_{\text{IPTAdoOnARTCD4low}} + I_{\text{IPTAdoOnARTCD4high}} + I_{\text{FNAdlHIVn}} + I_{\text{FNAdlPreART}} + I_{\text{FNAdlARTelig}} + I_{\text{FNAdlOnARTCD4low}} + I_{\text{FNAdlOnARTCD4high}} + I_{\text{IPTAdlHIVn}} + I_{\text{IPTAdlPreART}} + I_{\text{IPTAdlARTelig}} + I_{\text{IPTAdlOnARTCD4low}} + I_{\text{IPTAdlOnARTCD4high}}) + (\lambda_{\text{HIVp}} * L_{\text{nottreatedAdlPreART}})) + \sigma * I_{\text{CTAdlPreART}} - \chi_{\text{Adl}} * I_{\text{nottreatedAdlPreART}} - \mu_{\text{TBPreARTAdl}} * I_{\text{nottreatedAdlPreART}} - \varphi * I_{\text{nottreatedAdlPreART}}$

$dI_{\text{nottreatedAdlARTelig}} = \varepsilon * I_{\text{nottreatedAdoARTelig}} + \zeta_{\text{HIV}} * I_{\text{nottreatedAdlPreART}} - \eta_{\text{HIV}} * I_{\text{nottreatedAdlARTelig}} + \beta_{\text{TBadl}} * (1-$

$\rho_{\text{TBinfHIVp}}) * S_{\text{AdlARTelig}} * ((I_{\text{nottreatedAdoHIVn}} + I_{\text{nottreatedAdoPreART}} + I_{\text{nottreatedAdoARTelig}} + I_{\text{nottreatedAdoOnARTCD4low}} + I_{\text{nottreatedAdoOnARTCD4high}} + I_{\text{nottreatedAdlHIVn}} + I_{\text{nottreatedAdlPreART}} + I_{\text{nottreatedAdlARTelig}} + I_{\text{nottreatedAdlOnARTCD4low}} + I_{\text{nottreatedAdlOnARTCD4high}} + I_{\text{FNAdoHIVn}} + I_{\text{FNAdoPreART}} + I_{\text{FNAdoARTelig}} + I_{\text{FNAdoOnARTCD4low}} + I_{\text{FNAdoOnARTCD4high}} + I_{\text{IPTAdoHIVn}} + I_{\text{IPTAdoPreART}} + I_{\text{IPTAdoARTelig}} + I_{\text{IPTAdoOnARTCD4low}} + I_{\text{IPTAdoOnARTCD4high}} + I_{\text{FNAdlHIVn}} + I_{\text{FNAdlPreART}} + I_{\text{FNAdlARTelig}} + I_{\text{FNAdlOnARTCD4low}} + I_{\text{FNAdlOnARTCD4high}} + I_{\text{IPTAdlHIVn}} + I_{\text{IPTAdlPreART}} + I_{\text{IPTAdlARTelig}} + I_{\text{IPTAdlOnARTCD4low}} + I_{\text{IPTAdlOnARTCD4high}})) + (a * (1-$

$\rho_{\text{TBinfHIVp}}) * \beta_{\text{TBadl}} * L_{\text{nottreatedAdlARTelig}} * (I_{\text{nottreatedAdoHIVn}} + I_{\text{nottreatedAdoPreART}} + I_{\text{nottreatedAdoARTelig}} + I_{\text{nottreatedAdoOnARTCD4low}} + I_{\text{nottreatedAdoOnARTCD4high}} + I_{\text{nottreatedAdlHIVn}} + I_{\text{nottreatedAdlPreART}} + I_{\text{nottreatedAdlARTelig}} + I_{\text{nottreatedAdl}}$

$OnARTCD4low+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) + (\lambda_{HIVp}*L_{nottreatedAdlARTelig})) - \chi_{Adl}*I_{nottreatedAdlARTelig} + \sigma*I_{CTAdlARTelig} - \mu_{TBARTeligAdl}*I_{nottreatedAdlARTelig} - \varphi*I_{nottreatedAdlARTelig}$

$dI_{nottreatedAdlOnARTCD4low} = \varepsilon*I_{nottreatedAdoOnARTCD4low}+\eta_{HIV}*I_{nottreatedAdlARTelig} - \theta_{HIV}*I_{nottreatedAdlOnARTCD4low}+\beta_{TBadl}*(1-\rho_{TBinfHIVART})*S_{AdoOnARTCD4low}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) +(a*(1-\rho_{TBinfHIVART})*\beta_{TBAdl}*L_{nottreatedAdlOnARTCD4low}*(I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) + (\tau_{ART}*L_{nottreatedAdlOnARTCD4low})) - \chi_{Adl}*I_{nottreatedAdlOnARTCD4low} + \sigma_{CT}*I_{CTAdlOnARTCD4low} - \mu_{TBARTlowAdl}*I_{nottreatedAdlOnARTCD4low} - \varphi*I_{nottreatedAdlOnARTCD4low}$

$dI_{nottreatedAdlOnARTCD4high} = \varepsilon*I_{nottreatedAdoOnARTCD4high}+\theta_{HIV}*I_{nottreatedAdlOnARTCD4low} +\beta_{TBadl}*(1-\rho_{TBinfHIVART})*S_{AdlOnARTCD4high}*((I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high})) +(a*(1-\rho_{TBinfHIVART})*\beta_{TBadl}*L_{nottreatedAdlOnARTCD4high}*(I_{nottreatedAdoHIVn}+I_{nottreatedAdoPreART}+I_{nottreatedAdoARTelig}+I_{nottreatedAdoOnARTCD4low}+I_{nottreatedAdoOnARTCD4high}+I_{nottreatedAdlHIVn}+I_{nottreatedAdlPreART}+I_{nottreatedAdlARTelig}+I_{nottreatedAdlOnARTCD4low}+I_{nottreatedAdlOnARTCD4high} + I_{FNAdoHIVn} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{FNAdoOnARTCD4low} + I_{FNAdoOnARTCD4high} + I_{IPTAdoHIVn} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{IPTAdoOnARTCD4low} + I_{IPTAdoOnARTCD4high} + I_{FNAdlHIVn} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{FNAdlOnARTCD4low} + I_{FNAdlOnARTCD4high} + I_{IPTAdlHIVn} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + I_{IPTAdlOnARTCD4low} + I_{IPTAdlOnARTCD4high}) + (\tau_{AdlHIVART}*L_{nottreatedAdlOnARTCD4high})) + \sigma*I_{CTAdlOnARTCD4high} -$

$\chi_{Adl}*I_{nottreatedAdlOnARTCD4high} - \mu_{TBARThighAdl}*I_{nottreatedAdlOnARTCD4high} - \varphi*I_{nottreatedAdlOnARTCD4high}$

$dI_{CTAdlHIVn} = \varepsilon*I_{CTAdoHIVn} - I_{CTAdlHIVn}*\beta_{HIVInfAdl}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) + \chi_{Adl}*I_{nottreatedAdlHIVn} - \sigma*I_{CTAdlHIVn} - \gamma_{CT}*I_{CTAdlHIVn} - \psi*I_{CTAdlHIVn} + \phi*I_{IPTAdlHIVn} - \mu_{TBAdl}*I_{CTAdlHIVn}$

$dI_{CTAdlPreART} = \varepsilon*I_{CTAdoPreART} + I_{CTAdlHIVn}*\beta_{HIVInfAdl}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) - \zeta_{HIV}*I_{CTAdlPreART} + \chi_{Adl}*I_{nottreatedAdlPreART} - \sigma*I_{CTAdlPreART} - \gamma_{CT}*I_{CTAdlPreART} - \psi*I_{CTAdlPreART} - \mu_{TBPreARTAdl}*I_{CTAdlPreART}$

$dI_{CTAdlARTelig} = \varepsilon*I_{CTAdoARTelig} + \zeta_{HIV}*I_{CTAdlPreART} - \eta_{HIV}*I_{CTAdlARTelig} + \chi_{Adl}*I_{nottreatedAdlARTelig} - \sigma*I_{CTAdlARTelig} - \gamma_{CT}*I_{CTAdlARTelig} - \psi*I_{CTAdlARTelig} + \phi*I_{IPTAdlPreART} - \mu_{TBARTeligAdl}*I_{CTAdlARTelig}$

$dI_{CTAdlOnARTCD4low} = \varepsilon*I_{CTAdoOnARTCD4low} + \eta_{HIV}*I_{CTAdlARTelig} - \theta_{HIV}*I_{CTAdlOnARTCD4low} + \chi_{Adl}*I_{nottreatedAdlOnARTCD4low} - noncureCT*I_{CTAdlOnARTCD4low} - \theta_{CT}*I_{CTAdlOnARTCD4low} - \psi*I_{CTAdlOnARTCD4low} + \phi*I_{IPTAdlOnARTCD4low} - \mu_{TBARTlowAdl}*I_{CTAdlOnARTCD4low}$

$dI_{CTAdlOnARTCD4high} = \varepsilon*I_{CTAdoOnARTCD4high} + \theta_{HIV}*I_{CTAdlOnARTCD4low} + \chi_{Adl}*I_{nottreatedAdlOnARTCD4high} - \sigma_{CT}*I_{CTAdlOnARTCD4high} - \gamma_{CT}*I_{CTAdlOnARTCD4high} - \psi*I_{CTAdlOnARTCD4high} + \phi*I_{IPTAdlOnARTCD4high} - \mu_{TBARThighAdl}*I_{CTAdlOnARTCD4high}$

$dI_{FNAdlHIVn} = -I_{FNAdlHIVn}*\beta_{HIVInfAdl}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} +$

$I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) + L_{nottreatedAdlHIVn}*\vartheta_{HIVn} -I_{FNAdlHIVn}*\xi_{Adl} + \varepsilon*I_{FNAdlHIVn} - \mu_{TBAdl}*I_{FNAdlHIVn}$

$dI_{FNAdlPreART} = I_{FNAdlHIVn}*\beta_{HIViInfAdl}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) - \zeta_{HIV}*I_{FNAdlPreART} + L_{nottreatedAdlPreART}*\vartheta_{HIVp} - I_{FNAdlPreART}*\xi_{Adl} + \varepsilon*I_{FNAdlPreART} - \mu_{TBPreARTAdl}*I_{FNAdlPreART}$

$dI_{FNAdlARTelig} = \zeta_{HIV}*I_{FNAdlPreART} - \eta_{HIV}*I_{FNAdlARTelig} + L_{LnottreatedAdlARTelig}*\vartheta_{HIVp} -I_{FNAdlARTelig}*\xi_{Adl} + \varepsilon*I_{FNAdlARTelig} - \mu_{TBARTeligAdl}*I_{FNAdlARTelig}$

$dI_{FNAdlOnARTCD4low} = \eta_{HIV}*I_{FNAdlARTelig} - \theta_{HIV}*I_{FNAdlOnARTCD4low} + L_{nottreatedAdlOnARTCD4low}*\vartheta_{ART} - I_{FNAdlOnARTCD4low}*\xi_{Adl} + \varepsilon*I_{FNAdlOnARTCD4low} - \mu_{TBARTlowAdl}*I_{FNAdlOnARTCD4low}$

$dI_{FNAdlOnARTCD4high} = \theta_{HIV}*I_{FNAdlOnARTCD4low} + L_{nottreatedAdlOnARTCD4high}*\vartheta_{ART} - I_{FNAdlOnARTCD4high}*IPTtoacivesAdl + \varepsilon*I_{FNAdlOnARTCD4high} - \mu_{TBARThighAdl}*I_{FNAdlOnARTCD4high}$

$dI_{IPTAdlHIVn} = -I_{IPTAdlHIVn}*\beta_{HIVInfAdl}*((S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} + I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig})) + I_{FNAdlHIVn}*\xi_{Adl} - \phi*I_{IPTAdlHIVn} + ((\lambda_{HIVn}*L_{IPTAdlHIVn})) + \varepsilon*I_{IPTAdlHIVn} - \mu_{TBAdl}*I_{IPTAdlHIVn}$

$dI_{IPTAdlPreART} = I_{IPTAdlHIVn}*\beta_{HIVInfAdl}*(S_{AdoPreART} + S_{AdoARTelig} + L_{nottreatedAdoPreART} + L_{nottreatedAdoARTelig} + L_{IPTAdoPreART} + L_{IPTAdoARTelig} + I_{InottreatedAdoPreART} + I_{nottreatedAdoARTelig} + I_{CTAdoPreART} + I_{CTAdoARTelig} + S_{AdlPreART} + S_{AdlARTelig} + L_{nottreatedAdlPreART} + L_{nottreatedAdlARTelig} + L_{IPTAdlPreART} + L_{IPTAdlARTelig} + I_{nottreatedAdlPreART} + I_{nottreatedAdlARTelig} + I_{CTAdlPreART} + I_{CTAdlARTelig} +$

$I_{FNAdoPreART} + I_{FNAdoARTelig} + I_{IPTAdoPreART} + I_{IPTAdoARTelig} + I_{FNAdlPreART} + I_{FNAdlARTelig} + I_{IPTAdlPreART} + I_{IPTAdlARTelig} + R_{AdoPreART} + R_{AdoARTelig}) - \zeta_{HIV}*I_{IPTAdlPreART} + I_{FNAdlPreART}*\xi_{Adl} - \phi*I_{IPTAdlPreART} + ((\lambda_{HIVp}*L_{IPTAdlPreART})) + \varepsilon*I_{IPTAdlPreART} - \mu_{TBPreARTAdl}*I_{IPTAdlPreART}$

$dI_{IPTAdlARTelig} = \zeta_{HIV}*I_{IPTAdlPreART} - \eta_{HIV}*I_{IPTAdlARTelig} + I_{FNAdlARTelig}*I_{PTtoacivesAdl} - \phi*I_{IPTAdlARTelig} + ((activIPT*L_{IPTAdlARTelig})) + \varepsilon*I_{IPTAdlARTelig} - \mu_{TBARTeligAdl}*I_{IPTAdlARTelig}$

$dI_{IPTAdlOnARTCD4low} = \eta_{HIV}*I_{IPTAdlARTelig} - \theta_{HIV}*I_{IPTAdlOnARTCD4low} + I_{FNAdlOnARTCD4low}*IPTtoacivesAdl - \phi*I_{IPTAdlOnARTCD4low} + ((\lambda_{HIVp}*L_{IPTAdlOnARTCD4low})) + \varepsilon*I_{IPTAdlOnARTCD4low} - \mu_{TBARTlowAdl}* I_{IPTAdlOnARTCD4low}$

$dI_{IPTAdlOnARTCD4high} = \theta_{HIV}*I_{IPTAdlOnARTCD4low} + I_{FNAdlOnARTCD4high}*\xi_{Adl} - \phi*I_{IPTAdlOnARTCD4high} + ((\lambda_{HIVp}*L_{IPTAdlOnARTCD4high})) + \varepsilon*I_{IPTAdlOnARTCD4high} - \mu_{TBARThighAdl}*I_{IPTAdlOnARTCD4high}$

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