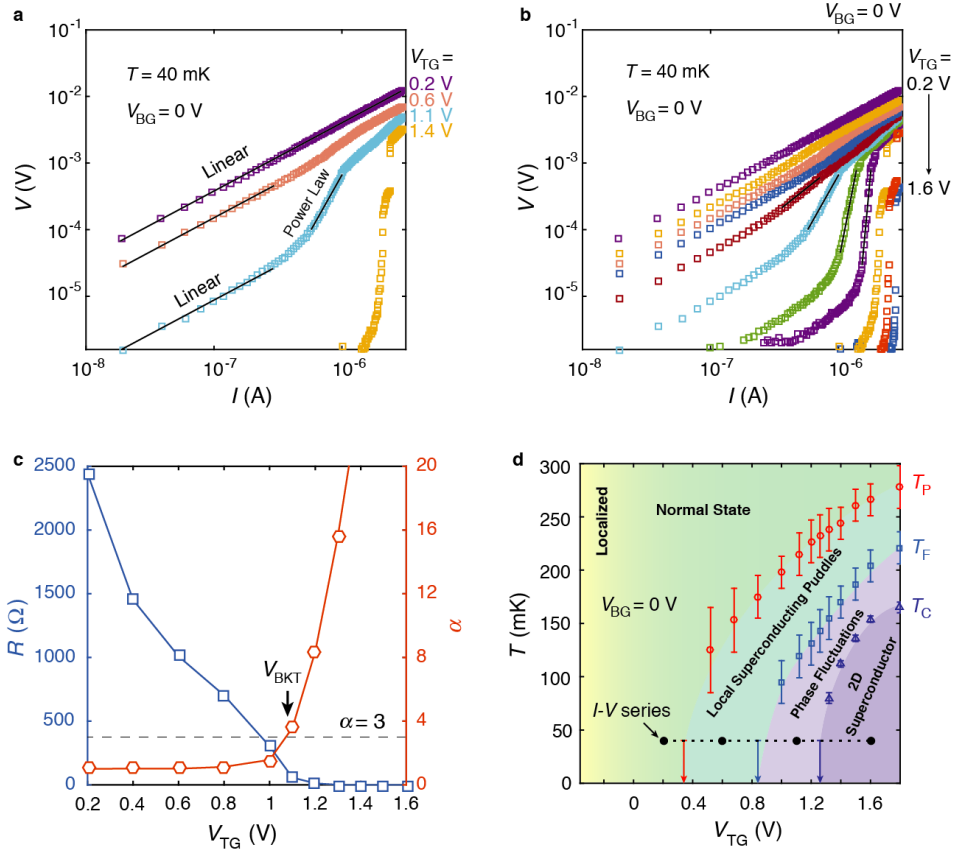


Supplementary Information

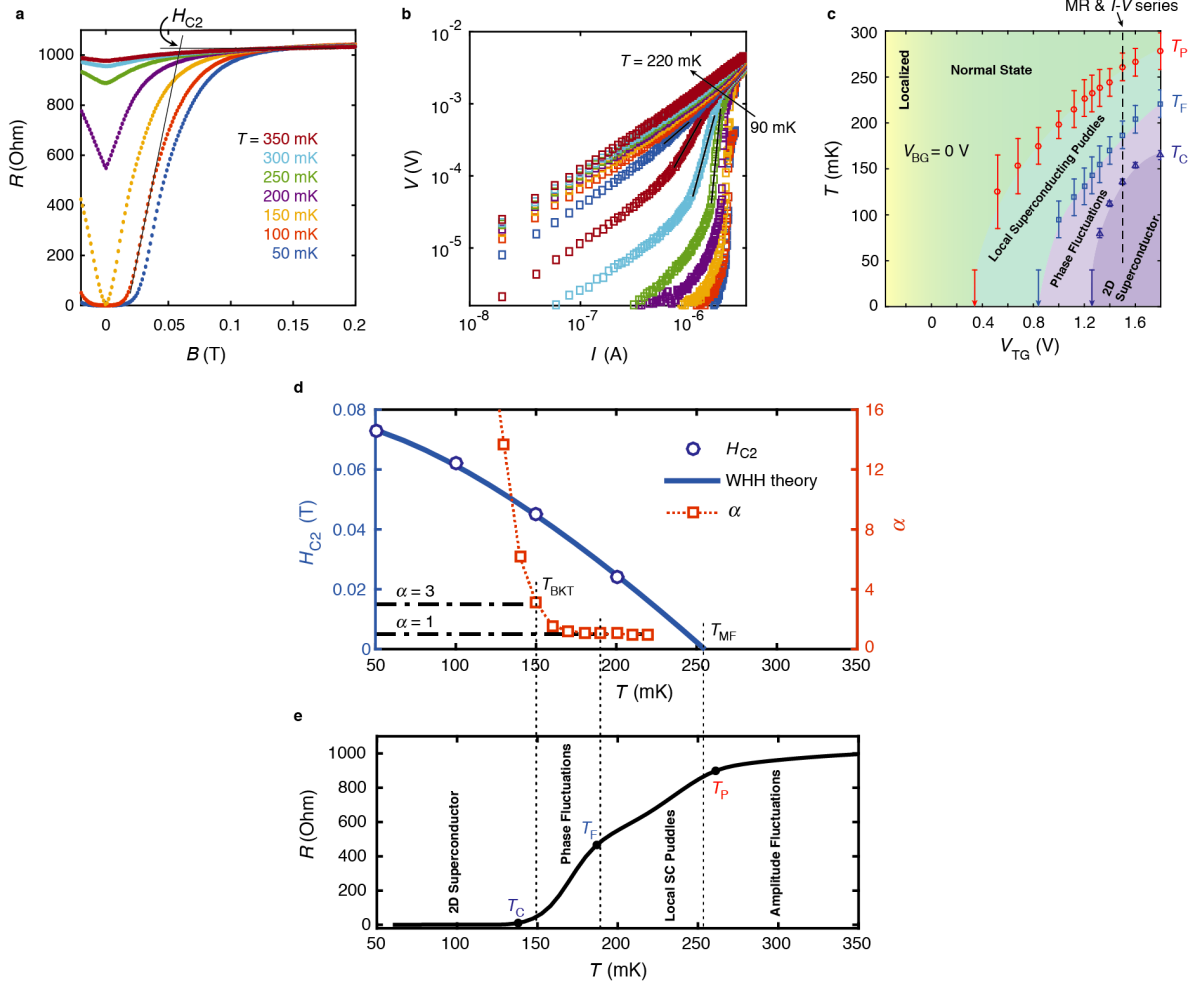
**Carrier density and disorder tuned superconductor-metal
transition in a two-dimensional electron system**

Chen et al.



Supplementary Figure 1 | Distinguishing different regimes tuned by V_{TG} via current-voltage curves.

a, Four representative current-voltage (I - V) curves in different regimes of the phase diagram as a function of V_{TG} with fixed V_{BG} (0 V) and T (40 mK). **b**, I - V curve series with denser V_{TG} intervals. Solid black lines indicate the power law regime in certain I - V curves. **c**, The left axis shows the resistance measured by the slope at the lowest currents as a function of V_{TG} . The right axis shows the exponent α of the power law regime as a function of V_{TG} . **d**, Reproduction of the phase diagram shown in Fig. 2c in the main text. Data points and error bars are defined in the same manner as in Fig. 2c. Background colors are guides to the eye indicating the different regimes. The horizontal dashed line indicates where the I - V series is measured in the phase diagram, and the representative I - V curves in **a** are marked with solid circles.



Supplementary Figure 2 | Nature of characteristic temperatures by combined analysis of magnetoresistance and I - V . **a**, Perpendicular magnetoresistance (MR) measured at different temperatures and fixed $V_{TG} = 1.5$ V, $V_{BG} = 0$ V. H_{C2} is indicated by the crossing of two fitted straight lines. **b**, I - V characteristics from $T = 90$ mK to 220 mK with 10 mK intervals at fixed $V_{TG} = 1.5$ V, $V_{BG} = 0$ V. The solid black lines fitted to the curves exhibit the regime for the power-law behavior. **c**, Reproduction of the phase diagram shown in Fig. 2c in the main text. Data points and error bars are defined in the same manner as in Fig. 2c. Background colors are guides to the eye indicating the different regimes. The vertical dashed line indicates where the MR and I - V series are measured in the phase diagram. **d**, Combined results of the MR and I - V analysis. Blue circles represent H_{C2} as defined in **a**. The solid blue curve is a fit with the Werthamer-Helfand-Hohenberg (WHH) form. Red squares represent the power-law exponent α extracted from the I - V curves. Dashed horizontal lines indicate $\alpha = 3$ and $\alpha = 1$. **e**, R - T curve at $V_{TG} = 1.5$ V, $V_{BG} = 0$ V. Solid circles show the characteristic temperatures T_P , T_F , and T_C as defined in the R - T curve. Dotted lines exhibit the correspondence between these temperatures and the characteristic temperatures extracted from MR and I - V analysis as shown in **d**.

Supplementary Note 1

In Supplementary Figure 1, we show current-voltage (I - V) characteristics as a function of V_{TG} at $T = 40$ mK, in which the nature of the four regimes in the phase diagrams shown in the main text can be clearly distinguished. The system goes through a gate-tuned Berezinskii–Kosterlitz–Thouless (BKT) transition indicated by the power-law behavior in I - V curves. The I - V characteristics series corresponds to a horizontal cut in the V_{TG} tuned phase diagram (Supplementary Figure 1d). Distinct I - V behavior in the four different regimes are shown in Supplementary Figure 1a. The I - V curve for $V_{\text{TG}} = 0.2$ V exhibits linear behaviour in the entire current range, indicating a normal state response. For $V_{\text{TG}} = 0.6$ V, the I - V curve shows a partial drop of resistance at around 1.2 μA , but returns to linear dependence quickly below that, indicating finite superconducting pairing but only on a local scale. The I - V curve for $V_{\text{TG}} = 1.1$ V not only displays a partial drop of resistance at 2.2 μA , but also a power law behavior $V \propto I^\alpha$ below 1.0 μA , indicating the onset of long-range phase coupling governed by BKT physics. Yet further lowering the current, it returns to linear, indicating that the phase fluctuations coexist with dissipation. The I - V curve for $V_{\text{TG}} = 1.4$ V shows typical global 2D superconducting behaviour with voltage lower than the measurement noise limit for current lower than ~ 1.5 μA . The exponent α of the power law regime changes continuously as a function of V_{TG} as shown in Supplementary Figure 1c, crossing $\alpha = 3$ at $V_{\text{TG}} = 1.07$ V. This indicates that the system goes through a BKT-driven transition with fixed temperature $T = 40$ mK and varying V_{TG} .

Supplementary Note 2

In Supplementary Figure 2, we present a combined analysis of perpendicular magnetoresistance (MR) and I - V as a function of T and fixed gate voltages. The analysis of H_{C2} extracted from MR yields the pairing temperature scale for superconductivity. The analysis of the power law exponent from I - V provides a BKT transition temperature, together with an onset temperature for superlinear power law. The correspondence between these extracted characteristic temperatures and the T_P , T_F , and T_C as defined in R - T curves identifies these regimes from a different approach. This supports the identification of the different regimes in the phase diagram from R - T analysis.

MR (Supplementary Figure 2a) in perpendicular field and I - V curves (Supplementary Figure 2b) are measured in a temperature series with fixed gate voltages, which corresponds to a vertical cut in the phase diagram shown in Supplementary Figure 2c. From the MR, we can experimentally define a characteristic field H_{C2} as illustrated in the figure. The results shown in Supplementary Figure 2d suggest that the behaviour of the H_{C2} can be fit by Werthamer-Helfand-Hohenberg (WHH) theory¹, which describes the mean-field behaviour of H_{C2} . Extrapolating the critical field H_{C2} to zero, we can extract a mean field temperature T_{MF} as a temperature scale for pair breaking. On the other hand, we can extract the exponent α from the power-law regime in the I - V curves. Shown in Supplementary Figure 2d, we can define the temperature at which $\alpha = 3$ to be the BKT transition temperature T_{BKT} . Interestingly, we found that the T_{MF} closely matches T_P defined in the R - T curve, as can be seen in Supplementary Figure 2e. T_{BKT} is slightly higher than the T_C extracted from the R - T curve (defined as 1% R_N), similar to the case for Supplementary Figure 1. Moreover, T_F closely matches the temperature for the onset of a superlinear power law (i.e. from $\alpha = 1$ to $\alpha > 1$). These correspondences independently support the identification of T_P as the pairing temperature; T_F as the onset of macroscopic phase fluctuations; and T_C for macroscopic phase coherence. The identification of the three characteristic temperatures divide the phase diagram into four regimes as discussed in the main text. Above T_P , the resistivity at zero field increases with increasing temperature, consistent with superconducting amplitude fluctuations. The MR in this regime can be well fitted with Aslamasov-Larkin (AL) theory² and Maki-Thompson (MT) theory^{3,4}, following similar procedures as in ref. 5.

Supplementary References

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