

1 **Supplementary Material 1**

2 **Fecundity assay**

3 A total of 210 uninfected individual female *Aedes aegypti* across nine laboratory populations and two
4 recently established populations from Yorkeys Knob and Gordonvale were isolated into 150 mL
5 plastic cylindrical fly vials (2.85 cm diameter). These females were isolated around 7-10 days old
6 from mass rearing cages straight after blood-feeding. The vials were filled with about 5 mL water
7 with a small cone shape filter paper placed on the water. Eggs were collected for four days, once daily
8 starting from the day after placing females in the fly vials. Filter paper with eggs was kept wet for
9 three days and then partially dried (moist to touch) to initiate egg quiescence as part of hatch rate
10 assessment (not described here). Drying eggs was not necessary for fecundity assessment, but drying
11 was initiated as part of some unpublished experiments. The eggs were then counted under the
12 microscope if there were few eggs (<50) or by taking photographs and doing the counting in ImageJ
13 manually by placing landmarks on each eggs. Counts were not approximated. Fecundity ranged from
14 2 to 111 eggs, with a mean of 59.61 and standard deviation of 20.99 eggs. When testing for normality
15 assumption of the fecundity numbers distribution using the Shapiro-Wilks Test, we found no
16 significant evidence of deviation from normality, $p > 0.4$. The 95% Confidence Interval (CI) of the
17 fecundity of uninfected females assuming normal distribution is (56.78, 62.45).

18 In a second pilot experiment with single individual females, we tested whether mating with
19 *Wolbachia*-infected males affected uninfected female fecundity. This was a necessary to avoid bias in
20 the estimate of non-random mating parameter, β due to greater/fewer eggs produced by the uninfected
21 female when mated to males of different infection status. Approximately 41 females crossed to
22 uninfected males and 28 females crossed to infected males. These females were then subjected to the
23 say protocol as the previous fecundity assay (see above). Fecundity of females mated to uninfected
24 males had a 95% CI of (67.57, 79.11) and 95% CI of females mated to infected males was (60.52,
25 73.27). There was no significant evidence of difference in female fecundity when mated to males of
26 different infection status (Welch Two Sample, $t = 1.4699$, $df = 61.524$, $p > 0.073$). Any effects of non-
27 random mating or sperm competition were not attributed to differences in female fecundity mated to
28 males of different infection status.

29 **Supplementary Material 2**

30 **Table S2.1** Simulated quantiles (0, 5, 80, 85, 90, 95 and 100%) for sample sizes 100, 120, 130, 150, 180, 200

n	100							120							130						
p	q0	q0.05	q0.80	q0.85	q0.90	q0.95	q1	q0	q0.05	q0.80	q0.85	q0.90	q0.95	q1	q0	q0.05	q0.80	q0.85	q0.90	q0.95	q1
0.05	0.000	0.017	0.067	0.073	0.077	0.086	0.127	0.005	0.020	0.065	0.070	0.076	0.083	0.121	0.000	0.020	0.065	0.069	0.075	0.084	0.117
0.10	0.023	0.052	0.123	0.131	0.139	0.150	0.239	0.019	0.055	0.121	0.128	0.137	0.148	0.197	0.027	0.057	0.122	0.127	0.134	0.147	0.217
0.15	0.056	0.091	0.180	0.188	0.198	0.214	0.277	0.054	0.096	0.176	0.182	0.191	0.206	0.269	0.058	0.098	0.176	0.183	0.191	0.206	0.267
0.20	0.077	0.134	0.234	0.243	0.253	0.270	0.346	0.104	0.141	0.232	0.239	0.249	0.266	0.319	0.091	0.141	0.231	0.237	0.246	0.262	0.315
0.25	0.121	0.179	0.286	0.295	0.307	0.324	0.405	0.143	0.184	0.283	0.290	0.299	0.316	0.363	0.126	0.187	0.282	0.292	0.303	0.315	0.360
0.30	0.160	0.225	0.336	0.347	0.361	0.380	0.438	0.180	0.228	0.335	0.341	0.353	0.367	0.434	0.149	0.230	0.333	0.340	0.351	0.368	0.424
0.35	0.216	0.269	0.391	0.401	0.412	0.433	0.486	0.220	0.275	0.386	0.394	0.404	0.419	0.483	0.198	0.276	0.385	0.393	0.403	0.421	0.452
0.40	0.253	0.316	0.443	0.452	0.464	0.488	0.580	0.263	0.323	0.437	0.445	0.458	0.474	0.539	0.240	0.325	0.435	0.445	0.460	0.476	0.528
0.45	0.286	0.364	0.493	0.504	0.517	0.539	0.650	0.310	0.367	0.486	0.496	0.507	0.528	0.584	0.272	0.374	0.485	0.496	0.508	0.528	0.621
0.50	0.344	0.412	0.545	0.555	0.569	0.587	0.671	0.356	0.420	0.539	0.547	0.559	0.579	0.655	0.349	0.423	0.537	0.548	0.558	0.578	0.652
0.55	0.370	0.463	0.597	0.607	0.617	0.634	0.727	0.411	0.470	0.591	0.600	0.609	0.625	0.690	0.370	0.476	0.588	0.597	0.605	0.623	0.697
0.60	0.422	0.509	0.644	0.652	0.667	0.682	0.745	0.434	0.520	0.638	0.646	0.659	0.676	0.723	0.450	0.523	0.638	0.646	0.657	0.675	0.749
0.65	0.480	0.565	0.692	0.700	0.715	0.730	0.795	0.488	0.577	0.689	0.696	0.707	0.724	0.774	0.488	0.577	0.689	0.697	0.707	0.721	0.779
0.70	0.557	0.616	0.744	0.753	0.762	0.778	0.850	0.551	0.625	0.737	0.744	0.753	0.769	0.831	0.527	0.625	0.738	0.746	0.755	0.767	0.811
0.75	0.604	0.673	0.790	0.799	0.807	0.824	0.885	0.629	0.682	0.784	0.790	0.800	0.815	0.889	0.620	0.684	0.785	0.792	0.801	0.812	0.868
0.80	0.656	0.729	0.839	0.845	0.854	0.866	0.913	0.693	0.740	0.833	0.840	0.850	0.860	0.905	0.701	0.740	0.832	0.838	0.847	0.858	0.917
0.85	0.721	0.788	0.882	0.889	0.897	0.909	0.962	0.752	0.795	0.879	0.885	0.894	0.903	0.944	0.731	0.797	0.878	0.883	0.892	0.902	0.952
0.90	0.753	0.849	0.926	0.931	0.938	0.948	0.984	0.811	0.853	0.925	0.929	0.936	0.943	0.981	0.798	0.855	0.924	0.929	0.935	0.944	0.965
0.95	0.861	0.911	0.968	0.971	0.976	0.983	1.000	0.879	0.914	0.968	0.971	0.975	0.981	1.000	0.892	0.916	0.968	0.971	0.976	0.980	1.000

n	150							180							200						
p	q0	q0.05	q0.80	q0.85	q0.90	q0.95	q1	q0	q0.05	q0.80	q0.85	q0.90	q0.95	q1	q0	q0.05	q0.80	q0.85	q0.90	q0.95	q1
0.05	0.005	0.022	0.065	0.068	0.073	0.079	0.115	0.007	0.024	0.063	0.066	0.071	0.076	0.127	0.004	0.025	0.062	0.065	0.069	0.079	0.105
0.10	0.035	0.060	0.120	0.124	0.131	0.142	0.192	0.039	0.064	0.118	0.122	0.128	0.137	0.203	0.038	0.065	0.117	0.122	0.128	0.136	0.209
0.15	0.070	0.100	0.175	0.181	0.188	0.201	0.251	0.071	0.104	0.172	0.177	0.185	0.199	0.253	0.083	0.112	0.170	0.176	0.183	0.194	0.251
0.20	0.105	0.144	0.228	0.235	0.243	0.255	0.306	0.125	0.150	0.224	0.231	0.240	0.250	0.294	0.124	0.156	0.223	0.230	0.236	0.248	0.295
0.25	0.151	0.192	0.279	0.287	0.295	0.310	0.374	0.152	0.197	0.277	0.283	0.291	0.305	0.365	0.157	0.202	0.275	0.281	0.289	0.300	0.344
0.30	0.185	0.236	0.331	0.339	0.349	0.363	0.417	0.198	0.242	0.327	0.334	0.343	0.356	0.392	0.211	0.247	0.327	0.333	0.342	0.352	0.391
0.35	0.224	0.283	0.382	0.389	0.399	0.414	0.461	0.235	0.289	0.379	0.385	0.395	0.410	0.460	0.252	0.295	0.377	0.385	0.395	0.405	0.439
0.40	0.271	0.333	0.435	0.443	0.452	0.468	0.529	0.296	0.337	0.429	0.438	0.447	0.463	0.518	0.303	0.340	0.429	0.435	0.444	0.458	0.503
0.45	0.330	0.382	0.486	0.494	0.503	0.519	0.592	0.324	0.387	0.479	0.489	0.500	0.513	0.569	0.344	0.388	0.480	0.486	0.495	0.508	0.579
0.50	0.379	0.428	0.537	0.545	0.556	0.570	0.634	0.388	0.435	0.531	0.539	0.549	0.564	0.614	0.388	0.439	0.531	0.537	0.546	0.558	0.618
0.55	0.406	0.478	0.585	0.592	0.603	0.619	0.682	0.424	0.488	0.580	0.587	0.597	0.612	0.668	0.414	0.487	0.581	0.588	0.597	0.607	0.673
0.60	0.470	0.531	0.635	0.642	0.651	0.668	0.724	0.480	0.537	0.629	0.635	0.644	0.658	0.701	0.474	0.541	0.629	0.636	0.646	0.660	0.714
0.65	0.513	0.582	0.686	0.694	0.702	0.717	0.774	0.531	0.588	0.679	0.687	0.694	0.706	0.746	0.531	0.591	0.679	0.686	0.695	0.705	0.755
0.70	0.563	0.634	0.733	0.739	0.747	0.762	0.813	0.575	0.638	0.730	0.735	0.743	0.755	0.798	0.585	0.644	0.728	0.734	0.742	0.754	0.815
0.75	0.627	0.687	0.781	0.786	0.795	0.808	0.863	0.643	0.692	0.777	0.783	0.790	0.801	0.844	0.651	0.694	0.776	0.782	0.789	0.800	0.840
0.80	0.693	0.744	0.828	0.835	0.844	0.856	0.903	0.710	0.749	0.826	0.831	0.839	0.850	0.886	0.706	0.750	0.826	0.832	0.838	0.848	0.882
0.85	0.766	0.801	0.876	0.883	0.890	0.899	0.933	0.768	0.804	0.873	0.878	0.885	0.895	0.921	0.774	0.811	0.874	0.878	0.885	0.892	0.930
0.90	0.823	0.857	0.920	0.925	0.932	0.940	0.973	0.816	0.861	0.919	0.925	0.931	0.938	0.964	0.809	0.864	0.920	0.924	0.929	0.934	0.954
0.95	0.885	0.918	0.966	0.969	0.972	0.978	0.994	0.886	0.920	0.964	0.967	0.971	0.977	0.997	0.889	0.921	0.964	0.966	0.970	0.975	1.000

32 As an example, at sample size = 100, if the test frequency is 0.75 and the true effective reduction is 0.6, observing a value below 0.673 would imply that the
33 true effective reduction had only a 5% (α) chance of being 0.75. 0.673 is within 90-95% (power, β) interval 0.6 (see q_{90} and q_{95}). Both values highlighted in
34 red.

35 **Supplementary Material 3**

36 **Table S3.1** Probability rejecting the null hypothesis that $\beta = 1$ under three different scenarios

β	Probability rejecting null hypothesis		
	1	2	3
2.0	1.0000	0.9985	0.5020
1.0	0.6038	0.0339	0.0424
0.9	0.9739	0.4526	0.1336
0.8	0.9999	0.9604	0.3586
0.7	1.0000	1.0000	0.6743
0.6	1.0000	1.0000	0.9218
0.5	1.0000	1.0000	0.9933

37

38 1 – 25 uninfected females per treatment and replicate, with three replicates of each control (0 and
 39 100% infected male), seven replicates of the 50% and 60% infected male treatment and eight
 40 replicates of the 20%, 30%, 40%, 70%, 80% and 90% infected male treatment. *P*-value of the
 41 regression test has a cutoff of 0.05. There is a high Type 1 error if using unadjusted *p*-value

42 2 – 25 uninfected females per treatment and replicate, with three replicates of each control (0 and
 43 100% infected male), seven replicates of the 50% and 60% infected male treatment and eight
 44 replicates of the 20%, 30%, 40%, 70%, 80% and 90% infected male treatment. *P*-value of the
 45 regression test has a cutoff of 5×10^{-5} . Adjusted *p*-value has Type 1 error rate of 0.0339.

46 2 – 25 uninfected females per treatment and replicate, with three replicates of each control (0 and
 47 100% infected male), with only one treatment; eight replicates of the 50% infected male treatment. *P*-
 48 value of the regression test has a cutoff of 5×10^{-5}

49 **Supplementary Material 4**

50 Here, we show (rather than prove) that both the non-random mating due to difference in male fitness
51 model and the non-random mating due to assortative mating will converge when the unstable
52 equilibrium is 0.5 for arbitrary values of s_f and s_h .

53 From equation 11,

54
$$0.5 = \frac{s_f}{s_f + \beta_{nr}(s_h - s_f)},$$

55
$$s_f + \beta_{nr}(s_h - s_f) = 2s_f,$$

56
$$\beta_{nr} = \frac{s_f}{(s_h - s_f)}.$$

57

58 From equation 12,

59
$$0.5 = \frac{\beta_{am}s_f + 1 - \beta_{am}}{2(1 - \beta_{am}) + s_f(\beta_{am} - 1) + \beta_{am}s_h},$$

60
$$2(1 - \beta_{am}) + s_f(\beta_{am} - 1) + \beta_{am}s_h = 2\beta_{am}s_f + 2 - 2\beta_{am},$$

61
$$s_f\beta_{am} - s_f + \beta_{am}s_h = 2\beta_{am}s_f,$$

62
$$\beta_{am}(s_h - s_f) = s_f,$$

63
$$\beta_{am} = \frac{s_f}{(s_h - s_f)}.$$

64 Hence at $\hat{p} = 0.5$ for arbitrary values of s_f and s_h .

65
$$\beta_{am} = \beta_{nr}$$

66 (SHOWN)

67

68

69 **Supplementary Material 5**

70 We demonstrate that we can transform the unstable equilibrium directly, except when there is
 71 maternal transmission leakage (see Supplementary Material 6). For the model with random mating
 72 and no sperm competition, assuming no maternal transmission leakage:

$$73 \quad \hat{p} = \frac{s_f}{s_h}.$$

74 Applying this to equation 6:

$$75 \quad \hat{p} = \frac{\frac{s_f}{s_h}}{\frac{s_f}{s_h} + \beta_{nr}\left(1 - \frac{s_f}{s_h}\right)},$$

$$76 \quad = \frac{\frac{s_f}{s_h}}{\frac{s_f + \beta_{nr}(s_h - s_f)}{s_h}},$$

$$77 \quad = \frac{s_f}{s_f + \beta_{nr}(s_h - s_f)} = \text{Equation 11}.$$

78 It should be apparent that since this transformation works for the non-random mating due to
 79 difference in male fitness model (NR), it should not work for the assortative mating model (AM).

80 Applying to equation 7:

$$81 \quad \hat{p}^2 \left(p_m - 2p_m \left(\frac{\beta_{sc}}{\beta_{sc} + 1} \right) \right) + \hat{p} \left(1 - \left[p_m - 2p_m \left(\frac{\beta_{sc}}{\beta_{sc} + 1} \right) \right] \right) - p_{sc} = 0,$$

$$82 \quad \hat{p}^2 \left(p_m - 2p_m \left(\frac{\beta_{sc}}{\beta_{sc} + 1} \right) \right) + \hat{p} \left(1 - \left[p_m - 2p_m \left(\frac{\beta_{sc}}{\beta_{sc} + 1} \right) \right] \right) - \frac{s_f}{s_h} = 0.$$

83 Multiply by s_h :

$$84 \quad \hat{p}^2 \left(s_h p_m - 2s_h p_m \left(\frac{\beta_{sc}}{\beta_{sc} + 1} \right) \right) + \hat{p} \left(s_h - \left[s_h p_m - 2s_h p_m \left(\frac{\beta_{sc}}{\beta_{sc} + 1} \right) \right] \right) - s_f = 0,$$

$$85 \quad \hat{p}^2 \left(\frac{\beta_{sc} s_h p_m + s_h p_m - 2s_h p_m \beta_{sc}}{\beta_{sc} + 1} \right) + \hat{p} \left(s_h - \frac{[\beta_{sc} s_h p_m + s_h p_m - 2\beta_{sc} s_h p_m]}{\beta_{sc} + 1} \right) - s_f = 0,$$

$$86 \quad \hat{p}^2 \left(s_h p_m \left(\frac{1 - \beta_{sc}}{1 + \beta_{sc}} \right) \right) + \hat{p} \left(s_h + s_h p_m \left(\frac{\beta_{sc} - 1}{1 + \beta_{sc}} \right) \right) - s_f = 0.$$

87 This is the same as equation 13 (SHOWN).

88 **Supplementary Material 6**

89 When there is maternal transmission leakage, the unstable equilibrium frequency under the random mating and no sperm competition model is

$$90 \quad \hat{p} = \frac{s_h + s_f - \sqrt{(s_h + s_f)^2 - 4s_h(1 - \mu(1 - s_f))(s_f + \mu(1 - s_f))}}{2s_h(1 - \mu(1 - s_f))},$$

91 while under the NR model, the unstable equilibrium can be obtained by solving the equation for the smaller positive value:

$$92 \quad \hat{p}^2 \left(\beta_{nr} s_h (1 - \mu(1 - s_f)) + s_f (1 - \beta_{nr}) \right) + \hat{p} \left(\beta_{nr} s_h + s_f (2 - \beta_{nr}) + \mu(1 - s_f)(\beta_{nr} - 1) \right) + s_f + \mu(1 - s_f) = 0.$$

93 We prove by contradiction that the transformation of the unstable equilibrium does not give an accurate estimate of the unstable equilibrium due to non-

94 random mating due to male (NR) and sperm competition (SC).

95 Suppose the transformation using equation 6 in the main text holds, then we have:

$$96 \quad \hat{p} = \frac{\frac{s_h + s_f - \sqrt{(s_h + s_f)^2 - 4s_h(1 - \mu(1 - s_f))(s_f + \mu(1 - s_f))}}{2s_h(1 - \mu(1 - s_f))}}{(1 - \beta_{nr}) \left(s_h + s_f - \sqrt{(s_h + s_f)^2 - 4s_h(1 - \mu(1 - s_f))(s_f + \mu(1 - s_f))} \right) + \beta_{nr} \left(2s_h(1 - \mu(1 - s_f)) \right)}{2s_h(1 - \mu(1 - s_f))}.$$

97 Say $\beta_{nr} = 0$:

$$98 \quad = \frac{s_h + s_f - \sqrt{(s_h + s_f)^2 - 4s_h(1 - \mu(1 - s_f))(s_f + \mu(1 - s_f))}}{\left(s_h + s_f - \sqrt{(s_h + s_f)^2 - 4s_h(1 - \mu(1 - s_f))(s_f + \mu(1 - s_f))} \right)} = 1.$$

99

100 Now from the actual derivation, if $\beta_{nr} = 0$:

101
$$\hat{p}^2(s_f) + \hat{p}(2s_f - \mu(1 - s_f)) + s_f + \mu(1 - s_f) = 0.$$

102 If we plug in $\hat{p} = 1$, we get

103
$$4s_f = 0,$$

104 implying that $\hat{p} = 1$ is not a solution to the actual derivation unless $s_f = 0$, hence a contradiction.

105

106 **Supplementary Material 7**

107 We show that for any fixed value of s_h and s_f , the non-random mating due to fitness differences in the NR male mating model will always lead to greater
 108 change in the unstable equilibrium frequency than the sperm competition model (SC). Recall from equation 13:

109
$$\hat{p}^2 \left(s_h p_m \left(\frac{1 - \beta_{sc}}{1 + \beta_{sc}} \right) \right) + \hat{p} \left(s_h - s_h p_m \left(\frac{1 - \beta_{sc}}{1 + \beta_{sc}} \right) \right) - s_f = 0.$$

110 Recall that when there is no sperm competition, $\beta_{sc} = 1$, the unstable equilibrium is s_f/s_h . Also, $p_m = 0$ the unstable equilibrium is s_f/s_h , i.e. no change, since
 111 sperm competition requires females to be able to multiply mate. Otherwise:

112
$$\hat{p} = \frac{-(1 + \beta_{sc})}{2p_m(1 - \beta_{sc})} + \frac{1}{2} \pm \sqrt{\frac{s_h(\beta_{sc} + 1 - p_m(1 - \beta_{sc}))^2 + 4s_f p_m(1 - \beta_{sc}^2)}{4p_m^2 s_h(1 - \beta_{sc})^2}}.$$

113 Note that, by taking the limit of $\beta_{sc} \rightarrow 1$ on the above equation and applying L'Hopital's Rule, we can also obtain the unstable equilibrium of s_f/s_h .

114 Firstly notice that when $s_h = s_f$, the unstable equilibrium will simplify to:

$$\hat{p} = \frac{-(1 + \beta_{sc})}{2p_m(1 - \beta_{sc})} + \frac{1}{2} \pm \frac{(1 + \beta_{sc}) + p_m(1 - \beta_{sc})}{2p_m(1 - \beta_{sc})} = \frac{1}{2} \pm \frac{1}{2} = 0 \text{ or } 1.$$

115

116 When $s_h > s_f$,

117
$$\frac{-(1 + \beta_{sc})}{2p_m(1 - \beta_{sc})} + \sqrt{\frac{s_h(\beta_{sc} + 1 - p_m(1 - \beta_{sc}))^2 + 4s_f p_m(1 - \beta_{sc}^2)}{4p_m^2 s_h(1 - \beta_{sc})^2}} > \frac{1}{2}.$$

118 The unstable equilibrium is between 0 and 1 since the square root portion of the unstable equilibrium is decreasing (i.e. making the positive value smaller, or
 119 negative value larger). This is because, as we can see above, the red portion of the square root is constant since s_h cancels out while the blue portion decreases
 120 since s_h is in the denominator.

121

122 However, when $s_h < s_f$, it will be increasing giving rise to invalid values of the unstable equilibrium. Hence, $s_h > s_f$ must be fulfilled else the system will always
 123 crash for any infection frequency less than 1 since the unstable equilibrium has to be one. It is unlikely that there are two unstable equilibria. Realistically
 124 speaking, it is biologically unreasonable that as $s_h \rightarrow s_f$ the unstable equilibrium will be zero, since it should become more difficult for the *Wolbachia*
 125 infection to spread to fixation as the cytoplasmic incompatibility effect is weakening. By this rationale, when $\beta_{sc} < 1$, the unstable equilibrium must take the
 126 positive value of the square root:

$$127 \quad \hat{p} = \frac{-(1 + \beta_{sc})}{2p_m(1 - \beta_{sc})} + \frac{1}{2} + \sqrt{\frac{s_h(\beta_{sc} + 1 - p_m(1 - \beta_{sc}))^2 + 4s_f p_m(1 - \beta_{sc}^2)}{4p_m^2 s_h(1 - \beta_{sc})^2}},$$

128 while when $\beta_{sc} > 1$, the unstable equilibrium must be computed with the negative value of the square root:

$$129 \quad \hat{p} = \frac{(1 + \beta_{sc})}{2p_m(\beta_{sc} - 1)} + \frac{1}{2} - \sqrt{\frac{s_h(\beta_{sc} + 1 + p_m(\beta_{sc} - 1))^2 - 4s_f p_m(\beta_{sc}^2 - 1)}{4p_m^2 s_h(\beta_{sc} - 1)^2}}.$$

130 **The unstable equilibrium is monotonic with respect to the remating frequency, p_m , under specific conditions**

131 We also find that for any value of p_m , for $\beta_{sc} < 1$, the unstable equilibrium is monotonically increasing with respect to p_m but for $\beta_{sc} > 1$, the unstable
 132 equilibrium is monotonically decreasing (see below). To show this we have to assess the (partial) derivative of the unstable equilibrium with respect to p_m . If

133 the derivative is positive across all values of p_m , the unstable equilibrium is monotonically increasing; if negative, it is monotonically decreasing. However, if
 134 it is a mix of positive and negative then the system is not monotonic. Assuming all other parameters except the unstable equilibrium and p_m are constants, let

$$135 \quad u = s_h(\beta_{sc} + 1 - p_m(1 - \beta_{sc}))^2 + 4s_f p_m(1 - \beta_{sc}^2),$$

$$136 \quad v = 4p_m^2 s_h(1 - \beta_{sc})^2,$$

$$137 \quad \frac{du}{dp_m} = -2s_h(1 - \beta_{sc}^2 - p_m(1 - \beta_{sc})^2) + 4s_f(1 - \beta_{sc}^2),$$

$$138 \quad \frac{dv}{dp_m} = 8p_m s_h(1 - \beta_{sc})^2,$$

$$139 \quad \frac{d\hat{p}}{dp_m} = \frac{(1 + \beta_{sc})}{2p_m^2(1 - \beta_{sc})} + \frac{1}{2} \left(\frac{u}{v} \right)^{-\frac{1}{2}} \left(\frac{v \frac{du}{dp_m} - u \frac{dv}{dp_m}}{v^2} \right).$$

140 Now we have the strict condition that $s_h > s_f$. When $s_h = s_f$ and $\beta_{sc} < 1$:

$$141 \quad u = s_f(\beta_{sc} + 1 + p_m(1 - \beta_{sc}))^2,$$

$$142 \quad v = 4p_m^2 s_f(1 - \beta_{sc})^2,$$

$$143 \quad \frac{du}{dp_m} = 2s_f(1 - \beta_{sc}^2 + p_m(1 - \beta_{sc})^2),$$

$$144 \quad \frac{dv}{dp_m} = 8p_m s_f(1 - \beta_{sc})^2,$$

145

$$\begin{aligned}
146 \quad \frac{d\hat{p}}{dp_m} &= \frac{(1 + \beta_{sc})}{2p_m^2(1 - \beta_{sc})} \\
147 \quad &+ \frac{1}{2} \left(\frac{(2p_m(1 - \beta_{sc}))}{(1 + \beta_{sc}) + p_m(1 - \beta_{sc})} \right) \left(\frac{8p_m s_f^2 (1 - \beta_{sc})^2 (p_m(1 - \beta_{sc}^2) + p_m^2(1 - \beta_{sc})^2 - (\beta_{sc} + 1)^2 - 2p_m(1 - \beta_{sc}^2) - p_m^2(1 - \beta_{sc})^2)}{16p_m^4 s_f^2 (1 - \beta_{sc})^4} \right), \\
148 \quad &= \frac{(1 + \beta_{sc})}{2p_m^2(1 - \beta_{sc})} + \frac{1}{2} \left(\frac{(2p_m(1 - \beta_{sc}))}{(1 + \beta_{sc}) + p_m(1 - \beta_{sc})} \right) \left(\frac{-(\beta_{sc} + 1)((1 + \beta_{sc}) + p_m(1 - \beta_{sc}))}{2p_m^3(1 - \beta_{sc})^2} \right), \\
149 \quad &= \frac{(1 + \beta_{sc})}{2p_m^2(1 - \beta_{sc})} - \frac{(1 + \beta_{sc})}{2p_m^2(1 - \beta_{sc})} = 0.
\end{aligned}$$

150 We know that

$$151 \quad \frac{(1 + \beta_{sc})}{2p_m^2(1 - \beta_{sc})} > 0,$$

152 and if we let s_h to increase from s_f ,

$$153 \quad \frac{(1 + \beta_{sc})}{2p_m^2(1 - \beta_{sc})} + \frac{1}{2} \left(\frac{u}{v} \right)^{-\frac{1}{2}} \left(\frac{v \frac{du}{dp_m} - u \frac{dv}{dp_m}}{v^2} \right) > 0.$$

154 To show this, for some real number, k , we let $s_h = ks_f \leq 1$, for $k > 1$:

$$\begin{aligned}
155 \quad \frac{1}{2} \left(\frac{u}{v} \right)^{-\frac{1}{2}} &= \frac{1}{2} \left(\frac{ks_f(\beta_{sc} + 1 - p_m(1 - \beta_{sc}))^2 + 4s_f p_m(1 - \beta_{sc}^2)}{4p_m^2 ks_f(1 - \beta_{sc})^2} \right)^{-\frac{1}{2}}, \\
156 \quad &= \frac{1}{2} \left(\frac{ks_f(\beta_{sc} + 1 - p_m(1 - \beta_{sc}))^2 + 4s_f p_m(1 - \beta_{sc}^2)}{4p_m^2 ks_f(1 - \beta_{sc})^2} \right)^{-\frac{1}{2}},
\end{aligned}$$

157 and by completing the squares:

$$= \frac{1}{2} \left(\frac{\left(\beta_{sc} + 1 + \frac{2-k}{k} p_m (1 - \beta_{sc}) \right)^2 + \left(\frac{4k-4}{k^2} \right) p_m^2 (1 - \beta_{sc})^2}{4p_m^2 (1 - \beta_{sc})^2} \right)^{\frac{1}{2}} > 0.$$

158 Next:

$$159 \left(\frac{v \frac{du}{dp_m} - u \frac{dv}{dp_m}}{v^2} \right)$$

$$160 = \frac{4p_m^2 k s_f (1 - \beta_{sc})^2 \left(-2k s_f (1 - \beta_{sc}^2 - p_m (1 - \beta_{sc})^2) + 4s_f (1 - \beta_{sc}^2) \right) - 8p_m k s_f (1 - \beta_{sc})^2 \left(k s_f (\beta_{sc} + 1 - p_m (1 - \beta_{sc}))^2 + 4s_f p_m (1 - \beta_{sc}^2) \right)}{16p_m^4 k^2 s_f^2 (1 - \beta_{sc})^4},$$

$$161 = \frac{8p_m^2 k^2 s_f^2 (1 - \beta_{sc})^2 \left(\frac{2-k}{k} (1 - \beta_{sc}^2) + p_m (1 - \beta_{sc})^2 \right) - 8p_m k^2 s_f^2 (1 - \beta_{sc})^2 \left(\left(\beta_{sc} + 1 + \frac{2-k}{k} p_m (1 - \beta_{sc}) \right)^2 + \left(\frac{4k-4}{k^2} \right) p_m^2 (1 - \beta_{sc})^2 \right)}{16p_m^4 k^2 s_f^2 (1 - \beta_{sc})^4},$$

162

$$163 = \frac{\left(\frac{2-k}{k} p_m (1 - \beta_{sc}^2) + p_m^2 (1 - \beta_{sc})^2 \right) - \left(\left(\beta_{sc} + 1 + \frac{2-k}{k} p_m (1 - \beta_{sc}) \right)^2 + \left(\frac{4k-4}{k^2} \right) p_m^2 (1 - \beta_{sc})^2 \right)}{2p_m^3 (1 - \beta_{sc})^2},$$

$$= \frac{-(\beta_{sc} + 1) \left((1 + \beta_{sc}) + \left(\frac{2-k}{k} \right) p_m (1 - \beta_{sc}) \right)}{2p_m^3 (1 - \beta_{sc})^2} > \frac{-(\beta_{sc} + 1) \left((1 + \beta_{sc}) + p_m (1 - \beta_{sc}) \right)}{2p_m^3 (1 - \beta_{sc})^2}.$$

164 Hence:

$$165 \quad \frac{1}{2} \left(\frac{u}{v} \right)^{-\frac{1}{2}} \left(\frac{v \frac{du}{dp_m} - u \frac{dv}{dp_m}}{v^2} \right) = \frac{1}{2} \left(\frac{\left(\beta_{sc} + 1 + \frac{2-k}{k} p_m (1 - \beta_{sc}) \right)^2 + \left(\frac{4k-4}{k^2} \right) p_m^2 (1 - \beta_{sc})^2}{4p_m^2 (1 - \beta_{sc})^2} \right)^{-\frac{1}{2}} \left(\frac{-(\beta_{sc} + 1) \left((1 + \beta_{sc}) + \left(\frac{2-k}{k} \right) p_m (1 - \beta_{sc}) \right)}{2p_m^3 (1 - \beta_{sc})^2} \right),$$

$$166 \quad = \frac{-(\beta_{sc} + 1) \left((1 + \beta_{sc}) + \left(\frac{2-k}{k} \right) p_m (1 - \beta_{sc}) \right)}{2p_m^2 (1 - \beta_{sc}) \sqrt{\left(\beta_{sc} + 1 + \frac{2-k}{k} p_m (1 - \beta_{sc}) \right)^2 + \left(\frac{4k-4}{k^2} \right) p_m^2 (1 - \beta_{sc})^2}},$$

$$167 \quad = \frac{-(\beta_{sc} + 1)}{2p_m^2 (1 - \beta_{sc}) \sqrt{\frac{\left(\beta_{sc} + 1 + \frac{2-k}{k} p_m (1 - \beta_{sc}) \right)^2 + \left(\frac{4k-4}{k^2} \right) p_m^2 (1 - \beta_{sc})^2}{\left((1 + \beta_{sc}) + \left(\frac{2-k}{k} \right) p_m (1 - \beta_{sc}) \right)^2}}}$$

$$168 \quad = \frac{-(\beta_{sc} + 1)}{2p_m^2 (1 - \beta_{sc}) \sqrt{1 + \frac{\left(\frac{4k-4}{k^2} \right) p_m^2 (1 - \beta_{sc})^2}{\left((1 + \beta_{sc}) + \left(\frac{2-k}{k} \right) p_m (1 - \beta_{sc}) \right)^2}}}$$

169 As k increases, the fraction containing k will increase in the numerator but decrease in the denominator, hence the magnitude of the whole expression will
 170 decrease (becoming less negative).

171 Recall that when $k = 1$ or $s_h = s_f$,

$$172 \quad \frac{1}{2} \left(\frac{u}{v} \right)^{-\frac{1}{2}} \left(\frac{v \frac{du}{dp_m} - u \frac{dv}{dp_m}}{v^2} \right) = -\frac{(1 + \beta_{sc})}{2p_m^2(1 - \beta_{sc})}.$$

173 Hence for $k > 1$, or $s_h > s_f$,

$$174 \quad \frac{1}{2} \left(\frac{u}{v} \right)^{-\frac{1}{2}} \left(\frac{v \frac{du}{dp_m} - u \frac{dv}{dp_m}}{v^2} \right) = \frac{-(\beta_{sc} + 1)}{2p_m^2(1 - \beta_{sc}) \sqrt{1 + \frac{\left(\frac{4k-4}{k^2}\right) p_m^2(1 - \beta_{sc})^2}{\left((1 + \beta_{sc}) + \left(\frac{2-k}{k}\right) p_m(1 - \beta_{sc})\right)^2}}} > -\frac{(1 + \beta_{sc})}{2p_m^2(1 - \beta_{sc})}.$$

175 This implies that:

$$176 \quad \frac{(1 + \beta_{sc})}{2p_m^2(1 - \beta_{sc})} + \frac{1}{2} \left(\frac{u}{v} \right)^{-\frac{1}{2}} \left(\frac{v \frac{du}{dp_m} - u \frac{dv}{dp_m}}{v^2} \right) = \frac{(1 + \beta_{sc})}{2p_m^2(1 - \beta_{sc})} + \frac{-(\beta_{sc} + 1)}{2p_m^2(1 - \beta_{sc}) \sqrt{1 + \frac{\left(\frac{4k-4}{k^2}\right) p_m^2(1 - \beta_{sc})^2}{\left((1 + \beta_{sc}) + \left(\frac{2-k}{k}\right) p_m(1 - \beta_{sc})\right)^2}}}$$

$$177 \quad > \frac{(1 + \beta_{sc})}{2p_m^2(1 - \beta_{sc})} - \frac{(1 + \beta_{sc})}{2p_m^2(1 - \beta_{sc})} = 0.$$

178 (SHOWN)

179 Hence when $s_h > s_f$, $\beta_{sc} < 1$:

180
$$\frac{d\hat{p}}{dp_m} > 0.$$

181 The unstable equilibrium is monotonic increasing with increasing p_m .

182

183 Next, while $\beta > 1$

184
$$\hat{p} = \frac{(1 + \beta_{sc})}{2p_m(\beta_{sc} - 1)} + \frac{1}{2} - \sqrt{\frac{s_h(\beta_{sc} + 1 + p_m(\beta_{sc} - 1))^2 - 4s_f p_m(\beta_{sc}^2 - 1)}{4p_m^2 s_h(\beta_{sc} - 1)^2}}.$$

185 Assuming all other parameters except the unstable equilibrium and p_m are constants, let

186
$$u = s_h(\beta_{sc} + 1 + p_m(\beta_{sc} - 1))^2 - 4s_f p_m(\beta_{sc}^2 - 1)$$

187
$$v = 4p_m^2 s_h(\beta_{sc} - 1)^2$$

188
$$\frac{du}{dp_m} = 2s_h(\beta_{sc}^2 - 1 + p_m(\beta_{sc} - 1)^2) - 4s_f(\beta_{sc}^2 - 1)$$

189
$$\frac{dv}{dp_m} = 8p_m s_h(\beta_{sc} - 1)^2$$

190
$$\frac{d\hat{p}}{dp_m} = \frac{-(\beta_{sc} + 1)}{2p_m^2(\beta_{sc} - 1)} - \frac{1}{2} \left(\frac{u}{v}\right)^{-\frac{1}{2}} \left(\frac{v \frac{du}{dp_m} - u \frac{dv}{dp_m}}{v^2} \right).$$

191 When $s_h = s_f$ and $\beta_{sc} < 1$:

192

$$\frac{d\hat{p}}{dp_m} = 0.$$

193

We let $s_h = ks_f \leq 1$, for $k > 1$

194

$$\frac{1}{2} \left(\frac{u}{v} \right)^{-\frac{1}{2}} = \frac{1}{2} \left(\frac{ks_f(\beta_{sc} + 1 + p_m(\beta_{sc} - 1))^2 - 4s_f p_m(\beta_{sc}^2 - 1)}{4p_m^2 ks_f(\beta_{sc} - 1)^2} \right)^{-\frac{1}{2}}$$

195

$$= \frac{1}{2} \left(\frac{\left(\beta_{sc} + 1 + \left(\frac{k-2}{k} \right) p_m(\beta_{sc} - 1) \right)^2 + \left(\frac{4k-4}{k^2} \right) p_m^2(\beta_{sc} - 1)^2}{4p_m^2(\beta_{sc} - 1)^2} \right)^{-\frac{1}{2}}$$

196

$$\left(\frac{v \frac{du}{dp_m} - u \frac{dv}{dp_m}}{v^2} \right)$$

197

$$= \frac{4p_m^2 ks_f(\beta_{sc} - 1)^2 (2ks_f(\beta_{sc}^2 - 1 + p_m(\beta_{sc} - 1)^2) - 4s_f(\beta_{sc}^2 - 1)) - 8p_m ks_f(\beta_{sc} - 1)^2 (ks_f(\beta_{sc} + 1 + p_m(\beta_{sc} - 1))^2 - 4s_f p_m(\beta_{sc}^2 - 1))}{16p_m^4 k^2 s_f^2 (1 - \beta_{sc})^4}$$

198

$$= \frac{\left(\frac{2-k}{k} p_m(\beta_{sc}^2 - 1) + p_m^2(\beta_{sc} - 1)^2 \right) - \left(\left(\beta_{sc} + 1 + \left(\frac{k-2}{k} \right) p_m(\beta_{sc} - 1) \right)^2 + \left(\frac{4k-4}{k^2} \right) p_m^2(\beta_{sc} - 1)^2 \right)}{2p_m^3(\beta_{sc} - 1)^2}$$

199

$$= \frac{-(\beta_{sc} + 1) \left((1 + \beta_{sc}) + \left(\frac{k-2}{k} \right) p_m(\beta_{sc} - 1) \right)}{2p_m^3(\beta_{sc} - 1)^2} > \frac{-(\beta_{sc} + 1) \left((1 + \beta_{sc}) + p_m(\beta_{sc} - 1) \right)}{2p_m^3(1 - \beta_{sc})^2}.$$

200 Hence:

$$201 \quad = \frac{(\beta_{sc} + 1) \left((1 + \beta_{sc}) + \left(\frac{k-2}{k} \right) p_m (\beta_{sc} - 1) \right)}{2p_m^3 (\beta_{sc} - 1)^2} < \frac{(\beta_{sc} + 1) \left((1 + \beta_{sc}) + p_m (\beta_{sc} - 1) \right)}{2p_m^3 (1 - \beta_{sc})^2}.$$

202

203 For $k > 1$, or $s_h > s_f$,

$$204 \quad \frac{1}{2} \left(\frac{u}{v} \right)^{-\frac{1}{2}} \left(\frac{v \frac{du}{dp_m} - u \frac{dv}{dp_m}}{v^2} \right) = \frac{1}{2} \left(\frac{\left(\beta_{sc} + 1 + \frac{k-2}{k} p_m (\beta_{sc} - 1) \right)^2 + \left(\frac{4k-4}{k^2} \right) p_m^2 (\beta_{sc} - 1)^2}{4p_m^2 (\beta_{sc} - 1)^2} \right)^{-\frac{1}{2}} \left(\frac{-(\beta_{sc} + 1) \left((1 + \beta_{sc}) + \left(\frac{k-2}{k} \right) p_m (\beta_{sc} - 1) \right)}{2p_m^3 (\beta_{sc} - 1)^2} \right)$$

$$205 \quad = \frac{-(\beta_{sc} + 1)}{2p_m^2 (\beta_{sc} - 1) \sqrt{1 + \frac{\left(\frac{4k-4}{k^2} \right) p_m^2 (\beta_{sc} - 1)^2}{\left((1 + \beta_{sc}) + \left(\frac{k-2}{k} \right) p_m (\beta_{sc} - 1) \right)^2}}} > -\frac{(\beta_{sc} + 1)}{2p_m^2 (\beta_{sc} - 1)}.$$

206 Hence:

$$207 \quad \frac{(\beta_{sc} + 1)}{2p_m^2 (\beta_{sc} - 1) \sqrt{1 + \frac{\left(\frac{4k-4}{k^2} \right) p_m^2 (\beta_{sc} - 1)^2}{\left((1 + \beta_{sc}) + \left(\frac{k-2}{k} \right) p_m (\beta_{sc} - 1) \right)^2}}} < \frac{(\beta_{sc} + 1)}{2p_m^2 (\beta_{sc} - 1)}.$$

208 This implies that:

$$\begin{aligned}
209 \quad & \frac{-(\beta_{sc} + 1)}{2p_m^2(\beta_{sc} - 1)} + \frac{1}{2} \left(\frac{u}{v}\right)^{-\frac{1}{2}} \left(\frac{v \frac{du}{dp_m} - u \frac{dv}{dp_m}}{v^2} \right) = \frac{-(\beta_{sc} + 1)}{2p_m^2(\beta_{sc} - 1)} + \frac{(\beta_{sc} + 1)}{2p_m^2(\beta_{sc} - 1) \sqrt{1 + \frac{\left(\frac{4k-4}{k^2}\right) p_m^2 (\beta_{sc} - 1)^2}{\left((1 + \beta_{sc}) + \left(\frac{k-2}{k}\right) p_m (\beta_{sc} - 1)\right)^2}}} \\
210 \quad & < \frac{-(1 + \beta_{sc})}{2p_m^2(\beta_{sc} - 1)} + \frac{(1 + \beta_{sc})}{2p_m^2(\beta_{sc} - 1)} = 0.
\end{aligned}$$

211 Since the square root value will always greater than 1, for any $k > 1$.

212 Hence when $s_h > s_f$; $\beta_{sc} > 1$:

$$213 \quad \frac{d\hat{p}}{dp_m} < 0.$$

214 The unstable equilibrium is monotonic decreasing with increasing p_m .

215

216 **Proving that the NR model transforms the unstable equilibrium more than the SC model**

217 Since all the unstable equilibria for any given value of p_m converge to a single s_f/s_h when $\beta_{sc}=1$, the largest value of p_m , i.e. 1, will give rise to the greatest
218 change in unstable equilibrium for all values of β_{sc} . This is because the unstable equilibrium increases the most at $p_m = 1$, when $\beta_{sc} < 1$, and decreases the most
219 at $p_m = 1$, when $\beta_{sc} > 1$. Thus the unstable equilibrium for the sperm competition model with maximum two male mates evaluated at $p_m = 1$, when $\beta_{sc} < 1$, is

$$220 \quad \hat{p}_{sc} = \frac{-(1 + \beta_{sc})}{2(1 - \beta_{sc})} + \frac{1}{2} + \sqrt{\frac{(s_h - s_f)\beta_{sc}^2 + s_f}{s_h(1 - \beta_{sc})^2}}$$

221 OR when $s_h = ks_f \leq 1$, for $k \geq 1$

222

$$\hat{p}_{sc} = \frac{\sqrt{\beta_{sc}^2 + \left(\frac{1}{k}\right)(1 - \beta_{sc}^2)} - \beta_{sc}}{(1 - \beta_{sc})},$$

223 while for $\beta_{sc} > 1$

224

$$\hat{p}_{sc} = \frac{\beta_{sc} - \sqrt{\beta_{sc}^2 + \left(\frac{1}{k}\right)(1 - \beta_{sc}^2)}}{(\beta_{sc} - 1)}.$$

225 Recall that the non-random mating due to difference in male fitness model, NR, that the unstable equilibrium is:

226

$$\hat{p}_{nr} = \frac{s_f}{s_f + \beta_{nr}(s_h - s_f)},$$

227

$$\hat{p}_{nr} = \frac{1}{1 + \beta_{nr}(k - 1)}.$$

228 Both models converge at $\beta_{sc} = \beta_{nr} = 1$ and when $k = 1$ or $k \rightarrow \infty$. To show that the NR model always change the unstable equilibrium at a higher rate than the

229 SC, we need to show that:

230 1) when $\beta_{sc} = \beta_{nr} = \beta < 1$

231

$$\hat{p}_{nr} > \hat{p}_{sc},$$

232 2) while when $\beta_{sc} = \beta_{nr} = \beta > 1$

233

$$\hat{p}_{nr} < \hat{p}_{sc}.$$

234 Here, we show a partial proof for the above. Knowing that the unstable equilibrium of both models only converge at one estimate of β , we proceed to show
 235 that both unstable equilibrium expressions are monotonic with increasing β . We then evaluate the unstable equilibrium at two particular values of β that are
 236 less than 1 and more than 1 respectively, to demonstrate the above statements to be true:

237
$$\frac{d\hat{\beta}_{nr}}{d\beta_{nr}} = \frac{1 - k}{1 + \beta_{nr}(k - 1)} < 0.$$

238 Since $k > 0$, the unstable equilibrium under the NR model is monotonically decreasing with increasing values of β .

239 For $\beta_{sc} < 1$,

240
$$\frac{d\hat{\beta}_{sc}}{d\beta_{sc}} = \frac{-1}{(1 - \beta_{sc})^2} + \frac{1}{(1 - \beta_{sc})^2} \left(\frac{\beta_{sc} + \frac{1}{k}(1 - \beta_{sc})}{\sqrt{\beta_{sc}^2 + \frac{1}{k}(1 - \beta_{sc}^2)}} \right).$$

241 Multiply the equation with β_{sc}/β_{sc} and add $1/k - 1/k$ to the RH most numerator:

242
$$\frac{d\hat{\beta}_{sc}}{d\beta_{sc}} = \frac{-1}{\beta_{sc}(1 - \beta_{sc})^2} + \frac{1}{\beta_{sc}(1 - \beta_{sc})^2} \left(\frac{\beta_{sc}^2 + \frac{1}{k}(1 - \beta_{sc}^2)}{\sqrt{\beta_{sc}^2 + \frac{1}{k}(1 - \beta_{sc}^2)}} + \frac{\frac{1}{k}(\beta_{sc} - 1)}{\sqrt{\beta_{sc}^2 + \frac{1}{k}(1 - \beta_{sc}^2)}} \right),$$

243
$$\frac{d\hat{\beta}_{sc}}{d\beta_{sc}} = \frac{-1}{\beta_{sc}(1 - \beta_{sc})^2} + \frac{1}{\beta_{sc}(1 - \beta_{sc})^2} \left(\sqrt{\beta_{sc}^2 + \frac{1}{k}(1 - \beta_{sc}^2)} + \frac{\frac{1}{k}(\beta_{sc} - 1)}{\sqrt{\beta_{sc}^2 + \frac{1}{k}(1 - \beta_{sc}^2)}} \right).$$

244 Note that for all values of $k > 1$, the red part of the equation will always be less than 1, while the blue portion will always be less than zero, since $\beta_{sc} < 1$,

245 hence the expression in the large bracket will always be less than 1 implying that:

246
$$\frac{d\hat{p}_{sc}}{d\beta_{sc}} < \frac{-1}{\beta_{sc}(1-\beta_{sc})^2} + \frac{1}{\beta_{sc}(1-\beta_{sc})^2} = 0.$$

247 Thus the SC model is monotonically decreasing with increasing values of β for $\beta < 1$.

248 We can easily show the same for $\beta_{sc} > 1$:

249
$$\frac{d\hat{p}_{sc}}{d\beta_{sc}} = \frac{-1}{(1-\beta_{sc})^2} - \frac{1}{(1-\beta_{sc})^2} \left(\frac{\beta_{sc} + \frac{1}{k}(1-\beta_{sc})}{\sqrt{\beta_{sc}^2 + \frac{1}{k}(1-\beta_{sc}^2)}} \right) < 0.$$

250 Since both NR and SC model are monotonically decreasing with increasing values of $\beta_{sc} = \beta_{nr} = \beta$, and since we know that the two models only converge

251 when $\beta_{sc} = \beta_{nr} = \beta = 1$, we just have to compute the unstable equilibrium of the two models at $\beta < 1$ and $\beta > 1$

252 Let us take $\beta = 0$:

253
$$\hat{p}_{nr} = 1$$

254
$$\hat{p}_{sc} = \sqrt{\frac{1}{k}}$$

255 For $k > 1$:

256
$$\hat{p}_{nr} > \hat{p}_{sc}.$$

257 SHOWN

258

259 Next let $\beta = 2$:

260

$$\hat{p}_{nr} = \frac{1}{2k-1},$$

261

$$\hat{p}_{sc} = 2 - \sqrt{4 - \frac{3}{k}}.$$

262 When $k = 2 > 1$,

263

$$\hat{p}_{nr} = \frac{1}{2k-1} = \frac{1}{3} < 0.4188612 = \hat{p}_{sc}.$$

264 SHOWN

265

266 **Supplementary Material 8**

267 To show numerically that if a female has a very large number of male mates and the remating
 268 frequency is 1, then the SC model will appear to converge close to the NR model.
 269 Using the model described in equation 5 of the main text, we simulate for a particular scenario (see
 270 table below). As the number of maximum male mates increase the effective infection frequency in the
 271 SC model, it moves closer towards the NR model.

272

273 **Table S8.1**– Effective infection frequency under non-random mating due to a difference in male
 274 fitness model (NR) vs sperm competition model (SC) at different maximum number of male mates, n ,
 275 given the actual infection frequency, p_I .

p_I	Sperm competition model (SC)					
	NR model	$n = 2$	$n = 3$	$n = 5$	$n = 10$	$n = 1000$
0.1000	0.0526	0.0700	0.0631	0.0585	0.0554	0.0527
0.2000	0.1111	0.1467	0.1328	0.1232	0.1169	0.1112
0.3000	0.1765	0.2300	0.2097	0.1952	0.1854	0.1766
0.4000	0.2500	0.3200	0.2944	0.2754	0.2622	0.2501
0.5000	0.3333	0.4167	0.3875	0.3648	0.3486	0.3335
0.6000	0.4286	0.5200	0.4896	0.4648	0.4464	0.4287
0.7000	0.5385	0.6300	0.6013	0.5767	0.5576	0.5387
0.8000	0.6667	0.7467	0.7232	0.7020	0.6848	0.6669
0.9000	0.8182	0.8700	0.8559	0.8425	0.8310	0.8183

276

277 $\beta = 0.5, p_m = 1$

278

279 **Supplementary Material 9**

280 Based on Turelli ¹, the asymptotic geometric growth rate of the uninfected female population, λ , can
 281 be expressed in terms of the frequency of infected males:

$$\lambda^{\tau_U+1} = v_U \lambda^{\tau_U} + F_U (1 - s_h p),$$

$$p = \frac{v_U \lambda^{\tau_U} + F_U - \lambda^{\tau_U+1}}{F_U s_h},$$

282 where τ_U is the length of pre-reproductive females in days, v_U is the daily survival rate of uninfected
 283 females, F_U is the expected number of reproductive uninfected females per day, s_h is the reduction in
 284 hatch rate due to incompatible cross with infected males, and p is the frequency of infected males to
 285 be maintained over long periods to achieve a population decline rate of λ . Because λ is asymptotic, we
 286 assume that p is maintained as constant over long periods of time (e.g. on-going releases). $\lambda > 1$
 287 implies population growth while $\lambda < 1$ signify population decline, and $\lambda = 1$ implies no growth or
 288 decline. Since p is the infection frequency in males, it can be replaced with p_{nr} and p_{sc} to estimate λ
 289 under AM/NR and SC respectively.

290 An economically relevant way to look at the impact of non-random mating on IIT is by calculating the
 291 relative decrease/increase in time required to achieve the same effective suppression. Let t be time in
 292 days elapsing since release of incompatible males, and r be the relative reduction in uninfected
 293 population size (at time t) to starting population size (when $t = 0$). Then, r can be quantified as $\lambda^t = r$,
 294 hence the time taken to reduce the uninfected population size by r times is $t = \log(r) / \log(\lambda)$. If Δ_3 is
 295 the percentage increase in time required to achieve reduction in an uninfected population by r times
 296 when there is non-random mating with parameter β relative to random mating:

$$\Delta_3 = \frac{\frac{\log(r)}{\log(\lambda(p|\beta))} - \frac{\log(r)}{\log(\lambda(p|\beta = 1))}}{\frac{\log(r)}{\log(\lambda(p|\beta = 1))}},$$

$$= \begin{cases} \left(\frac{\log(\lambda(p|\beta = 1))}{\log(\lambda(p|\beta))} - 1 \right) \times 100\% & , \quad \lambda(q|\beta) < 1 \\ +\infty & , \quad \lambda(q|\beta) \geq 1 \end{cases}$$

297 If $\lambda(p|\beta) \geq 1$, suppression is impossible if the infection is maintained at frequency p , thus Δ_3 must be
298 undefined for $\lambda(p|\beta) \geq 1$.

299 The estimate in the text is based on $\beta = 0.6410$ and release ratio of 20 infected males to 1 (i.e. $p =$
300 $20/21$). We can then estimate the effective infection frequency under the NR model based on equation
301 4 in the main text for which we get $p_{nr} = 0.9276$. Using the estimates from Turelli ¹, of $\tau_U = 19$, $\nu_U =$
302 0.85 , $F_U = 0.269$, $s_h = 1$, we estimate using the first equation in this section the two population growth
303 rate, λ , when $p = 20/21$ and $p_{nr} = 0.9276$. This gives $\lambda(p|\beta=1)=0.9168$ and $\lambda(p|\beta) = 0.9290$. Then using
304 the equation above, the increase in time require to achieve the same level of suppression is 17.91%.

305

306 REFERENCE

- 307 1. Turelli M, 2010. Cytoplasmic incompatibility in populations with overlapping generations.
308 Evolution 64: 232-241.

309