Battiston et al, Multiplex core-periphery organization of the human connectome
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Supplementary text S1

Robustness of the rich-core detection

To test the robustness of our method, we have implemented a simulation model allowing random fluctuations in the distributions of values of the multiplex richness μ^+. In particular, we have modified the value of each node i so that the new $\mu_i^+ = \mu_i^+ + \eta_i \mu_{i,\max}^+$. Here, $\mu_{i,\max}^+$ is the maximum value of the original richness and η_i is a random variable within the range $[-\eta_{\max}, \eta_{\max}]$, where $\eta_{\max}$ is a tunable parameter ranging from 0 to 1. Hence, when $\eta_{\max} = 0$, the richness of the nodes is not altered; when $\eta_{\max} = 1$, the richness of the nodes is independently and maximally altered by a random factor within the range $[-\mu_{i,\max}^+, \mu_{i,\max}^+]$.
We have applied this simulation model to the multiplex richness values of the brain networks illustrated in the Figure 3 of the main text and we have checked the composition (i.e., the size) of the core as a function of $\eta_{\max}$. Notably, for each value of $\eta_{\max}$ we generate 100 random samples. Results show that the average core size is relatively stable for a broad range of $\eta_{\max}$ values (Fig. 1a). Notably, fluctuations are completely negligible until $\eta_{\max} = 0.04$, and have a high chance to significantly modify the identified rich core only when they are larger than 0.1 (Fig. 1b).

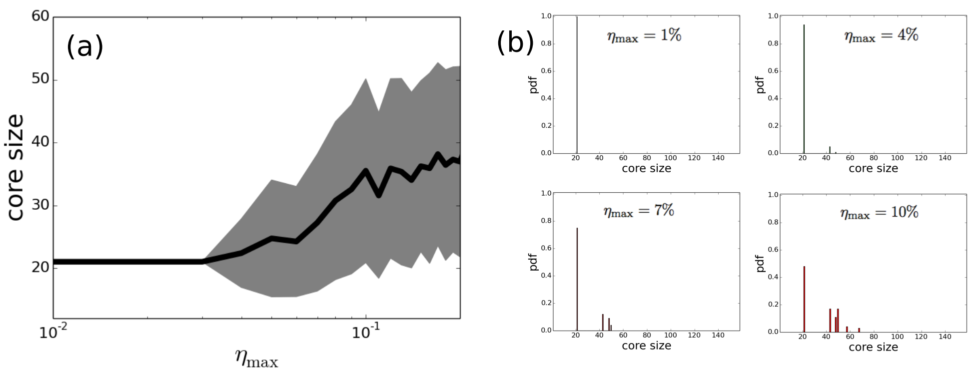

Figure 1: (a) Average core size in the multiplex brain network with $\langle k \rangle = 7$ as a function of the fluctuations parameter $\eta_{\max}$. (b) Cumulative density function of the core size for four selected values in the range $0.01 \leq \eta_{\max} \leq 0.1$.

Finally, we have compared the main results with those obtained through an alternative approach where the core-periphery threshold is selected according to a statistical criterion. To this purpose, we have generated 100 degree-constrained random networks from both structural and brain networks. We have then normalized the actual μ^+ values with respect to those obtained from the random samples according to a standard

Z-score $z(\mu_i^+) = \frac{\mu_i^+ - \bar{\mu}^+}{\sigma(\mu^+)}$. We eventually report that the regional coreness (on which all the main results are based) is relatively stable regardless whether we have considered the maximum from the actual or normalized values of μ^+ (Fig. 2).

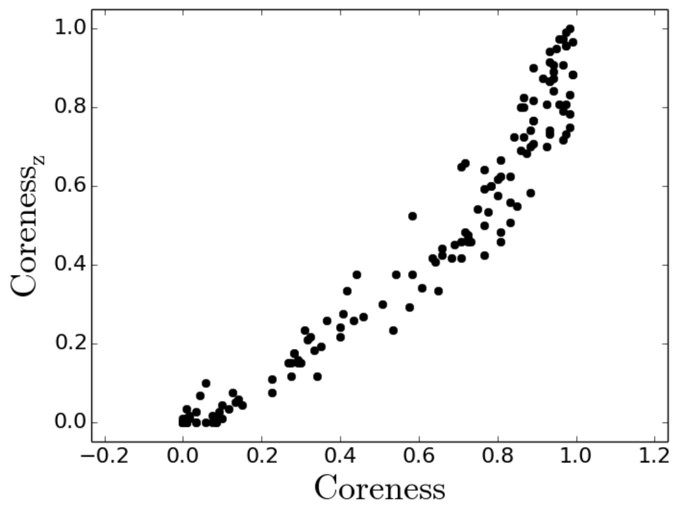

Figure 2: Scatterplot of the multiplex coreness obtained by looking at the maximum values of μ^+ and that obtained from the maximum of the corresponding Z-scores (Coreness z). The two measures are extremely correlated, $\rho_s = 0.97, p = 1.04 \times 10^{-80}$.