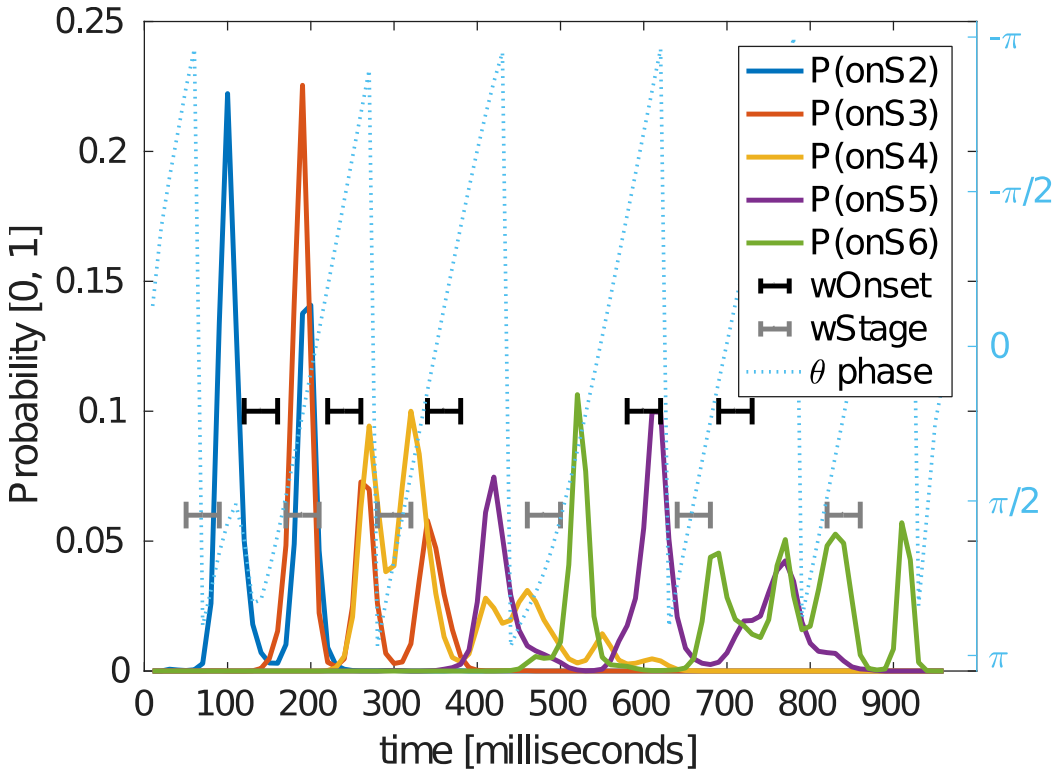
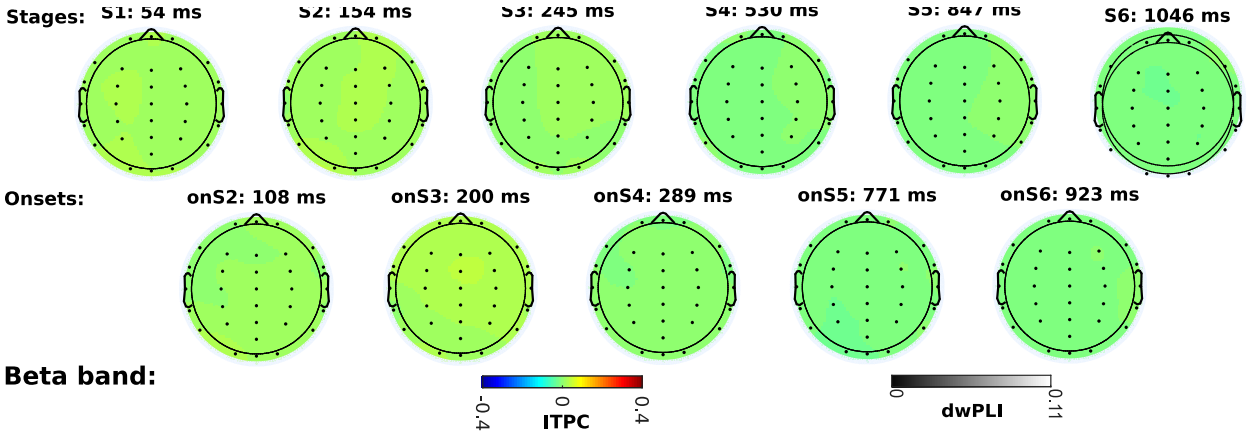


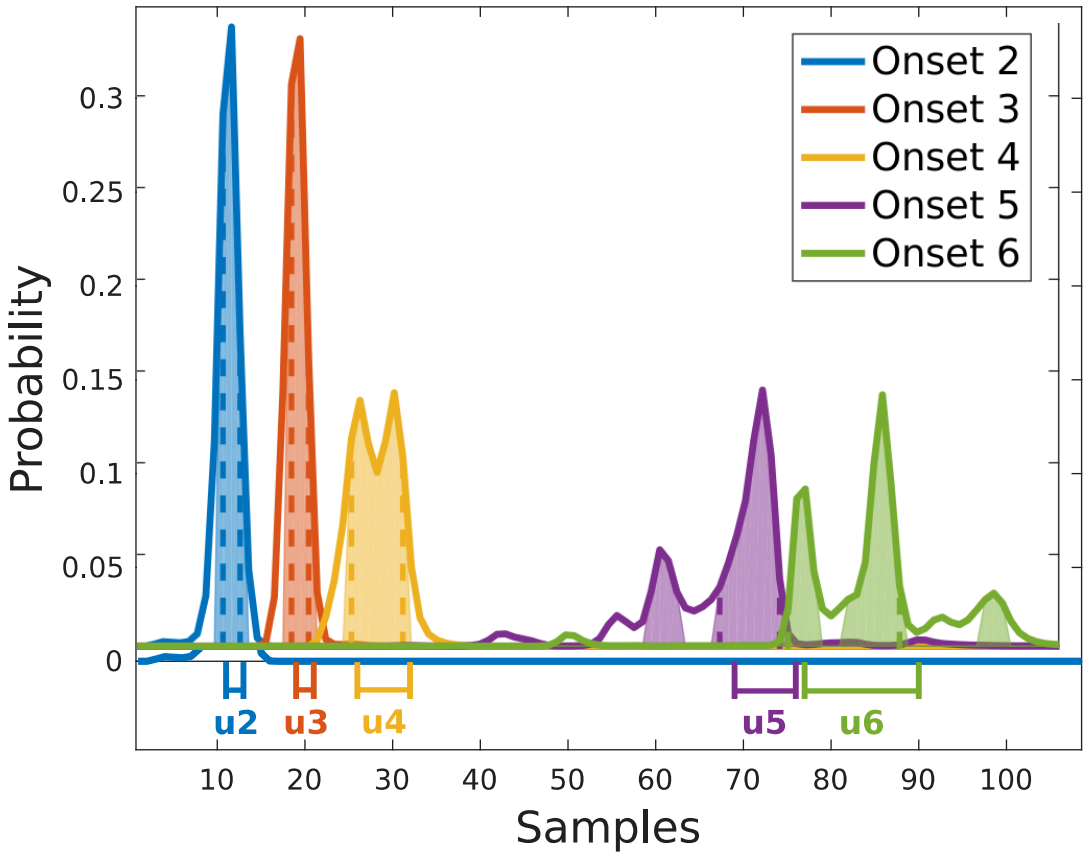

**Supplementary Figure S1**.

Example of one trial EEG phase (dotted, light blue), the probability distributions for the location of cognitive stages' onsets (solid lines), and the analysis windows of relevant cognitive events (brackets). The blue-dashed line is the theta band phase during a trial in one electrode. The solid lines are the probability distributions given by the MVPA-HSMM about where the onsets of cognitive stages are located (*P(onSX)*). The probability distributions of the onsets have multimodal distributions. Black indicators (*wOnset*) show the five-sample windows around the expected location of an onset used to compute Power, ITPC, and dwPLI. Similarly, gray indicators show the five-sample windows that define the respective stages (*wStage*). Estimating these time windows may be inaccurate given multimodal probability distributions.



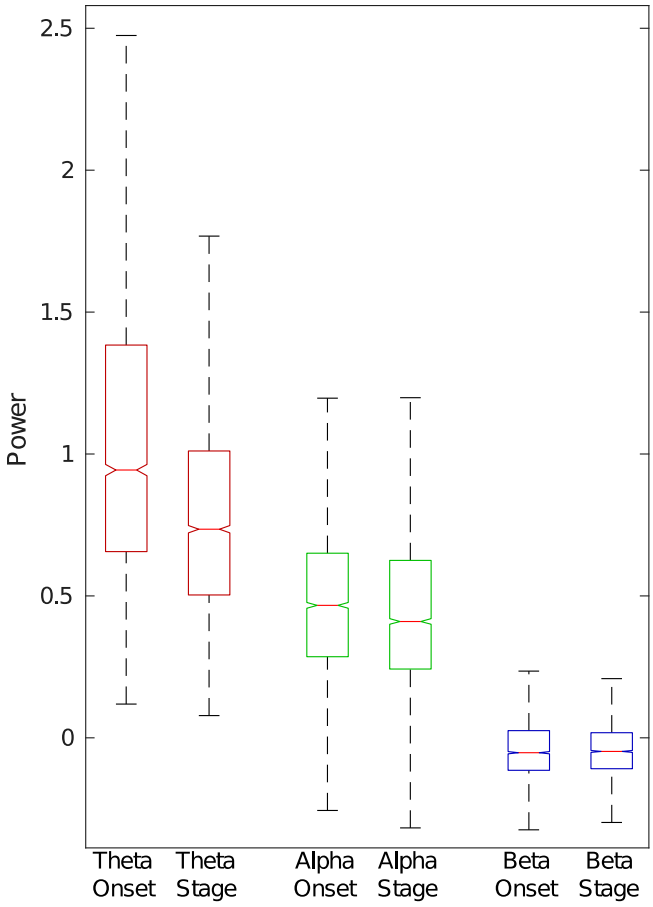

**Supplementary Figure S2.**

Beta band phase locking (*ITPC*: inter-trial phase clustering), and phase connectivity (*dwPLI*: de-biased weighted phase-lag index). It shows scalp profiles of grand-averaged ITPC values subtracted from baseline values. The row above has the centers of stages, and the row below the onset of stages. ITPC have the same color scale as Figures 5 and 6. There is not any phase connectivity edge that is consistent across a minimum of subjects (i.e. 75 percent of the subjects). The mean temporal location of each scalp map is depicted above them (e.g. *S1: 54 ms*: The stage one lays on average 54 millisecond after the onset of the trial)




**Supplementary Figure S3.**

Example of the measurement of uncertainty in stage onset locations on one trial. Solid color lines represent the probability of finding the onset of a stage over the trial samples. The semi-transparent area underneath a probability distribution with the same color is the core of the modes in the probability distribution. The core of the modes show the samples that remain after removing the samples that account for the first 25 percent of the probabilities sorted in ascending order. The dashed color lines mark the boundaries of the mode cores that represent at least the most likely 75 percent of the total area under all mode cores in a trial. The uncertainty about the location of an onset is the distance in samples between the two dashed lines of the same color. These uncertainties are marked with the symbols u2, u3,…,u6 on the same color as the probability distribution that they belong to. First and second onsets have sharp probability distributions, so their uncertainty is low. The third onset has higher uncertainty because its probability distribution is wider. The fourth probability distribution has two modes, but the area of the second mode represents more than 75 percent of the total area. So the second mode marks the boundaries for the uncertainty measurement. The fifth onset location has three modes, of which two are needed to account for the 75 percent of the area. So the two most extreme samples of these two modes mark the boundaries of the uncertainty measurement.




**Supplementary Figure S4**

Box plot comparing the power in the onset (*Onset*) of the stages and the power in the interval between onsets or trial edges (*Stage*). Red color boxes represent the theta band power. Green color boxes represent the alpha band power. Blue color boxes represent the beta band power. The upper and lower limits of a box represent the upper and lower quartiles respectively. The bright red line in the middle shows the median power. The notches around the median mark the 95% confidence intervals over the median. The whiskers represent the most extreme values not considered outliers. The box plot was drawn with the *boxplot* function in Matlab.