S.1. ESTIMATION OF FISHER INFORMATION

To trace the Fisher Information over a logarithmic scale, we substitute

$$u = \log_{10}(\nu) \tag{S.1}$$

so that

$$\frac{du}{d\nu} = \frac{1}{\nu \log 10} = \frac{10^{-u}}{\log(10)}. \tag{S.2}$$

This substitution is chosen for smoothness and resilience to noise as the difference between distributions becomes significant, the effect of noise on the calculation of the Fisher Information is diminished.

The Fisher Information of I, C and the joint random variable (I, C) with respect to ν can be expressed using the reparametrisation of the Fisher Information (15) as follows:

$$F_{I_M}(\nu) = F_{I_M}(\langle I^* \rangle) \left(\frac{d\langle I^* \rangle}{du}\right)^2 \left(\frac{du}{d\nu}\right)^2 \tag{S.3}$$

$$F_{I_O}(\nu) = F_{I_O}(u) \left(\frac{du}{d\nu}\right)^2 \tag{S.4}$$

Substituting equations (S.2) and (16) with $n = V$ and $q = \langle I^* \rangle$ into equations (S.3) and (S.4) yields

$$F_{I_M}(\nu) = \frac{V}{\langle I^* \rangle (1 - \langle I^* \rangle)} \left(\frac{d\langle I^* \rangle}{du}\right)^2 \left(\frac{1}{\nu \log 10}\right)^2 \tag{S.5}$$

and

$$F_{I_O}(\nu) = F_{I_O}(u) \left(\frac{1}{\nu \log 10}\right)^2 \tag{S.6}$$

When there is no closed form expression for the Fisher Information, it can be estimated numerically using suitable discretisations over system states $x_1, \ldots, x_n$. The derivative with respect to λ is calculated using a finite difference method with step length $\Delta\lambda$. In this paper we will use the backwards finite difference method to approximate the Fisher Information, denoted $\hat{F}_X(\lambda)$:

$$\hat{F}_X(\lambda) = \sum_{i=1}^{N} p(x_i; \lambda) \left(\frac{\log\left(p(x_i; \lambda)\right) - \log\left(p(x_i; \lambda - \Delta\lambda)\right)}{\Delta\lambda}\right)^2 \tag{S.7}$$

S.2. THERMODYNAMIC EFFICIENCY OF COMPUTATION η

To determine the thermodynamic efficiency of computation η, as defined in equation (24), we firstly identify the zero-response point(23). For our system it is simply zero, as there is no work expended in changing the transmission probability near zero, that is, $\nu^* = 0$. Hence, using (22), we obtain:

$$\frac{d\langle \beta W_{gen} \rangle}{d\nu} = -\int_0^{\nu} F_{I,C}(\nu') d\nu' \tag{S.8}$$

Again, we substitute $u = \log_{10}(\nu)$, resulting in:

$$\frac{dS}{d\nu} = \frac{dS}{du}\frac{du}{d\nu} = \frac{dS}{du}\frac{10^{-u}}{\log 10} \tag{S.9}$$

and

$$\begin{aligned} \frac{d\langle \beta W_{gen} \rangle}{d\nu} &= -\int_{u(0)}^{u(\nu)} F_{I,C}(\nu'(u')) \frac{d\nu'}{du'} du' \\ &= -\int_{u(0)}^{u(\nu)} F_{I,C}(\nu'(u')) 10^{u'} \log(10) du' \end{aligned} \tag{S.10}$$

These quantities are both calculated numerically for 100 values of u between $-4 \leq u \leq -2$. With $\frac{dS}{d\nu}$ calculated numerically using a backwards difference method, and $d\langle \beta W_{gen} \rangle / d\nu$ calculated using a cumulative trapezoidal numerical integration.

The entropy in proximity of the critical point was approximated using a nonlinear least-squares fit to a logistic growth curve. The fitted values for the entropy were used exclusively for the calculation of the numerical derivative dS/du in the calculation of η in the interval $\nu = [1.26 \times 10^{-4}, 2.21 \times 10^{-4}]$.

We approximate the entropy of (I, C) as a logistic growth curve for $\nu = 1 \times 10^{-4}$ to 2.9×10^{-4}. Explicitly, we fit

$$S_{fit} = \frac{L}{1 + e^{-k(\nu - \nu_0)}}. \tag{S.11}$$

Using the MATLAB curve-fitting tool, we obtain a curve of best fit with $L = 6.12$, $k = 1.67 \times 10^5$ and $\nu_0 = 1.855 \times 10^{-4}$, see Fig. S.1. For the data on $\nu = 1 \times 10^{-4}$ to 2.9×10^{-4}, this curve has an R-squared value of 0.99.

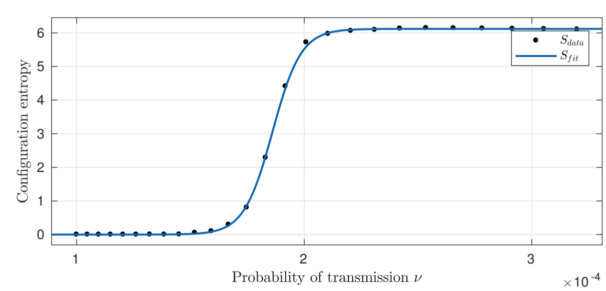

FIG. S.1: Comparison of configuration entropy of the joint random variable (I, C) and the best fit S_{fit}.

S.3. RELATING R_0 TO PROBABILITY OF INFECTION

We show that, for the system described in this study, the reproductive ratio R_0 can be expressed analytically as a function of ν, δ and k, the average degree of the graph. Let Y be the event when some infected individual x, before recovering from the infection, infects a neighbour y. Let us also denote the degree of x as k_x. Then

$$R_0 = E[k_x E[Y]]. \tag{S.12}$$

In the case of the Watts-Strogatz graph, the average degree within the population is equal to k, and so

$$R_0 = kE[Y] \tag{S.13}$$

In this case, a closed form of $P(Y)$ is given by

$$\begin{aligned} P(Y) &= \nu + \nu(1-\nu)(1-\delta) + \nu(1-\nu)^2(1-\delta)^2 + \dots \\ &= \nu \sum_{j=0}^{\infty} (1-\nu)^j (1-\delta)^j \end{aligned} \tag{S.14}$$

where the j^{th} term of the sum represents the probability of j consecutive unsuccessful infections of y by x, and j unsuccessful recoveries of x, followed by one successful infection. Since ν is in the interval $[0,1]$ and δ is in the interval $[0,1]$, this geometric series converges to

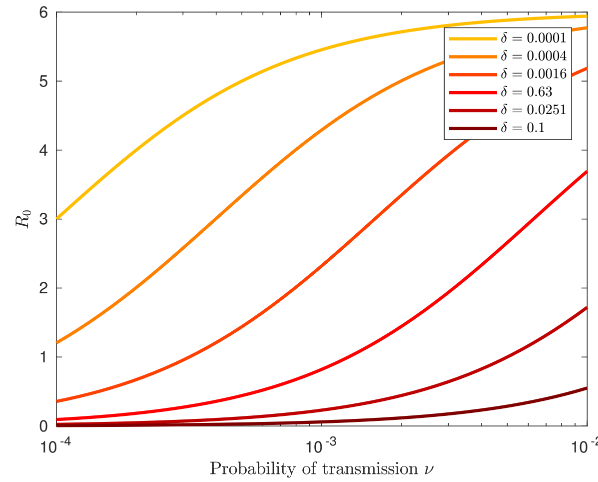

FIG. S.2: The reproductive ratio R_0 as a function of ν varying δ

$$\frac{\nu}{1-(1-\nu)(1-\delta)} = \frac{\nu}{\nu+\delta-\nu\delta} \tag{S.15}$$

Thus,

$$R_0 = \frac{k\nu}{\nu+\delta-\nu\delta} \tag{S.16}$$