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Supplementary Materials for

Field responsive mechanical metamaterials

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The PDF file includes:

Other Supplementary Material for this manuscript includes the following:

(available at advances.sciencemag.org/cgi/content/full/4/12/eaau6419/DC1)

Movie S1 (.mov format). Video of infilling a cuboctahedron lattice composed of a 2 by 2 by 2 arrangement of unit cells.
Movie S2 (.mp4 format). Video of a cuboctahedron lattice with a 10-g mass placed on its top surface and the magnetic field strength gradually lowered by slowly removing a magnet.

Supplementary Text

Modeling approach

Our modeling approach assumes the following about the response of the fluid-filled polymer tubes:

1) the AM tube material remains linear elastic;
2) the struts are cylindrical;
3) there is no fluid flow inside the structures – the system is sealed and the fluid responds only by developing hydrostatic pressure;
4) the fluid is incompressible, relative to the polymer shell;
5) the fluid is treated as a viscoelastic material with complex moduli controlled by the external applied magnetic field because it does not flow;
6) the materials are used and tested in a quasi-static environment, so the viscoelasticity of the fluid can be ignored in favor of a linear elastic treatment and all time-dependent magnetic properties of the MR fluid will be dealt with in the static limit;
7) the response of the fluid to applied field strength can be approximated as linear in the magnetic field ranges we are investigating;
8) the bending, torsional, and axial modes of deformation of the individual struts can be decoupled;
9) a simple mixing law is used to account for directional effects.

From these assumptions, this section develops a model for the response of an individual strut that can be calibrated to experimental data.

This model takes the form of a frame element – a structural theory that describes the response of a slender member to axial and transverse forces combined with bending and torsional moments. Unlike typical frame elements used to model the response of AM lattice materials (*10)*, the elastic stiffness of this element changes with the application of an external magnetic field. Individual frame elements can then be assembled into a system model of a MR-filled cell by the direct stiffness method of structural analysis. The system model predicts the linear elastic response of a unit cell under a combination of mechanical loading and external, applied magnetic field.

Strut model

Figure S1 illustrates the assumptions made when creating the frame element describing the response of a single strut. The figure illustrates (1) the field direction assumed to have the greatest effect on the material response for each type of loading (torsional, axial, bending), (2) the corresponding chain orientations inside the fluid, and (3) the equivalent linear elastic material structure used to model the deformation of the strut at a given field strength.

A complete, 3D frame element must consider four deformation modes: axial deformation, bending in two orthogonal directions, and torsion. Figure S1 illustrates three types of deformation – the final model uses the same formulation for bending into the page as it does for bending in the page.

The model assumes the magnetic field will not affect the torsional response of the cylindrical struts. The torsional response of the section is then the response of a circular annulus. This is given by

$$M = \frac{\pi G_{bulk}}{2L}(r_o^4 - r_i^4)\theta$$

with M the torsional moment, G_{bulk} the shear modulus of the as-cured polymer resin, L the length of the strut, r_o and r_i the outer and inner radii of the tube, and θ the relative twist angle between the ends of the strut.

For axial deformation, the model describes the effect of the MR fluid with a composite strut consisting of the outer polymer shell acting together with an effective linear elastic material representing the effect of the fluid. The Young's modulus of the effective material representing the fluid increases linearly with field strength. Even with the field off, the effective fluid material will have some axial stiffness because the fluid exists in a sealed system and will therefore develop hydrostatic stress under load. The axial effective Young's modulus of the fluid is modeled by

$$E_{MR} = E_{min} + k_E B_E^{eff}$$

with E_{MR} the effective Young's modulus, an B_E^{eff} effective scalar magnetic field strength, described below, and E_{min} and k_E empirical constants, measured experimentally.
In the system, this effective material representing the fluid will act together with the AM polymer tube. The formula

$$F = \pi\{(r_o^2 - r_i^2)E_{bulk} + r_i^2 E_{MR}\}\frac{d}{L}$$

describes the composite action of the system under axial load. Here F is the axial force in the strut, E_{bulk} is the as-cured polymer Young's modulus, d is the relative axial displacement of the ends of the strut, and the other quantities are defined previously.

The term B_E^{eff} is designed to account for the magnetic field being applied in some direction other than the direction of maximum effect. This situation will occur in a practical unit cell – not all the struts can be aligned with the field direction. Figure 2a-2b illustrates the assumption made for calculating the effective field in the axial case: a magnetic field applied transverse to the strut will produce no increase in axial stiffness, regardless of the field strength. Figure 2 also illustrates the reasoning behind this assumption. A transverse field will produce transverse chains spanning the width of the section. These chains, presumably, will not axially stiffen the member, though they may influence the buckling properties of the system. A reasonable model for the effective field strength, given this assumption, is

$$B_E^{eff} = |\boldsymbol{B} \cdot \boldsymbol{n}|$$

with $\boldsymbol{B}$ the vector describing the applied magnetic field and $\boldsymbol{n}$ the strut normal direction (see fig. S1).

The model assumes the Euler-Bernoulli model for the strut response to transverse forces and bending moments. With this model the product of the effective section Young's modulus times the effective section moment of inertia $(EI)_{eff}$ controls the bending response. The model assumes the changing effective shear modulus of the fluid with applied magnetic field controls the effect of the fluid on the member bending response. For an incompressible fluid, a change in shear modulus will manifest as a change in axial stiffness through the relation

$$E_{bend} = \lim_{K_{MR}\to\infty} \frac{9K_{MR}G_{MR}}{3K_{MR} + G_{MR}} = 3G_{MR}$$

Assuming the fluid will act as an effective linear elastic material with a shear modulus G_{MR} controlled by the applied magnetic field and using the method of transformed sections to calculate the effective properties of the composite section yields an effective bending constant of

$$(EI)_{eff} = \frac{\pi}{4}\left(E_{bulk} r_o^4 - (E_{bulk} - 3G_{MR}) r_i^4\right)$$

The model represents the effect of the applied field on the fluid effective shear modulus similarly to the axial effect

$$G_{MR} = G_{min} + k_G B_G^{eff}$$

with G_{min} and k_G empirical constants and B_G^{eff} the effective field strength accounting for directional effects. Figure 2 illustrates the assumed model for the influence of field direction on bending stiffness: maximal effect if the field is aligned with the bending displacement direction and zero effect if it is applied orthogonal to this direction. This geometric effect can be modeled with

$$B_G^{eff} = |\boldsymbol{B} \cdot \boldsymbol{s}|$$

with $\boldsymbol{s}$ the direction shown in fig. S1. This assumes the increase in bending stiffness is primarily due to shear effects. A more complete model would directly represent the effect of shear stiffening on bending response using Timoshenko's theory.

A complete frame element combines the individual models for torsion, axial deformation, and bending in two orthogonal directions into a matrix describing the complete response of the strut under any combination of forces and bending moments. It assumes the responses are uncoupled -- axial deformation does not affect bending, and so on – and that the same empirical constants describe bending in both orthogonal directions.

Calibrating the strut model to experimental data

Calibrating the model requires determining six parameters: E_{bulk}, G_{bulk}, E_{min}, k_E, G_{min}, and k_G. The E_{bulk} and G_{bulk} are material properties of the as-cured polymer resin. The remaining parameters are dependent both on the properties of the MR fluid and on the geometry of the struts – both cross section shape and strut length – because the strut geometry controls the length and spacing of the MR fluid chains. Therefore, we calibrated all the parameters empirically to simple, single-strut experiments, as described in the main body of the paper.

Lattice model

The main text describes extending this single strut model to a model of a periodic lattice structure. The final model describes the response of a structure composed of MR fluid filled struts given the applied forces, displacements, and magnetic field vector.

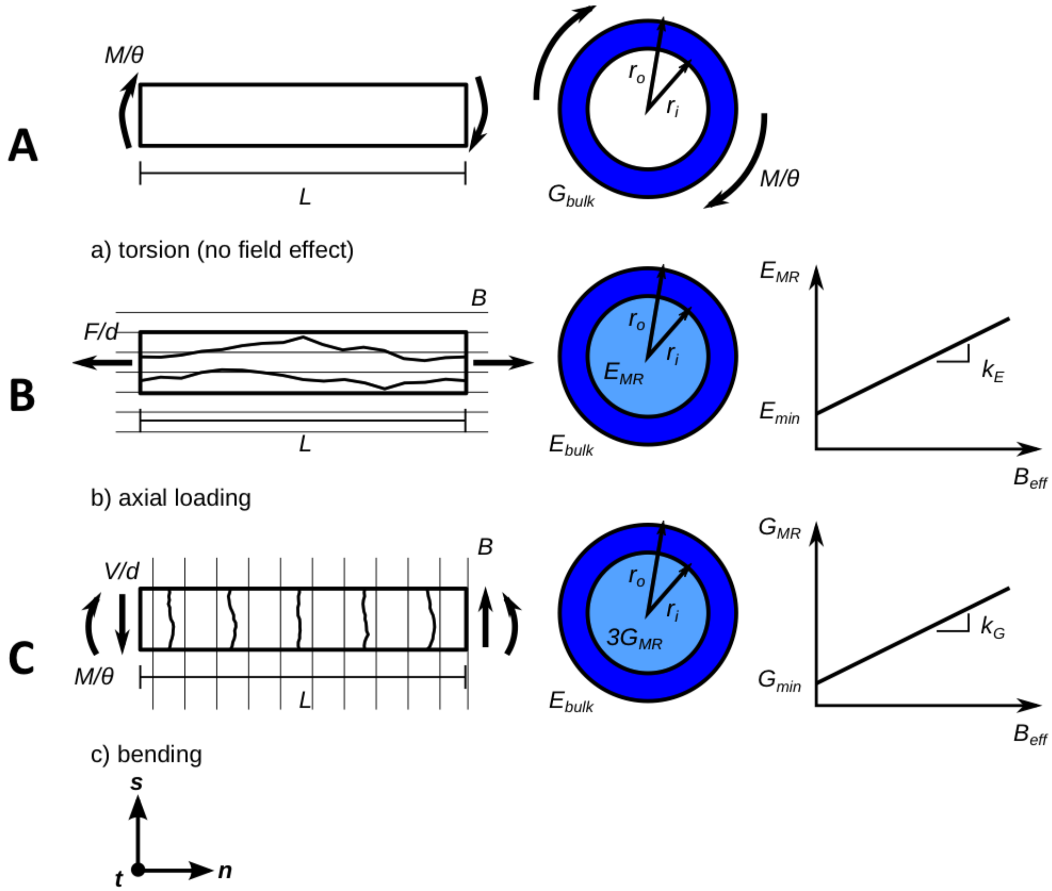

Fig. S1. Assumptions made when assembling the frame element describing a single strut. (**A**-**C**) Schematic illustration of the magnetic field effects. (A) No field effect when the struts are in torsion. (B) Axial field effect when the chains are aligned with the direction of the force. (C) Bending field effect when the chains are aligned with the direction of the applied force.

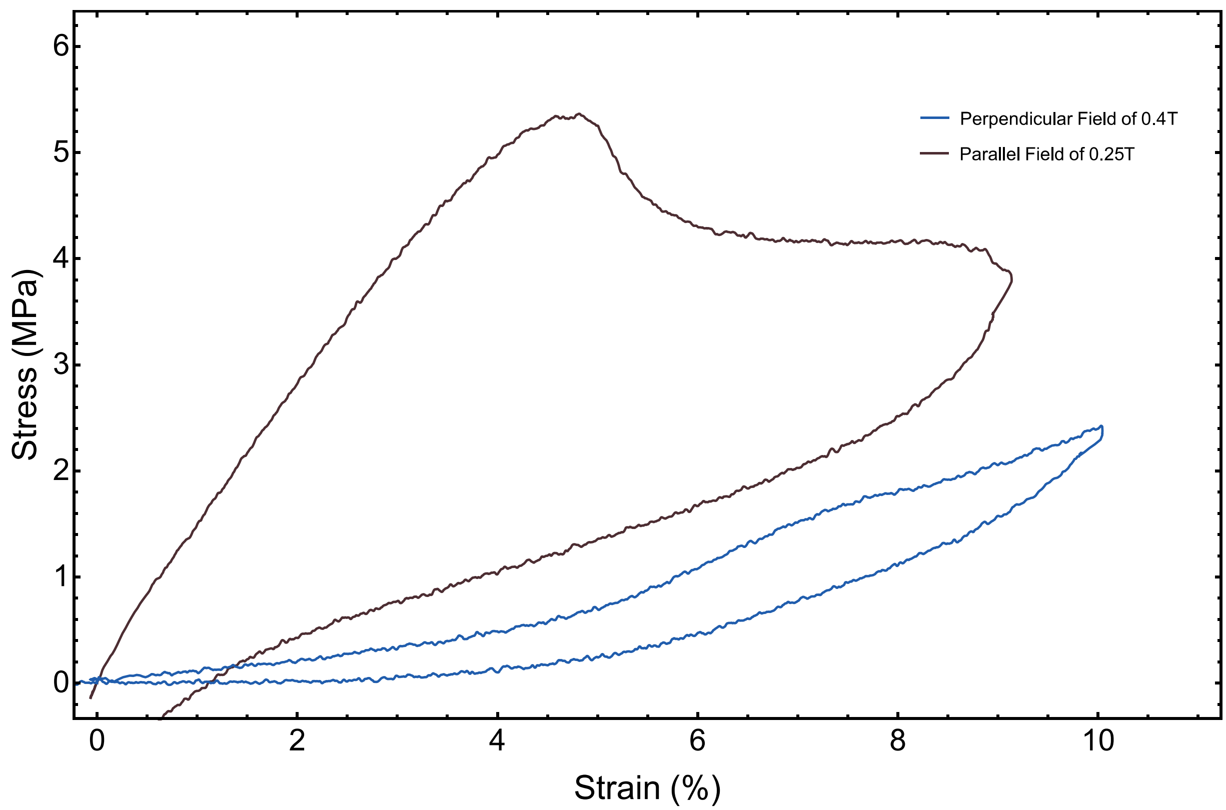

Fig. S2. Mechanical testing of MR fluid–filled struts under varying magnetic field orientations. Stress vs. strain plot of a polymer strut with a 1mm inner diameter, 50 µm thick walls and 1 mm length (1:1 aspect ratio to create similar chain lengths in all directions) under uniaxial compression with a magnetic field applied parallel and perpendicular to the direction of the force.

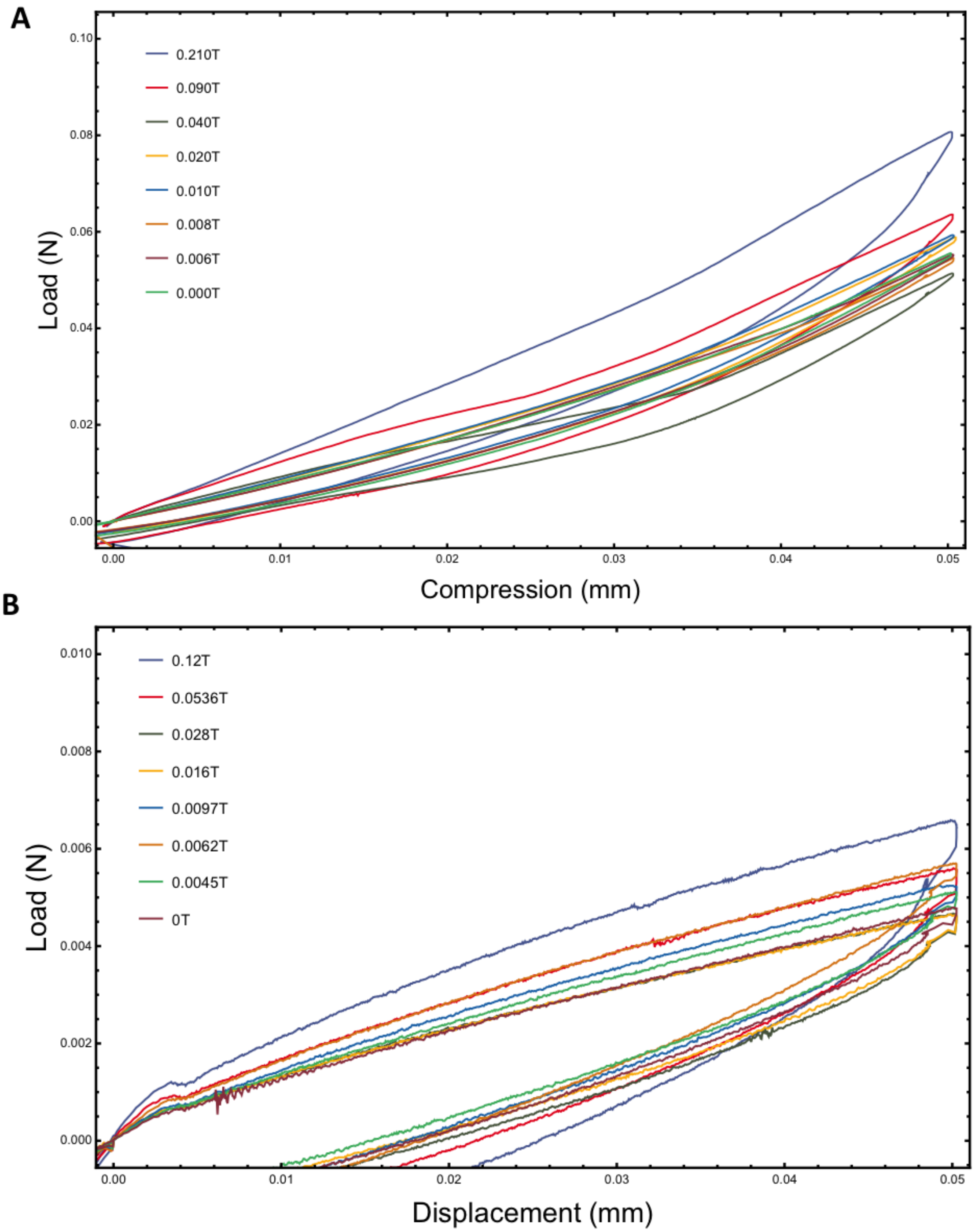

Fig. S3. Mechanical testing of MR fluid–filled struts under varying magnetic field strengths. (**A**) Load versus compression plot for one example of a strut tested under uniaxial compression. (**B**) Load versus displacement plot for one example of a strut tested under cantilevered bending.

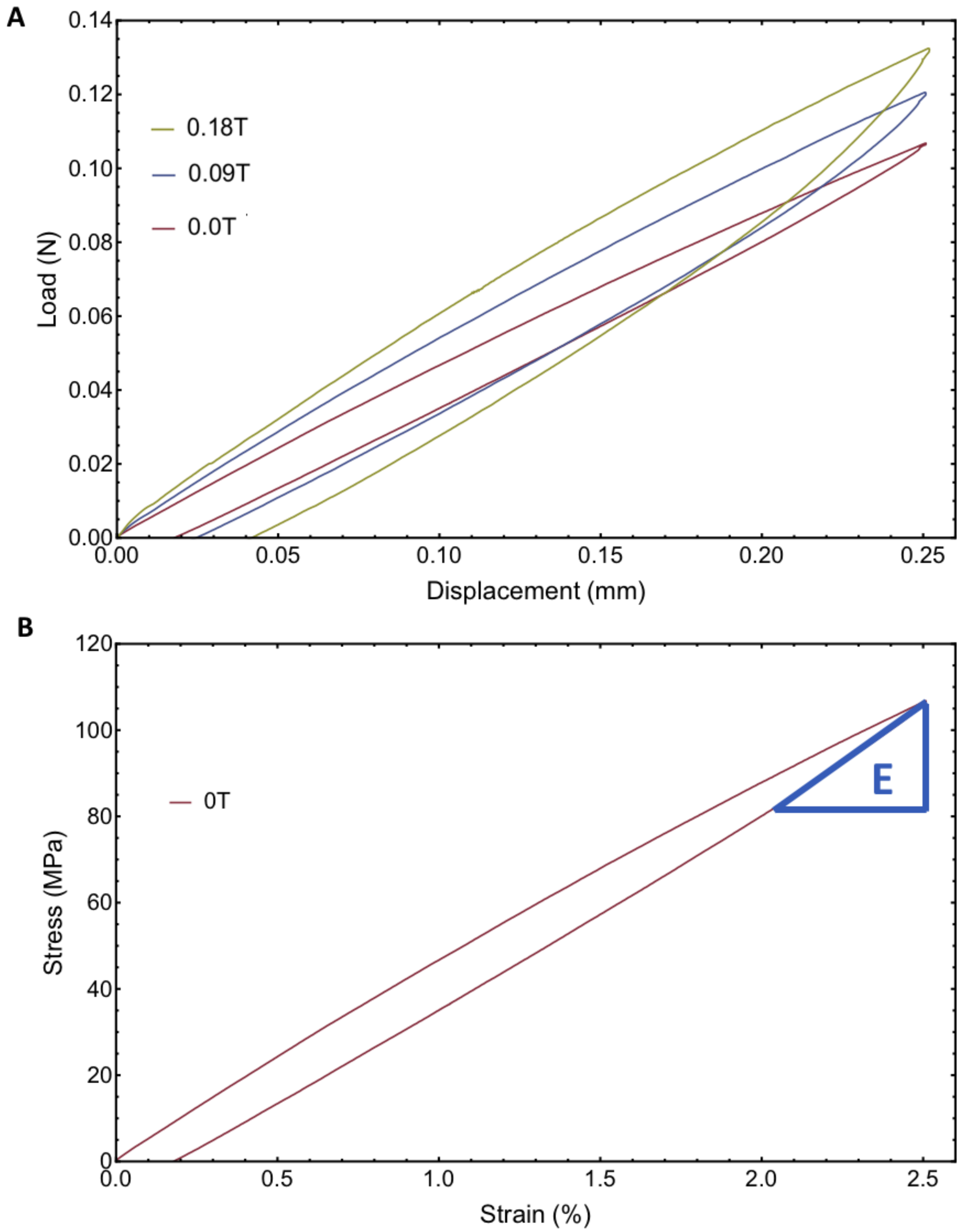

Fig. S4. Mechanical testing of MR fluid–filled cuboctahedron unit cells under varying magnetic field strengths. (**A**) Load versus compression plot for one example of a unit cell tested under uniaxial compression. (**B**) Example of how the Young's modulus was calculated by the unloading curve of the stress versus strain graph.

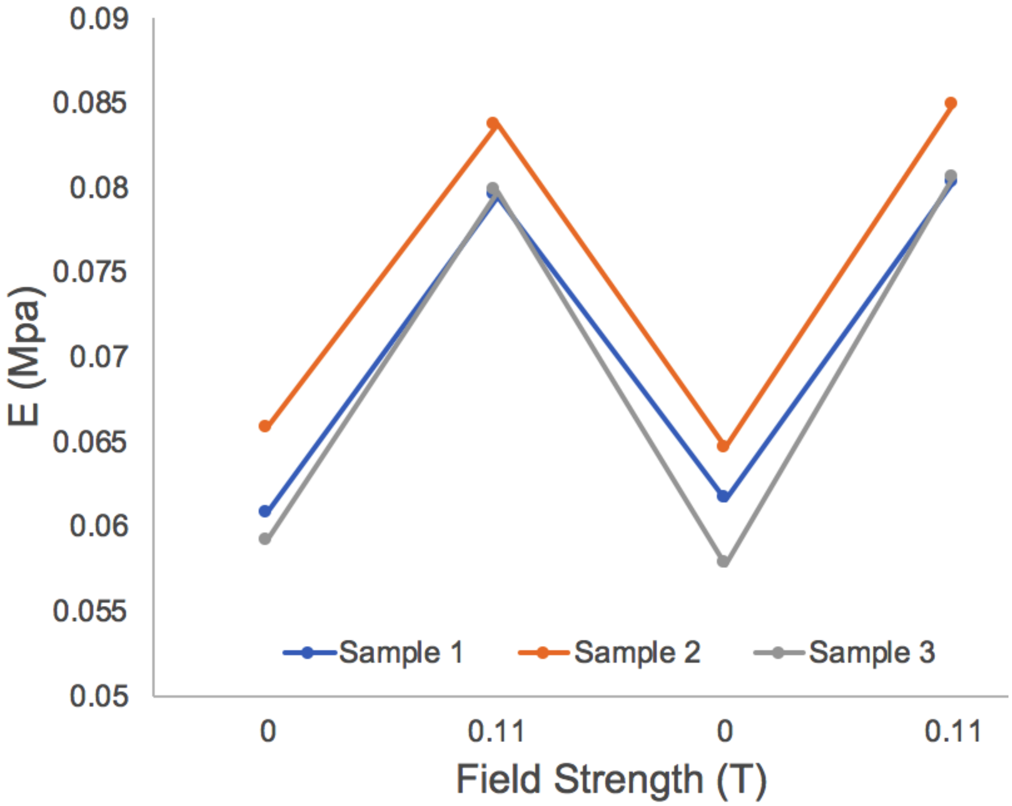

Fig. S5. Mechanical testing for reversibility of MR fluid–filled cuboctahedron unit cell. Three polymer cuboctahedron unit cells were cycled through "on" and "off" field states to display reversibility.

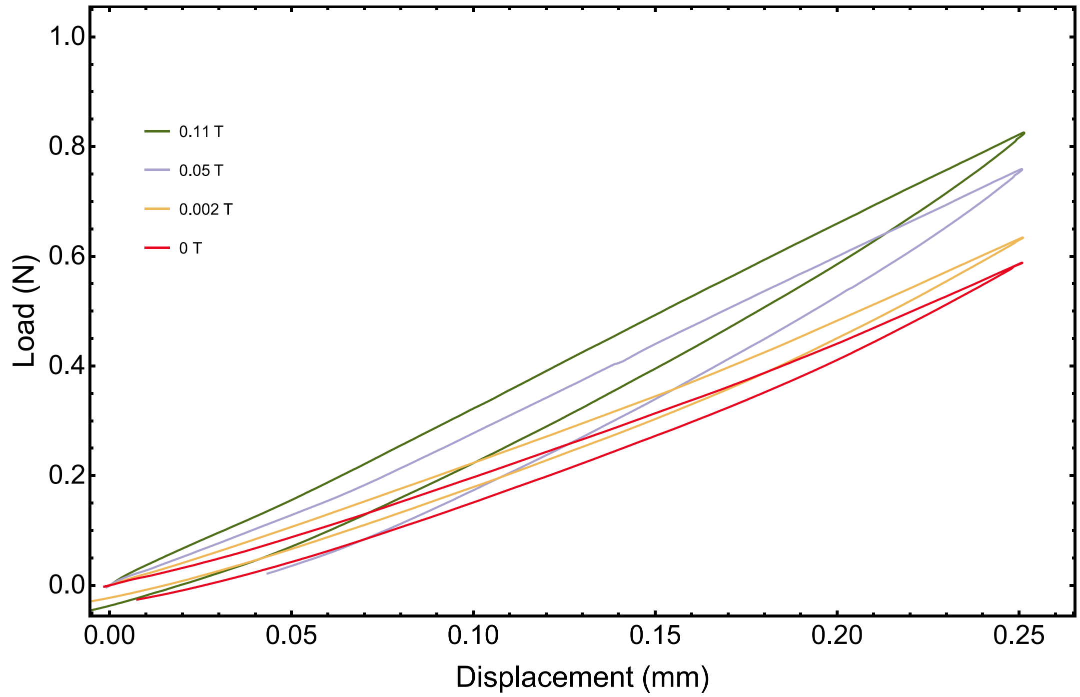

Fig. S6. Mechanical testing of MR fluid–filled cuboctahedron lattice under varying magnetic field strengths. Load versus compression plot for one example of a lattice tested under uniaxial compression.

Movie S1. Video of infilling a cuboctahedron lattice composed of a 2 by 2 by 2 arrangement of unit cells. LLNL-VIDEO-751076.

Movie S2. Video of a cuboctahedron lattice with a 10-g mass placed on its top surface and the magnetic field strength gradually lowered by slowly removing a magnet. LLNL-VIDEO-751076.