

S1 Appendix.

Here we show how repeatedly solving Eq. 3 is equivalent to solving one instance of Eq. 2. The relationship between these two equations can be seen by examining the relevant quantities from an elementwise perspective,

$$\|V - WX\|_F^2 = \sum_{i=1}^m \sum_{j=1}^n \left(V_{ji} - \sum_{l=1}^k W_{jl} X_{li} \right)^2 = \sum_{i=1}^m \|V_i - WX_i\|_2^2 \quad (1)$$

Note that minimizing each term in the sum on the right hand side of Eq. 1 as a function of X_i amounts to solving an instance of Eq. 3 – the \mathbf{q} in Eq. 3 is equivalent to X_i in Eq. 1. Each term in the sum on the right hand side of Eq. 1 depends only on one column of X , and each column of X appears in only one term. Therefore, when these terms are simultaneously minimized, the left hand side of Eq. 1 will be minimized as a function of X . Since minimizing the left hand side of Eq. 1 as a function of X is equivalent to solving Eq. 2, we can conclude that repeatedly solving Eq. 3 is equivalent to solving Eq. 2.