Appendix B. Additional Tables for Simulation Results

Tables showing the coefficients used for generating missing data B.6 and the power of the Wald test for testing $H_0 : \beta_1 = 0$ are provided below (See Table B.7 to B.11).

Table B.6: Parameters for simulation study

Method	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
10% missing data							
MCAR $^\mathcal{L}$	-2.2	0.0	0.0	0.0	0.0	0.0	0.0
MAR(Y) $^\sharp$	-3.4	0.0	0.0	0.0	0.0	0.0	1.8
MAR(X2)$^\flat$	-3.4	0.0	0.8	0.0	0.0	0.0	0.0
MAR(Y,X2)†	-3.5	0.0	1.2	0.0	0.0	0.0	-0.6
MNAR(X1)‡	-3.4	0.8	0.0	0.0	0.0	0.0	0.0
MNAR(X1,X2)$^\natural$	-3.4	0.4	0.4	0.0	0.0	0.0	0.0
MNAR(X1,Y)§	-3.4	1.2	0.0	0.0	0.0	0.0	-0.6
30% missing data							
MCAR $^\mathcal{L}$	-0.9	0.0	0.0	0.0	0.0	0.0	0.0
MAR(Y) $^\sharp$	-1.9	0.0	0.0	0.0	0.0	0.0	1.8
MAR(X2)$^\flat$	-2.0	0.0	0.8	0.0	0.0	0.0	0.0
MAR(Y,X2)†	-2.2	0.0	0.8	0.0	0.0	0.0	0.8
MNAR(X1)‡	-2.0	0.8	0.0	0.0	0.0	0.0	0.0
MNAR(X1,X2)$^\natural$	-2.0	0.4	0.4	0.0	0.0	0.0	0.0
MNAR(X1,Y)§	-2.4	0.8	0.0	0.0	0.0	0.0	0.8

$\mathcal{L}$ *missing completely at random*

$\sharp$ *missing at random conditional on* Y

$\flat$ *missing at random conditional on* $X2$

$\dagger$ *missing at random conditional on* $Y, X2$

$\ddagger$ *missing not at random conditional on* $X1$

$\natural$ *missing not at random conditional on* $X1, X2$

$\S$ *missing not at random conditional on* $X1, Y$

Table B.7: Power of Wald test for testing $H_0 : \beta_1 = 0$, where β_1=0.69, β_2=0, $pr(X_1 = 1) = 0.5$, n=200, 1000 simulations.

Pr(Miss)		CCA	MI	EM	LLMI	LCMI-2	LCMI-3
10%	MCAR $^\pounds$	0.750	0.780	0.820	0.780	0.790	0.810
	MAR(Y) $^\sharp$	0.750	0.780	0.810	0.780	0.780	0.820
	MAR(X2)$^\flat$	0.750	0.800	0.820	0.800	0.790	0.830
	MAR(Y,X2)†	0.740	0.770	0.800	0.770	0.790	0.820
	MNAR(X1)‡	0.750	0.800	0.800	0.800	0.810	0.810
	MNAR(X1,X2)$^\natural$	0.740	0.790	0.820	0.790	0.800	0.820
	MNAR(X1,Y)§	0.780	0.820	0.840	0.820	0.820	0.830
30%	MCAR $^\pounds$	0.510	0.660	0.750	0.660	0.700	0.740
	MAR(Y) $^\sharp$	0.470	0.650	0.740	0.650	0.650	0.720
	MAR(X2)$^\flat$	0.480	0.670	0.780	0.670	0.660	0.740
	MAR(Y,X2)†	0.550	0.660	0.760	0.660	0.660	0.720
	MNAR(X1)‡	0.530	0.640	0.750	0.640	0.660	0.740
	MNAR(X1,X2)$^\natural$	0.520	0.670	0.760	0.670	0.670	0.730
	MNAR(X1,Y)§	0.340	0.470	0.600	0.470	0.470	0.690
50%	MCAR $^\pounds$	0.270	0.500	0.730	0.500	0.500	0.640
	MAR(Y) $^\sharp$	0.320	0.520	0.740	0.520	0.550	0.670
	MAR(X2)$^\flat$	0.210	0.470	0.680	0.470	0.480	0.610
	MAR(Y,X2)†	0.300	0.520	0.700	0.520	0.520	0.660
	MNAR(X1)‡	0.310	0.490	0.680	0.490	0.490	0.610
	MNAR(X1,X2)$^\natural$	0.310	0.500	0.730	0.500	0.560	0.650
	MNAR(X1,Y)§	0.180	0.360	0.560	0.360	0.340	0.570

$\pounds$ *missing completely at random*
$\sharp$ *missing at random conditional on* Y
$\flat$ *missing at random conditional on* $X2$
$\dagger$ *missing at random conditional on* $Y, X2$
$\ddagger$ *missing not at random conditional on* $X1$
$\natural$ *missing not at random conditional on* $X1, X2$
$\S$ *missing not at random conditional on* $X1, Y$

Table B.8: Estimates of $\hat{\beta}_1$ and its standard error (when $\beta_1 = 0$) for 1:1 matched study with n=200, $pr(X_1 = 1) = 0.5$ and $pr(X_2 = 1) = 0.5$, 1000 simulations.

	CCA		MI		EM		LLMI		LCMI-2		LCMI-3	
	beta	ASE	beta	ASE	beta	ASE	beta	ASE	beta	ASE	beta	ASE
	Pr(Missing)=10%											
MCAR [£]	0.008	0.284	0.005	0.270	0.002	0.254	0.005	0.270	0.002	0.269	0.001	0.261
MAR(Y) [♯]	0.013	0.285	0.005	0.271	0.002	0.253	0.005	0.271	0.004	0.268	0.002	0.260
MAR(X2) [♭]	0.004	0.284	0.004	0.271	0.010	0.254	0.004	0.271	0.002	0.269	0.002	0.262
MAR(Y,X2) [†]	0.003	0.291	0.002	0.274	0.004	0.253	0.002	0.274	0.003	0.271	0.000	0.262
MNAR(X1) [‡]	0.005	0.284	0.000	0.273	0.001	0.258	0.000	0.273	0.001	0.273	0.002	0.262
MNAR(X1,X2) [♮]	0.000	0.283	0.002	0.270	0.001	0.256	0.002	0.270	0.003	0.271	0.002	0.261
MNAR(X1,Y) [§]	0.033	0.288	0.029	0.276	0.035	0.260	0.029	0.276	0.034	0.275	0.012	0.263
	Pr(Missing)=30%											
MCAR [£]	0.036	0.374	0.016	0.316	0.010	0.255	0.016	0.316	0.006	0.306	0.003	0.282
MAR(Y) [♯]	0.001	0.383	0.009	0.318	0.012	0.253	0.009	0.318	0.003	0.310	0.004	0.280
MAR(X2) [♭]	0.003	0.389	0.007	0.324	0.010	0.254	0.007	0.324	0.002	0.317	0.004	0.289
MAR(Y,X2) [†]	0.008	0.389	0.007	0.322	0.010	0.254	0.007	0.322	0.024	0.312	0.014	0.284
MNAR(X1) [‡]	0.012	0.382	0.006	0.330	0.004	0.271	0.006	0.330	0.001	0.325	0.002	0.286
MNAR(X1,X2) [♮]	0.006	0.379	0.005	0.318	0.008	0.262	0.005	0.318	0.003	0.317	0.008	0.281
MNAR(X1,Y) [§]	0.137	0.393	0.141	0.335	0.133	0.272	0.141	0.335	0.134	0.328	0.043	0.289
	Pr(Missing)=50%											
MCAR [£]	0.000	0.570	0.000	0.390	0.010	0.260	0.000	0.390	0.010	0.370	0.000	0.320
MAR(Y) [♯]	0.020	0.560	0.000	0.380	0.010	0.260	0.000	0.380	0.010	0.370	0.010	0.310
MAR(X2) [♭]	0.010	0.650	0.010	0.420	0.000	0.260	0.010	0.420	0.010	0.430	0.000	0.350
MAR(Y,X2) [†]	0.020	0.570	0.020	0.380	0.010	0.260	0.020	0.380	0.010	0.370	0.020	0.310
MNAR(X1) [‡]	0.020	0.560	0.000	0.410	0.000	0.290	0.000	0.410	0.010	0.400	0.010	0.320
MNAR(X1,X2) [♮]	0.020	0.560	0.010	0.390	0.010	0.270	0.010	0.390	0.000	0.380	0.000	0.310
MNAR(X1,Y) [§]	0.140	0.580	0.150	0.420	0.180	0.290	0.150	0.420	0.160	0.420	0.050	0.330

£ missing completely at random
♯ missing at random conditional on Y
♭ missing at random conditional on $X2$
† missing at random conditional on $Y, X2$
‡ missing not at random conditional on $X1$
♮ missing not at random conditional on $X1, X2$
§ missing not at random conditional on $X1, Y$

Table B.9: Estimates of $\hat{\beta}_1$ and its standard error (when $\beta_1 = 0.69$) for 1:1 matched study with n=200, $pr(X_1 = 1) = 0.5$ and $pr(X_2 = 1) = 0.5$, 1000 simulations.

	CCA		MI		EM		LLMI		LCMI-2		LCMI-3	
	beta	ASE	beta	ASE	beta	ASE	beta	ASE	beta	ASE	beta	ASE
	Pr(Missing)=10%											
MCAR $^{\pounds}$	0.720	0.280	0.720	0.270	0.720	0.250	0.720	0.270	0.710	0.260	0.720	0.260
MAR(Y) $^{\sharp}$	0.720	0.280	0.710	0.270	0.710	0.250	0.710	0.270	0.710	0.270	0.710	0.260
MAR(X2)$^{\flat}$	0.720	0.280	0.720	0.270	0.710	0.250	0.720	0.270	0.710	0.260	0.710	0.260
MAR(Y,X2)†	0.720	0.290	0.710	0.270	0.710	0.250	0.710	0.270	0.720	0.270	0.720	0.260
MNAR(X1)‡	0.720	0.280	0.710	0.270	0.710	0.250	0.710	0.270	0.720	0.270	0.710	0.260
MNAR(X1,X2)$^{\natural}$	0.720	0.280	0.720	0.270	0.720	0.250	0.720	0.270	0.720	0.260	0.720	0.260
MNAR(X1,Y)§	0.740	0.280	0.750	0.270	0.750	0.250	0.750	0.270	0.740	0.270	0.730	0.260
	Pr(Missing)=30%											
MCAR $^{\pounds}$	0.720	0.370	0.720	0.310	0.710	0.250	0.720	0.310	0.720	0.300	0.720	0.280
MAR(Y) $^{\sharp}$	0.740	0.380	0.710	0.310	0.710	0.250	0.710	0.310	0.700	0.310	0.710	0.280
MAR(X2)$^{\flat}$	0.730	0.380	0.720	0.310	0.740	0.250	0.720	0.310	0.710	0.310	0.720	0.280
MAR(Y,X2)†	0.750	0.380	0.710	0.310	0.710	0.250	0.710	0.310	0.700	0.310	0.700	0.280
MNAR(X1)‡	0.750	0.380	0.730	0.320	0.720	0.260	0.730	0.320	0.720	0.310	0.720	0.280
MNAR(X1,X2)$^{\natural}$	0.740	0.370	0.720	0.320	0.720	0.260	0.720	0.320	0.730	0.310	0.710	0.280
MNAR(X1,Y)§	0.610	0.380	0.590	0.320	0.600	0.260	0.590	0.320	0.590	0.310	0.670	0.280
	Pr(Missing)=50%											
MCAR $^{\pounds}$	0.760	0.560	0.730	0.380	0.750	0.260	0.730	0.380	0.710	0.370	0.720	0.320
MAR(Y) $^{\sharp}$	0.800	0.560	0.720	0.370	0.720	0.250	0.720	0.370	0.710	0.360	0.710	0.310
MAR(X2)$^{\flat}$	0.810	0.650	0.720	0.400	0.750	0.260	0.720	0.400	0.690	0.390	0.720	0.340
MAR(Y,X2)†	0.830	0.570	0.720	0.380	0.740	0.260	0.720	0.380	0.710	0.360	0.710	0.310
MNAR(X1)‡	0.830	0.560	0.730	0.390	0.740	0.270	0.730	0.390	0.730	0.380	0.710	0.320
MNAR(X1,X2)$^{\natural}$	0.800	0.550	0.730	0.380	0.740	0.260	0.730	0.380	0.730	0.360	0.710	0.310
MNAR(X1,Y)§	0.640	0.570	0.570	0.390	0.580	0.270	0.570	0.390	0.560	0.390	0.670	0.320

$\pounds$ missing completely at random
$\sharp$ missing at random conditional on Y
$\flat$ missing at random conditional on $X2$
$\dagger$ missing at random conditional on $Y, X2$
$\ddagger$ missing not at random conditional on $X1$
$\natural$ missing not at random conditional on $X1, X2$
$\S$ missing not at random conditional on $X1, Y$

Table B.10: Estimates of $\hat{\beta}_2$ and its standard error (when $\beta_2 = 0$) for 1:1 matched study with n=200, $pr(X_1 = 1) = 0.5$ and $pr(X_2 = 1) = 0.5$, 1000 simulations.

	CCA		MI		EM		LLMI		LCMI-2		LCMI-3	
	beta	ASE	beta	ASE	beta	ASE	beta	ASE	beta	ASE	beta	ASE
	Pr(Missing)=10%											
MCAR $^{\pounds}$	0.004	0.155	0.001	0.140	0.000	0.138	0.001	0.140	0.000	0.139	0.000	0.139
MAR(Y) $^{\sharp}$	0.000	0.156	0.001	0.140	0.000	0.138	0.001	0.140	0.001	0.139	0.000	0.139
MAR(X2)$^{\flat}$	0.000	0.155	0.001	0.140	0.002	0.138	0.001	0.140	0.000	0.139	0.000	0.139
MAR(Y,X2)†	0.003	0.160	0.001	0.140	0.001	0.138	0.001	0.140	0.001	0.139	0.000	0.139
MNAR(X1)‡	0.003	0.152	0.000	0.139	0.000	0.138	0.000	0.139	0.000	0.139	0.001	0.139
MNAR(X1,X2)$^{\natural}$	0.003	0.153	0.001	0.139	0.000	0.138	0.001	0.139	0.001	0.139	0.001	0.139
MNAR(X1,Y)§	0.004	0.152	0.005	0.139	0.006	0.138	0.005	0.139	0.006	0.139	0.002	0.139
	Pr(Missing)=30%											
MCAR $^{\pounds}$	0.009	0.205	0.003	0.143	0.001	0.139	0.003	0.143	0.001	0.141	0.000	0.140
MAR(Y) $^{\sharp}$	0.014	0.210	0.001	0.143	0.002	0.138	0.001	0.143	0.000	0.141	0.001	0.140
MAR(X2)$^{\flat}$	0.010	0.212	0.003	0.144	0.001	0.138	0.003	0.144	0.002	0.142	0.001	0.140
MAR(Y,X2)†	0.000	0.215	0.000	0.144	0.002	0.139	0.000	0.144	0.003	0.142	0.003	0.140
MNAR(X1)‡	0.006	0.193	0.001	0.141	0.001	0.137	0.001	0.141	0.000	0.140	0.001	0.140
MNAR(X1,X2)$^{\natural}$	0.003	0.199	0.001	0.142	0.002	0.138	0.001	0.142	0.001	0.141	0.001	0.140
MNAR(X1,Y)§	0.007	0.197	0.021	0.141	0.020	0.137	0.021	0.141	0.019	0.140	0.007	0.140
	Pr(Missing)=50%											
MCAR $^{\pounds}$	0.000	0.310	0.000	0.150	0.000	0.140	0.000	0.150	0.000	0.150	0.000	0.140
MAR(Y) $^{\sharp}$	0.000	0.310	0.000	0.150	0.000	0.140	0.000	0.150	0.000	0.150	0.000	0.140
MAR(X2)$^{\flat}$	0.010	0.350	0.010	0.150	0.000	0.140	0.010	0.150	0.000	0.150	0.000	0.140
MAR(Y,X2)†	0.020	0.320	0.010	0.150	0.000	0.140	0.010	0.150	0.000	0.150	0.000	0.140
MNAR(X1)‡	0.000	0.270	0.000	0.140	0.000	0.140	0.000	0.140	0.000	0.140	0.000	0.140
MNAR(X1,X2)$^{\natural}$	0.010	0.290	0.000	0.150	0.000	0.140	0.000	0.150	0.000	0.140	0.000	0.140
MNAR(X1,Y)§	0.010	0.270	0.020	0.140	0.030	0.140	0.020	0.140	0.020	0.140	0.010	0.140

$\pounds$ missing completely at random
$\sharp$ missing at random conditional on Y
$\flat$ missing at random conditional on $X2$
$\dagger$ missing at random conditional on $Y, X2$
$\ddagger$ missing not at random conditional on $X1$
$\natural$ missing not at random conditional on $X1, X2$
$\S$ missing not at random conditional on $X1, Y$

Table B.11: 95% confidence interval coverage (95CI) and Type-I error rate (TE) of Wald test for testing $H_0 : \beta_1 = 0$, where β_1=0.69, β_2=0, $pr(X_1 = 1) = 0.5$ and $pr(X_2 = 1) = 0.5$ for 1:1 matched study with n=200, 1000 simulations.

	CCA		MI		EM		LLMI		LCMI-2		LCMI-3	
	95CI	TE	95CI	TE	95CI	TE	95CI	TE	95CI	TE	95CI	TE
Pr(Missing=10%)												
MCAR [£]	0.970	0.030	0.942	0.058	0.926	0.074	0.942	0.058	0.952	0.048	0.956	0.044
MAR(Y) [♯]	0.960	0.040	0.952	0.048	0.922	0.078	0.952	0.048	0.948	0.052	0.960	0.040
MAR(X2) [♭]	0.960	0.040	0.952	0.048	0.930	0.070	0.952	0.048	0.956	0.044	0.960	0.040
MAR(Y,X2) [†]	0.972	0.028	0.954	0.046	0.920	0.080	0.954	0.046	0.960	0.040	0.960	0.040
MNAR(X1) [‡]	0.956	0.044	0.960	0.040	0.946	0.054	0.960	0.040	0.948	0.052	0.950	0.050
MNAR(X1,X2) [♮]	0.948	0.052	0.966	0.034	0.954	0.046	0.966	0.034	0.970	0.030	0.962	0.038
MNAR(X1,Y) [§]	0.964	0.036	0.958	0.042	0.944	0.056	0.958	0.042	0.956	0.044	0.954	0.046
Pr(Missing=30%)												
MCAR [£]	0.952	0.048	0.938	0.062	0.842	0.158	0.938	0.062	0.940	0.060	0.960	0.040
MAR(Y) [♯]	0.952	0.048	0.956	0.044	0.820	0.180	0.956	0.044	0.946	0.054	0.958	0.042
MAR(X2) [♭]	0.964	0.036	0.956	0.044	0.854	0.146	0.956	0.044	0.938	0.062	0.956	0.044
MAR(Y,X2) [†]	0.962	0.038	0.940	0.060	0.822	0.178	0.940	0.060	0.944	0.056	0.972	0.028
MNAR(X1) [‡]	0.950	0.050	0.932	0.068	0.866	0.134	0.932	0.068	0.944	0.056	0.954	0.046
MNAR(X1,X2) [♮]	0.942	0.058	0.958	0.042	0.850	0.150	0.958	0.042	0.948	0.052	0.962	0.038
MNAR(X1,Y) [§]	0.930	0.070	0.938	0.062	0.820	0.180	0.938	0.062	0.926	0.074	0.968	0.032
Pr(Missing=50%)												
MCAR [£]	0.930	0.070	0.930	0.070	0.760	0.240	0.930	0.070	0.950	0.050	0.950	0.050
MAR(Y) [♯]	0.960	0.040	0.940	0.060	0.690	0.310	0.940	0.060	0.950	0.050	0.970	0.030
MAR(X2) [♭]	0.960	0.040	0.960	0.310	0.930	0.070	0.930	0.070	0.940	0.060	0.960	0.040
MAR(Y,X2) [†]	0.940	0.060	0.940	0.060	0.710	0.290	0.940	0.060	0.930	0.070	0.970	0.030
MNAR(X1) [‡]	0.950	0.050	0.940	0.060	0.760	0.240	0.940	0.060	0.940	0.060	0.960	0.040
MNAR(X1,X2) [♮]	0.940	0.060	0.940	0.060	0.740	0.260	0.940	0.060	0.930	0.070	0.960	0.040
MNAR(X1,Y) [§]	0.950	0.050	0.930	0.070	0.740	0.260	0.930	0.070	0.920	0.080	0.960	0.040

£ *missing completely at random*
♯ *missing at random conditional on* Y
♭ *missing at random conditional on* $X2$
† *missing at random conditional on* $Y, X2$
‡ *missing not at random conditional on* $X1$
♮ *missing not at random conditional on* $X1, X2$
§ *missing not at random conditional on* $X1, Y$