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## 2 **Supplementary Information for**

### 3 **Imaging electron-density fluctuations by multidimensional X-ray photon-coincidence** 4 **diffraction**

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#### 8 **This PDF file includes:**

9     Supplementary text  
10    Figs. S1 to S2

## 11 Supporting Information Text

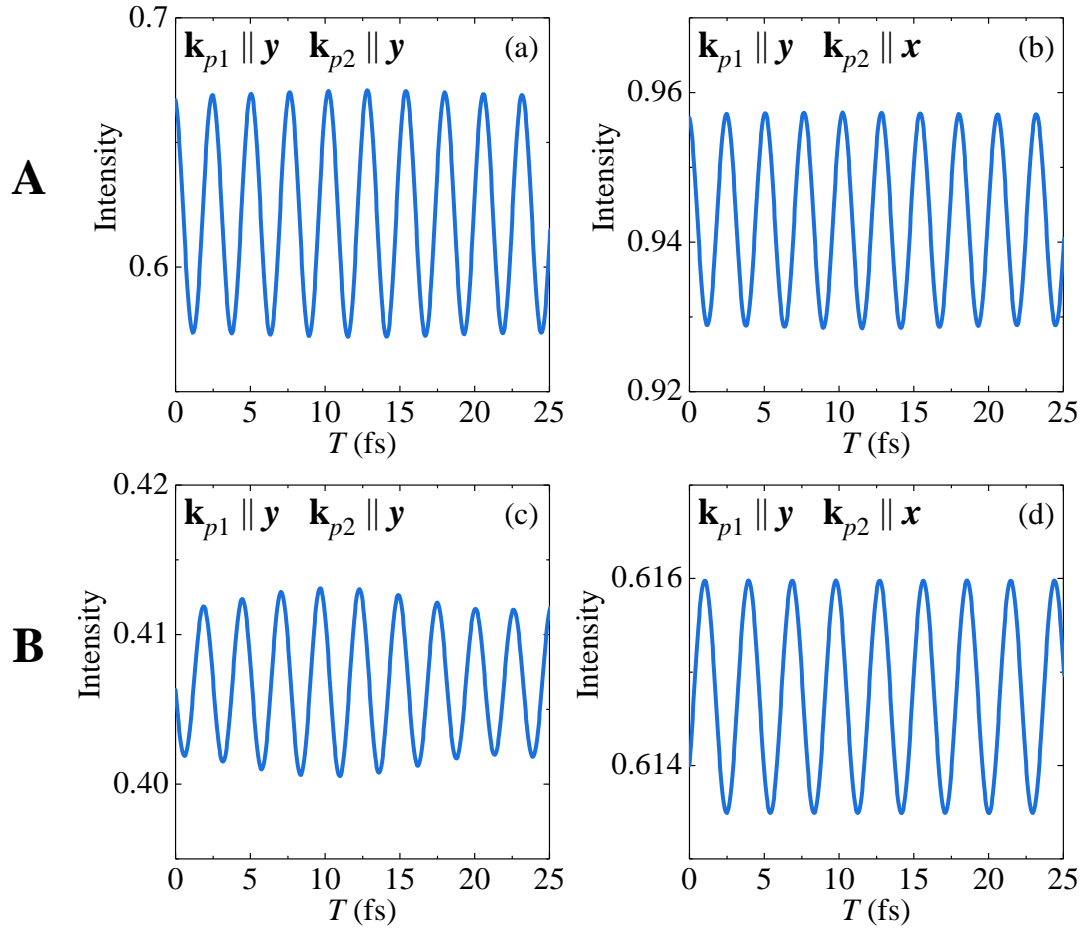
12 **Scattered intensities and the dominating pathways.** To investigate the dynamics of the signals in momentum space, we selected  
13 a  $\mathbf{q}_2$  point  $[(q_y, q_z)$  for  $yy$  configuration and  $(q_x, q_z)$  for  $yx$  configuration]  $(-0.19 \text{ \AA}^{-1}, 0.59 \text{ \AA}^{-1})$  from the  $\mathbf{q}_2$  scattering pattern  
14 in Fig. 4 of the main text. The corresponding time-dependent scattered intensities for pulse configurations  $yy$  and  $yx$  at two  $\mathbf{q}_1$   
15 points A and B are plotted in Fig. S1(a). The sum-over-states expression Eq. (3) allows to identify the pathways that dominate  
16 the signal. For example, at the  $\mathbf{q}_1$  point A for the  $yy$  configuration in Fig. 4 of the main text, with the interaction of the two  
17 temporally separated pulses  $\mathbf{k}_{p1}$  and  $\mathbf{k}_{p2}$  along  $y$ , the dominating pathways  $|g\rangle\langle g| \rightarrow |2\rangle\langle 2| \rightarrow |2\rangle\langle 2|$ ,  $|g\rangle\langle g| \rightarrow |2\rangle\langle 2| \rightarrow |8\rangle\langle 8|$   
18 and  $|g\rangle\langle g| \rightarrow |3\rangle\langle 3| \rightarrow |9\rangle\langle 9|$  are time independent, because the  $\mathbf{k}_{p1}$  pulse prepares the molecule in the populations. The  
19 time-independent pathways can be subtracted by looking at the signal difference  $S(T) - S(T = 0)$ . The time-dependence of the  
20 signal is dominated by the pathways  $|g\rangle\langle g| \rightarrow |5\rangle\langle 2| \rightarrow |8\rangle\langle 8|$  and  $|g\rangle\langle g| \rightarrow |6\rangle\langle 3| \rightarrow |9\rangle\langle 9|$  (and their complex conjugates).

21 The scattered intensities exhibit a simple time-dependent feature. The oscillation period is mainly determined by the  
22 energy differences  $(E_5 - E_2)$  and  $(E_6 - E_3)$ , indicating that the molecule is promoted to the coherence  $|5\rangle\langle 2|$  and  $|6\rangle\langle 3|$  (and  
23 the complex conjugates) by the first pulse. As displayed in Fig. 1(d), the excited states  $E_2, E_3$  and  $E_5, E_6$  are energetically  
24 degenerate. However, it is found that the coherences  $|6\rangle\langle 2|$  and  $|5\rangle\langle 3|$  and their complex conjugates do not contribute to  
25 the signal. Because the corresponding transition charge densities  $\sigma_{83}, \sigma_{86}, \sigma_{92}, \sigma_{85}$  are all zero. Figure S1(b) depicts the  
26 scattered intensities for pulse configuration  $yx$  at  $\mathbf{q}_1$  point A, where the oscillating behaviors are similar to those in Fig. S1(a).  
27 Analysis of the signal shows that the same scattering pathways as those in Fig. S1(a) dominate the signal, except that the  
28 time-independent pathway  $|g\rangle\langle g| \rightarrow |2\rangle\langle 2| \rightarrow |2\rangle\langle 2|$  has only neglectable contributions.

29 The beating patterns are observed in Fig. S1(c), indicating the interference between various pathways. Analyzing signals in  
30 detail shows that two different energy differences  $(E_4 - E_2)$  and  $(E_5 - E_2)$  determine the oscillation period. The dominating  
31 time-dependent pathways are  $|g\rangle\langle g| \rightarrow |5\rangle\langle 2| \rightarrow |2\rangle\langle 2|$  and  $|g\rangle\langle g| \rightarrow |4\rangle\langle 2| \rightarrow |8\rangle\langle 8|$  (and their complex conjugates).  
32 Figure S1(d) can be analyzed similarly. It is found that the time-dependence of the signal is dominated by the pathway  
33  $|g\rangle\langle g| \rightarrow |4\rangle\langle 2| \rightarrow |8\rangle\langle 8|$  and its complex conjugate. The oscillation amplitudes for configurations  $yy$  and  $yx$  at point B are  
34 smaller compared to those for point A, because the  $\mathbf{q}_1$  scattered intensity (see Fig. 2) and the dominating density matrix  
35 elements (see Fig. 3) for point B are smaller than those for points A.

36 **Charge densities in real space.** Figure S2 depicts the charge densities  $\sigma_{ec}(\mathbf{r})$  that contribute to the dominating correlation  
37 functions  $g_{eced}(\mathbf{R}_2)$  for the real-space signal Eq. (4) of the main text. If a pathway is time-independent, then the superposition  
38 coefficient  $f_{ecc}(\mathbf{q}_1)g_{cgcg}(\mathbf{q}_1)$  is real and time-independent. Therefore, the corresponding charge-density correlation function  
39  $g_{ecc}(\mathbf{R}_2)$  appears only in the real part of the signal  $S(\mathbf{q}_1, \mathbf{R}_2, T)$ .

40 As an example, for  $yy$  configuration at  $\mathbf{q}_1$  point A, the signals  $\text{Im}[S(\mathbf{q}_1, \mathbf{R}_2, T)]$  are cylindrically symmetric along  $x$ , since  
41 that all the contributing transition charge densities and thus the correlation functions  $g_{eced}(\mathbf{R}_2)$  possess the cylindrical  
42 symmetry along  $x$ . Due to the degeneration of the valence excited states  $E_2 = E_3, E_5 = E_6$  and  $E_8 = E_9$ , the corresponding  
43 transition charge densities are identical, *i.e.*  $\sigma_{82}(\mathbf{r}_2) = \sigma_{93}(\mathbf{r}_2)$  and  $\sigma_{85}(\mathbf{r}_2) = \sigma_{96}(\mathbf{r}_2)$ , as shown in Fig. S2. The real-space  
44 two-photon coincidence signal thus presents the dynamics of the correlation functions of the transition charge densities  
45  $g_{8582}(\mathbf{R}_2) = g_{9693}(\mathbf{R}_2)$  and  $g_{8285}(\mathbf{R}_2) = g_{9396}(\mathbf{R}_2)$ . This provides a real-space visualization of the time-dependent pathways  
46 that dominate the signals for  $yy$  configuration at  $\mathbf{q}_1$  point A in Fig. 4.



**Fig. S1.** The scattered intensities Eq. (3) as a function of time delay  $T$  at the  $\mathbf{q}_2$  point  $(-0.19 \text{ \AA}^{-1}, 0.59 \text{ \AA}^{-1})$  in Fig. 4 of the main text.

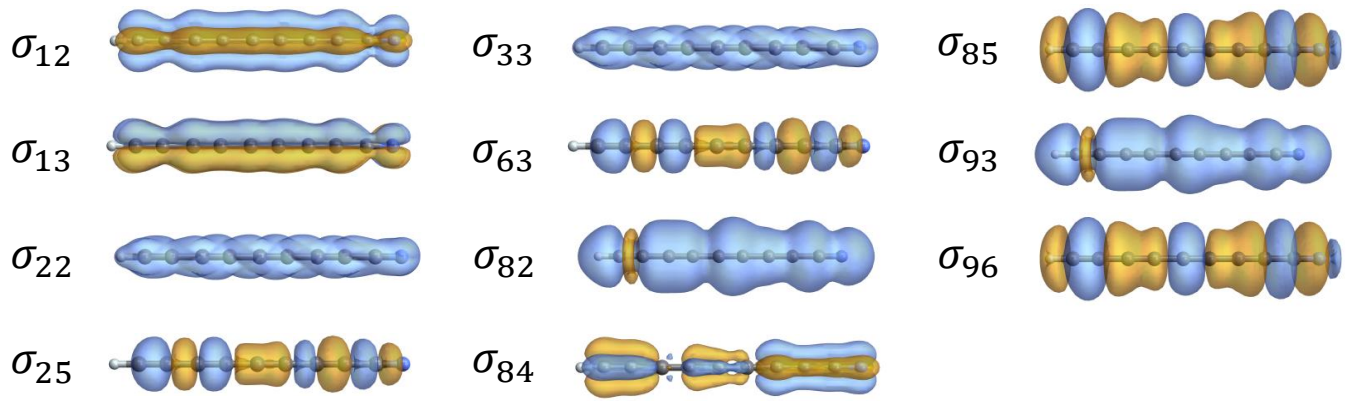


Fig. S2. The charge densities  $\sigma_{ec}(\mathbf{r})$  that contribute to the dominating correlation functions  $g_{eed}(\mathbf{r})$ .