

1 Share, but unequally: A plausible mechanism for
2 emergence and maintenance of intratumor
3 heterogeneity - Supplementary Information

4 Xin Li¹ and D. Thirumalai¹

5 ¹Department of Chemistry, University of Texas, Austin, Texas,
6 78712, United States

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8 In the Supplementary Information (SI) we provide a number of different
9 aspects of the results that are relevant to the results in the main text. Because
10 the SI covers diverse topics we provide a roadmap. In section 1, we describes
11 the procedure used to extract the parameters using the non-linear feedback
12 function used to analyze the experimental data given in [1]. In sections (2-
13 4) we provide all the details needed to not only quantitatively describe the
14 experiments on Glioblastoma using the same theory for the role of IGF-II in
15 maintaining heterogeneity in neuroendocrine pancreatic cancer but also makes
16 testable predictions.

17 **1 Qualitative results for maintenance of ITH do**
18 **not depend on the precise form of non-linear**
19 **fitness function**

20 Insulin-like growth factor II (IGF-II), overexpressed in many cancers, stimulates
21 cell proliferation, and prevents apoptosis[5, 6]. The growth rate of -/- cells
22 (tumor cells with the deletion of the IGF-II gene) as a function of the exogenous
23 IGF-II concentration has been measured systematically[1]. There is a nonlinear
24 relation between the growth rate and the concentration of IGF-II (see the open
25 squares in Fig. S2), which is perfectly described by the Hill function defined in
26 Eq. (10) (with $f_+ = 0$) in the main text (see the blue dashed line in the same
27 figure). The growth rate of -/- cells is strongly influenced by the exogenous
28 IGF-II concentration. In contrast, the proliferation rate of +/+ cells, derived
29 from the Eq. (12) (with $f_+ = 1$, $a = 80$, and $p_0 = 4.65$) in the main text, is quite
30 insensitive to this parameter and only small changes are observed at very high
31 IGF-II concentrations (see the red solid line in Fig. S2). This too is consistent

32 with experimental observations[1]. The reason is that +/+ cells can produce the
 33 IGF-II proteins by themselves to sustain and promote their growth. This also
 34 explains the slower growth rate for -/- cells compared with +/+ cells in Fig. S2
 35 in the absence of exogenous IGF-II. However, -/- cells grow faster than +/+ cell
 36 at very high IGF-II concentrations, which is a consequence of +/+ cells having
 37 to pay a cost for the production of IGF-II. The results in the main text were
 38 obtained using the Hill function for the dependence of the growth rate on the
 39 concentration of IGF-II. In order to assess the robustness of our conclusions we
 40 repeated the calculations using the logistic function (see Eq. (S.1) below).

41 It may seem that a different nonlinear relation (the logistic function),

$$w_- = \frac{a_1}{1 + a_2 e^{-\gamma_1 c}}, \quad (\text{S.1})$$

42 could fit the growth curve of -/- cells as a function IGF-II concentration. The
 43 best fit yields $a_1 = 19.97$, $a_2 = 6.07$, and $\gamma_1 = 0.28$ (see the red solid line
 44 in Fig. S3). However, the logistic function does not give as good a fit to the
 45 measured IGF-II dependent growth rate of -/- cells as the Hill function used in
 46 Eq. (10) in the main text. Nevertheless, Eq. (S.1) also captures the nonlinear
 47 growth profile of -/- cells. To demonstrate that this is indeed the case, we
 48 followed the same procedure described in the main text, and assumed that the
 49 public goods allocation strategy for -/- and +/+ cells are given by Eq. (11)
 50 and (13), respectively, except that w_- is given by Eq. (S.1). Reassuringly,
 51 we found qualitatively the same behavior as observed in Fig. 2 (see Fig. S4).
 52 Figs. S4A and S4B show that equal ($b/a = 1$) or no ($b/a=0$) share of public
 53 goods (generated by producer cells) cannot maintain a stable heterogeneous
 54 state. A stable coexisting state with both +/+ and -/- cells can only be reached
 55 when +/+ cells are allocated more resources than -/- cells ($0 < b/a < 1$)
 56 as illustrated in Fig. S4C. Therefore, the exact form of the growth curve is
 57 not critical in arriving qualitatively at the same conclusions that establish the
 58 presence of a stable heterogeneous system, as long as it is a nonlinear function
 59 of the resources, and a suitable allocation strategy for the public goods is used.

60 2 The evolution of glioblastoma tumor size con- 61 taining a mixture of wt and Δ cells

62 Here, we provide the details needed to establish the conditions for coexistence
 63 of different cell types (Δ and wt) in glioblastoma using the same theoretical
 64 framework to obtain the results for neuroendocrine pancreatic cancer. The
 65 present application is intended to show the generality of the theory by analyzing
 66 the experiments described elsewhere[7]. In the absence of Δ cells, the wt cells
 67 alone cannot induce the tumor growth in mouse[7] (see the pink down-triangles
 68 in Fig. 6 in the main text). In the experiments, consisting of wt and Δ cells,
 69 the authors did not consider the consequences of exogenous public goods (IL-6).
 70 Therefore, we take a simple non-linear fitness function (w_-) for wt cells similar

71 to the one in Eq. (10) ($\alpha = 1$) in the main text which leads to,

$$w_- = bf_+/(1 + bf_+). \quad (\text{S.2})$$

72 Note that the fitness function w_- is zero if the fraction of Δ cells, $f_+ = 0$, and,
 73 hence the constant c_0 in Eq. (11) is no longer required (no exogenous public
 74 goods). Similarly, the fitness function (w_+) for Δ cells can be written as

$$w_+ = af_+/(1 + af_+) - p_0, \quad (\text{S.3})$$

75 where p_0 is the price paid by the Δ cells. We take different parameters a and
 76 b ($a \neq b$, in general) in the above equations to illustrate the consequences of
 77 unequal sharing of public goods (produced by the Δ cells) between the two cell
 78 types. The evolution of the tumor size is described by Eq. (4) in the main text.
 79 Given $f_+ = 1$, the average fitness of the system is given by $\langle w \rangle = w_+$ (see
 80 Eq. (3)). Therefore, the tumor size N grows exponentially with $N = N_0 e^{w_+ t}$.
 81 By fitting the growth curve (blue) shown in the upper panel of Fig. 6 in the
 82 main text to an exponential function, we obtain,

$$\frac{a}{(1 + a)} - p_0 \approx 0.335. \quad (\text{S.4})$$

83 Thus, we obtain $w_+ \approx 0.335$ from the fit. It was noted in the experiments[7]
 84 that these two types of cells grow at the same rate ($f_+ = f_-$), which leads to

$$\frac{0.5a}{(1 + 0.5a)} - p_0 = \frac{0.5b}{(1 + 0.5b)}. \quad (\text{S.5})$$

85 The average fitness $\langle w \rangle = 0.5(w_+ + w_-) = w_+ = w_-$ given $f_+ = f_-$ and the
 86 evolution of the system size N can be described by $N = N_0 e^{w_- t}$. Similarly, the
 87 tumor growth curve (green line) in the upper panel of Fig. 6, results in,

$$\frac{0.5b}{(1 + 0.5b)} \approx 0.321, \quad (\text{S.6})$$

88 with the constant 0.321 obtained from the exponential fit. Thus, the three
 89 parameters a, b , and p_0 can be calculated from Eqs. (S.4)-(S.6), which yield to
 90 $a = 68.4$, $b = 0.946$, and $p_0 = 0.651$. The relation $a \gg b$ derived here using
 91 the experimental data shows that more public goods are allocated to producers
 92 than non-producers, which is the prerequisite for the maintenance of a stable
 93 heterogeneous system predicted from the theory.

94 After obtaining the values for all the three free parameters in Eqs. (S.2)
 95 and (S.3), the evolution of the tumor size for different initial fractions of Δ
 96 cells can be predicted. First, the evolution of f_+ and f_- can be calculated
 97 from Eqs. (1) and (2) in the main text given the initial fraction $f_+(0)$. Then,
 98 the evolution of the tumor size can be derived from Eq. (4) directly with $\langle w \rangle$
 99 given by Eq. (3) in the main text. Two examples are shown in the lower panel
 100 of Fig. 6 with the fraction of Δ cells given by 10% and 90% separately. The
 101 theoretical predictions (see the dash-dotted and solid lines) are in excellent
 102 agreement with experimental results (purple and green symbols, respectively).
 103 Additional experiments can be carried out to further test the predictions of our
 104 theory.

105 **3 The growth rate of tumors at different frac-**
 106 **tions of a subpopulation**

107 It is frequently found in the experiments that the mean growth rate of tumors is
 108 a non-monotonic function of the fraction of one cell type. A maximum growth
 109 rate is often observed in the middle fraction of producer cells[1, 7]. Generally
 110 speaking, the growth rate of a tumor at certain fractions of producer cells is
 111 difficult to measure accurately because the tumor evolves with time, and the
 112 fraction of different cells also changes. Therefore, the growth rate as a function
 113 of fraction of a cell type has to be measured over relatively short time interval
 114 because otherwise large fluctuation would be expected.

115 In contrast, it is relatively easy to calculate this quantity theoretically. The
 116 mean growth rate is just the average fitness as given by Eq. (3). One example
 117 is shown in Fig. 7 in the main text for the system composed of wt and Δ
 118 cells. The dotted and dashed lines correspond to the growth rate of wt and
 119 Δ cells described by Eqs. (S.2) and (S.3), respectively. The mean velocity as
 120 demonstrated by the solid line (lying between the dotted and dashed lines)
 121 reaches a maximum value at the fraction 0.77 of Δ cells (see also the inset in
 122 Fig. 7). The parameter values for a, b , and p_0 used in this figure are the same
 123 as discussed in the last section. Therefore, the relation observed here provides
 124 a direct explanation for the surprising finding shown in Fig. 6 that the tumor
 125 grows faster as the fraction of Δ cell is 90% compared with the case with 100%
 126 of Δ cell.

127 **4 The evolution of the fraction of Δ cells with**
 128 **and without exogenous resources.**

129 From the Eqs. (S.2), (S.3), combined with Eqs. (1) and (2), the evolution of
 130 the fraction $f_+(t)$ of Δ cells (without any supply of exogenous cytokines) can
 131 be calculated as shown in Fig. S5A. We found a very stable heterogeneous
 132 state (composed of both wt and Δ cells) as $f_+(0)$ is varied from 0.1 to 0.9. A
 133 poor prognosis for recovery would be expected for patients with such a stable
 134 heterogeneous tumor as long as a small fraction of Δ cells is present in the
 135 tumor.

136 Adding exogenous cytokines (c_0), such as IL-6 or LIF into the tumor, the
 137 Eqs. (S.2), and (S.3) change to the following forms,

$$w_- = (bf_+ + c_0)/(1 + bf_+ + c_0), \quad (\text{S.7})$$

138

$$w_+ = (af_+ + c_0)/(1 + af_+ + c_0) - p_0. \quad (\text{S.8})$$

139 A stable homogeneous tumor consisting only of wt cells can be obtained rapidly
 140 irrespective of $f_+(0)$ (see the evolution of $f_+(t)$ in Fig. S5B). It should not go
 141 unnoticed that the predictions in Fig. S5 are amenable to experimental tests,
 142 along the lines conducted in [1].

143 **References**

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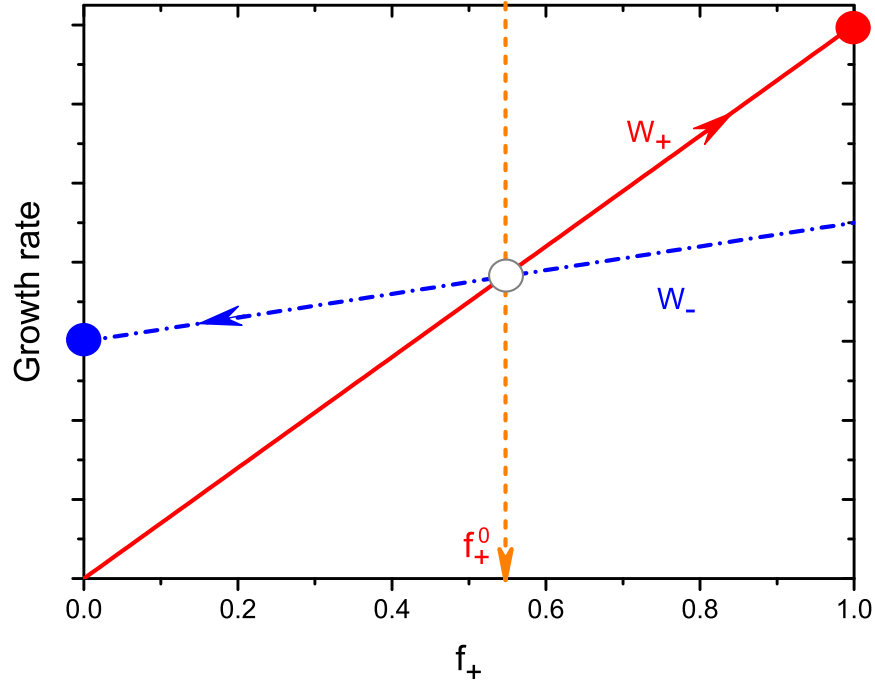


Figure S1: A schematic figure for the growth rate of the producer and non-producer as a function of the fraction (f_+) of the producer with a linear fitness function. The red solid line shows the the growth rate (w_+) of the producer described by Eq. (7) and the blue dash-dotted line represents the growth rate (w_-) of the non-producer given by Eq. (8) in the main text. The parameter $k_+ > k_-$ and the internal unstable state is indicated by the open circle with $f_+ = f_+^0$. The blue and red filled circles show the homogeneous state consisting of only non-producers and producers, respectively. The figure illustrates that upon an infinitesimal perturbation the flow from the internal state (open circle) to the stable states (filled circles) occurs depending on the value of f_+ .

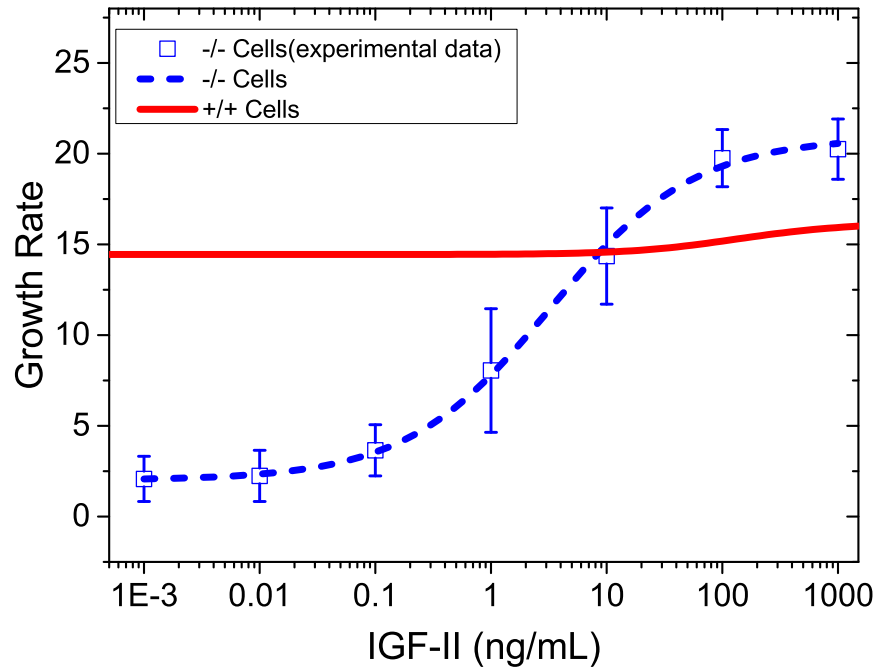


Figure S2: Growth rate of the +/+ and -/- cells as a function of IGF-II concentration. Experimental results[1] for -/- cells are represented by open squares which can be perfectly fitted by a Hill function as described by the Eq. (10) in the main text (see the blue dashed line). The red solid red line shows the growth of +/+ cells derived from the Eq. (12) in the main text. The error bars represent the standard deviation. The growth rate is defined the same as in Fig. 2 in the main text.

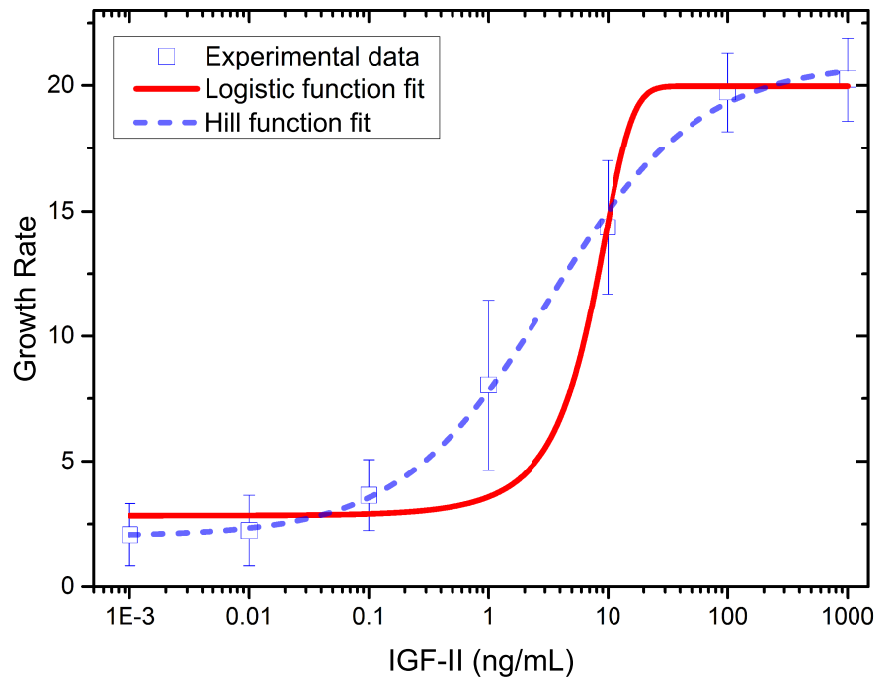


Figure S3: Growth rate of the $-/-$ cells as a function of IGF-II concentration. Experimental results[1] for $-/-$ cells are represented by open squares. The red solid red line shows the fit using a logistic function given Eq. (S.1). For comparison, we also show the blue dashed line, is obtained by fitting to the Hill function (see Fig. S1). The growth rate is defined the same as in Fig. 2 in the main text.

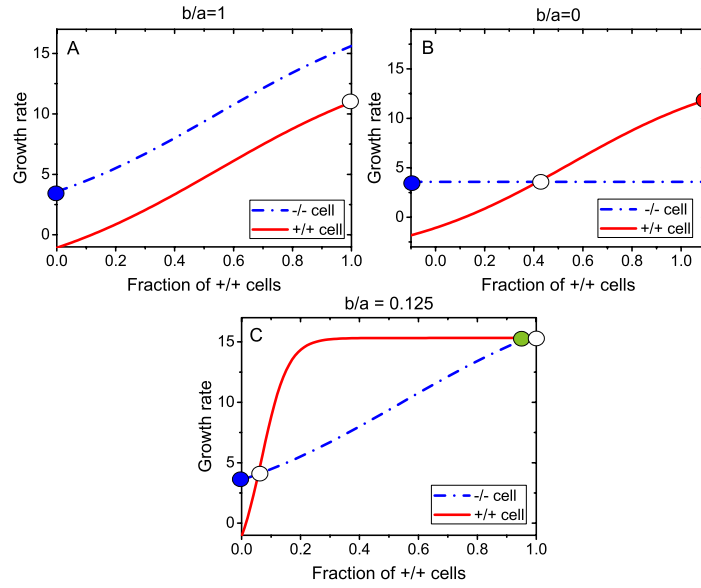


Figure S4: Growth rate of the +/+ and -/- cells as a function of the fraction of +/+ cells under different allocation strategies of IGF-II produced by the +/+ cells. (A) Equal share of IGF-II ($b = a = 10$), (B) no share ($b = 0$, and $a = 10$), (C) a small portion ($b = 10$, and $a = 80$) is allocated to -/- cells. The value of $c_0 = 1$, $p_0 = 4.65$ (same as used in Fig. 2 in the main text) in Eqs. (11) and (13) in the main text. The growth rate of +/+ cells are shown in solid red lines while dot-dashed blue lines describe the growth rate of -/- cells. The filled and empty circles are used to indicate a stable or unstable state, respectively. A stable state consisting of only +/+ (-/-) cells is indicated in red (blue) color. The green filled circle shows a stable heterogeneous state. The growth rate is defined the same as in Fig. 2 in the main text.

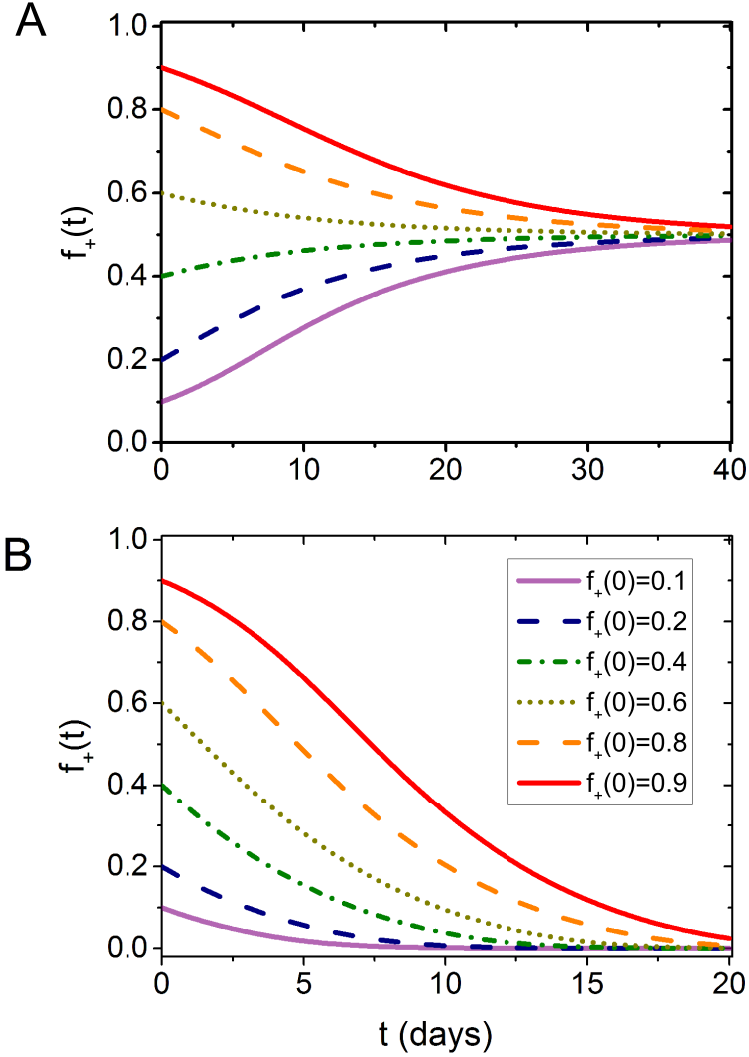


Figure S5: The evolution of the fraction $f_+(t)$ of Δ cells in GBM tumor as a function of time for distinct initial conditions specified by $f_+(0)$. (A) Without supply of exogenous public goods w_- and w_+ are given by Eqs. (S.2) and (S.3); (B) With supply of exogenous public goods w_- and w_+ are described by Eqs. (S.7) and (S.8) with $c_0 = 1.0$. Other parameters are the same as used in Fig. 6 in the main text. The labels for different $f_+(0)$ are the same in (A) and (B).