

1 **Supplementary Information for**
2
3 **Pareto Optimality Explanation of the Glycolytic Alternatives in Nature**

4
5 Chiam Yu Ng, Lin Wang, Anupam Chowdhury, and Costas D. Maranas

6
7 Corresponding author: Costas D. Maranas

8 Email: costas@psu.edu

9

10

11 **This PDF file includes:**

12

13 Supplementary Information Text

14 Figs. S1 to S8

15 Tables S1 to S2

16 References for SI reference citations

17

18 **Other supplementary materials for this manuscript include the following:**

19

20 Supplementary Data Files S1 to S6

21 **Supplementary Information Text**

22 **Integer cut constraints and the runtime reduction of optStoic.**

23 Integer cut constraints are introduced in the optStoic formulation to exhaustively identify
 24 alternate optimal pathways that satisfy the design equation. Herein, we define k as the
 25 number of iteration for running optStoic algorithm ($k \in \{1, 2, \dots, \kappa\}$).

26

27 The formulation of optStoic in the method section (see main text) was reformulated as a
 28 combination of linear relations by introducing two non-negative real number (or integer)
 29 variables for each v_j as followed:

$$v_j = v_j^f - v_j^r, \quad \forall j \in J$$

where $v_j \in \mathbb{Z}, v_j^f \in \mathbb{Z}_{\geq 0}$ and $v_j^r \in \mathbb{Z}_{\geq 0}$
 or $(v_j \in \mathbb{R}, v_j^f \in \mathbb{R}_{\geq 0}$ and $v_j^r \in \mathbb{R}_{\geq 0})$
 $|v_j| = v_j^f + v_j^r$

30

31 To this end, binary variables y_j^f and y_j^r are defined as followed:

$$y_j^f = \begin{cases} 1, & \text{if reaction } j \text{ carries non-zero flux in the forward direction } (v_j^f > 0) \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$y_j^r = \begin{cases} 1, & \text{if reaction } j \text{ carries non-zero flux in the reverse direction } (v_j^r > 0) \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

32 Likewise, $y_j^{f^k}$ and $y_j^{r^k}$ are binary variables associated with the solution from the k -th
 33 iteration. At iteration $k = \kappa$, the following constraints are added to the modified optStoic
 34 formulation:

$$\sum_{j \in J | y_j^{f^k} + y_j^{r^k} = 1} (1 - y_j^f - y_j^r) + \sum_{j \in J | y_j^{f^k} + y_j^{r^k} = 0} (y_j^f + y_j^r) \geq 1 \quad k = 1, 2, \dots, \kappa - 1 \quad (3)$$

$$y_j^f + y_j^r \leq 1 \quad (4)$$

$$y_j^f \varepsilon \leq v_j^f \leq y_j^f M \quad (5)$$

$$y_j^r \varepsilon \leq v_j^r \leq y_j^r M \quad (6)$$

35 Constraint 3 is the integer cut constraint that ensures that at least one of reaction j that was
 36 identified in the previous iteration k is inactive in the current iteration. Constraint 4
 37 enforces that only one of the binary variables (corresponding to the flux directions) for
 38 each reaction j is active. Finally, constraints 5 and 6 restrict the flux (in forward or reverse
 39 direction) to be strictly positive whenever the corresponding binary variable is active. The
 40 parameter ε is a user-defined small positive real number. The MILP problems were solved
 41 using the CPLEX v.12.6.1 solver accessed through the GAMS (v24.4.1) modeling system
 42 and Gurobi Optimizer v6.5.1 using Python 2.7.

43

44 The runtime of the modified optStoic algorithm depends on the size of the search space
 45 (i.e. database size). Therefore, blocked reactions (i.e., reactions incapable of carrying flux)

46 were identified first upon imposing the bounds on exchange fluxes (see constraint 4 in main
 47 text) and excluded from the search space before running the algorithm. We also observed
 48 that the runtime of optStoic significantly increases when more integer cuts are added. This
 49 is caused by a large number of integer variables introduced in the second term of constraint
 50 3 at each iteration k :

$$51 \quad \sum_{j \in J \mid y_j^{f^k} + y_j^{r^k} \neq 1} (y_j^f + y_j^r)$$

52 Upon removal of the blocked reactions, there are still over 3,000 reactions exist in the set
 53 $\{j \in J \mid y_j^{f^k} + y_j^{r^k} = 0\}$. To solve this issue, we imposed an additional constraint on the
 54 objective function as followed:

$$55 \quad \sum_{j \in J_{\text{exchange}}} |v_j| = z^*$$

57 and used the following integer cut to prevent the same pathway from being identified for
 58 the same objective value z^* .

$$59 \quad \sum_{j \in J \mid y_j^{f^k} + y_j^{r^k} = 1} (1 - y_j^f - y_j^r) \geq 1, \quad \forall k = 1, 2, 3, \dots$$

61 We run the modified optStoic algorithm in parallel for each fixed objective value z^* to
 62 further reduce the total runtime.

63 **Assessing the thermodynamic feasibility of a pathway.**

64 The thermodynamic feasibility of each pathway under physiological concentration ranges
 65 are assessed using the max-min driving force (MDF) formulation (1).

66 *Step 1:* The $\Delta_r G_j^{\prime\circ}$ for each reaction involved in a pathway ($j \in J_{\text{path}}$) is estimated using
 67 the Component Contribution method (2) at pH 7, 25°C and ionic strength of 0.1 M (3, 4).

68 *Step 2:* The MDF problem is solved for each pathway, which minimizes the maximum
 69 $\Delta_r G_j^{\prime}$ of a pathway by optimizing over the concentrations of all metabolites in the pathway.

70 The optimization formulation is given by:

$$\max_{c_i} \quad \min_j \{-\Delta_r G_j^{\prime}\} \quad (MDF) \quad (7)$$

$$\text{subject to } \Delta_r G_j^{\prime} = \Delta_r G_j^{\prime\circ} + RT \sum_{i \in I_{\text{path}}} S_{ij}^T \ln c_i, \quad \forall j \in J_{\text{path}} \quad (8)$$

$$\ln c_i^{\min} \leq \ln c_i \leq \ln c_i^{\max}, \quad \forall i \in I_{\text{path}} \quad (9)$$

$$\ln r_p^{\min} \leq \ln r_p \leq \ln r_p^{\max}, \quad \forall p \in \mathbf{P} \quad (10)$$

71 where I_{path} is the set of all metabolites and J_{path} is the set of all reactions in a pathway, c_i
 72 is the concentration of metabolite i , R is the gas constant, T is the temperature, r_p is the
 73 concentration ratio for an ordered pair of metabolites p (e.g., $p =$
 74 (ATP, ADP) , $r_{(ATP, ADP)} = c_{ATP}/c_{ADP}$), and \mathbf{P} is a set of metabolite pairs (e.g., $\mathbf{P} \in$
 75 $\{(ATP, ADP), (NADPH, NADP^+), (NADH, NAD^+)\}$). Note that the S matrix here refers to
 76 the stoichiometric matrix of the pathway with $S \in \mathbb{R}^{|I_{\text{path}}| \times |J_{\text{path}}|}$.
 77

78

79 Constraint 8 relates the Gibbs free energy of reaction ($\Delta_r G'_j$) with the standard Gibbs free
80 energy of reaction ($\Delta_r G'^{\circ}_j$) and the mass action ratio. The concentrations of all metabolites
81 are allowed to vary between 1 μM (c_i^{min}) and 100 mM (c_i^{max}) in constraint 9.
82 Concentration ratios of common cofactor pairs (e.g., NADPH/NADP⁺, NADH/NAD⁺ and
83 ATP/ADP) play an important role in a cell as they determine the driving force of a large
84 number of biosynthesis reactions (5). The concentration ratios of energy and redox
85 cofactors are therefore allowed to vary within the maximum and minimum values found in
86 the literature (6-8) and the Bionumbers database (9) in constraint 10. Constraint 9 is
87 optional depending on the case study. Herein, we assumed that the designed pathway
88 operates at steady-state and within a single compartment of a cell at a temperature (T) of
89 25°C, an ionic strength of 0.1 M and pH 7.0. The pathway with a positive objective
90 function (MDF) indicates that it is thermodynamically infeasible within the given
91 physiological concentration (and ratio) ranges is omitted from the subsequent step.
92 Importantly, the objective function of the enzyme cost minimization problem is convex
93 only when all $\Delta_r G'_j < 0$ in a pathway. The MDF problem is solved using Gurobi Optimizer
94 v6.5.1 solver and Python script modified from the Component Contribution Python
95 package (2).

96 We have previously found that when imposing the metabolite concentration ranges
97 (constraint 9) strictly to experimentally measured metabolite levels, the MDF formulation
98 is often over-constrained and may become infeasible due to several factors. They include:
99 (i) measurement errors of the absolute intracellular metabolite concentrations, (ii)
100 estimation errors of Gibbs free energy from group-contribution based approaches, (iii) the
101 MDF formulation assumes that metabolite concentrations are homogenous (i.e.,
102 compartmentalization of metabolites and potential occurrence of substrate-channeling are
103 ignored), and (iv) MDF analysis is performed with a pathway and not on the entire
104 metabolic network. In addition, when we added the uncertainties in $\Delta_r G'_j^{\circ}$ to the MDF
105 formulation (i.e., allowing the $\Delta_r G'_j^{\circ}$ of each reaction to vary between a range given by
106 $\Delta_r G'_j^{\circ} \pm SE_j$ in constraint 8), we found that MDF analysis will identify the more optimistic
107 solution given the degree of freedom to have a more negative $\Delta_r G'_j^{\circ}$. Since, we are
108 comparing between pathways, the relative contribution of the errors of $\Delta_r G'_j^{\circ}$ estimation
109 has a lesser contribution towards the overall analysis and would not affect our conclusions.
110 Therefore, MDF analysis of the pathways is studied within a larger physiological
111 metabolite concentration ranges, and the errors of Gibbs free energy are not considered in
112 the study.

113 **Minimization of protein cost.**

114 The minimal enzyme demand in units of mg protein/mmol glucose/h for each one of the
115 thermodynamically feasible pathways is then estimated based on the enzyme cost
116 minimization (ECM) method (10, 11). The formulation is as followed:

$$\text{minimize PC} = \frac{1}{v_{EX_glc}} \sum_j M_{E,j} \lambda_{E,j} \quad (ECM) \quad (11)$$

subject to $\lambda_{E,j}$

$$= \frac{v_j}{k_{cat,j}^+} \left(1 - \exp\left(\frac{\Delta_r G_j'}{RT}\right) \right)^{-1} \left(1 + \prod_{i \in I_{re,j}} \left(\frac{K_{M,ij}}{c_i}\right)^{q_{ij}^+} \right), \forall j \quad (12)$$

$\in J_{path}$

$$\Delta_r G_j' = \Delta_r G_j'^{\circ} + RT \ln \frac{\prod_{i \in I_{pr,j}} c_i^{q_{ij}^-}}{\prod_{i \in I_{re,j}} c_i^{q_{ij}^+}}, \quad \forall j \in J_{path} \quad (13)$$

$$\Delta_r G_j' < 0, \quad \forall j \in J_{path} \quad (14)$$

$$c_i^{min} \leq c_i \leq c_i^{max}, \quad \forall i \in I_{path} \quad (15)$$

$$r_p^{min} \leq r_p \leq r_p^{max}, \quad \forall p \in P \quad (16)$$

117

118 where $M_{E,j}$ is the molecular weight of enzyme per active site for reaction j , v_{EX_glc} is the
 119 glucose uptake flux (mmol Glucose/h), $\lambda_{E,j}$ is the enzyme level for reaction j , v_j is the flux
 120 through reaction j , $k_{cat,j}^+$ is the turnover number of the reaction in the forward direction,
 121 $I_{re,j}$ is the set of reactants in reaction j , $I_{pr,j}$ is the set of products in reaction j , the set of all
 122 metabolites in the pathway I_{path} is the union of $I_{re,j}$ and $I_{pr,j}$, $K_{M,ij}$ is the Michaelis-Menten
 123 constant of the enzyme for reaction j towards metabolite i , q_{ij}^+ and q_{ij}^- is the stoichiometric
 124 coefficient of metabolite i in reaction j . $q_{ij}^+ > 0$ if metabolite i is a reactant in reaction j
 125 and $q_{ij}^+ = 0$ otherwise, whereas $q_{ij}^- > 0$ if metabolite i is a product in reaction j and $q_{ij}^- =$
 126 0 otherwise. Note that in the preprocessing step, all the reactions are re-arranged such that
 127 flux v_j through each of them is strictly positive.

128

129 The objective function (equation 11) involves the minimization of the sum of the enzymatic
 130 cost ($\mu\text{g Protein/ mmol Glucose/ h}$) for each reaction in the pathway normalized by the
 131 glucose uptake rate. Constraint 12 defines the enzyme level for a reaction j as a function
 132 derived from the reversible Michaelis-Menten kinetic equation (10). Constraint 13 is
 133 equivalent to constraint 8 recasted using concentrations. Constraint 14 ensures that all
 134 reactions have a negative change in free energy and prevents division by zero in equation
 135 12. Constraints 15 and 16 impose the bounds on the concentration ranges and concentration
 136 ratio ranges. The above formulation can be simplified by substituting the concentration
 137 variable c_i with logarithmic concentrations $x_i = \ln c_i$ and thus converting the product term
 138 into a summation.

139

140 According to Flamholz *et al.* (10), the enzyme cost minimization (ECM) formulation can
 141 be rewritten by substituting $x_i = \ln c_i$ as followed:

142

$$\text{minimize PC} = \frac{1}{v_{EX_glc}} \sum_j M_{E,j} \lambda_{E,j} \quad (ECM) \quad (17)$$

$$\text{subject to } \lambda_{E,j} = \frac{v_j}{k_{cat,j}^+} \frac{1 + \exp\left(\sum_{i \in I_{re,j}} q_{ij}^+ (\ln K_{M,ij} - x_i)\right)}{1 - \exp\left(\sum_{i \in I_{path}} S_{ij}^T x_i + \frac{\Delta_r G_j'^0}{RT}\right)}, \forall j \in J_{path} \quad (18)$$

$$\Delta_r G_j' = \Delta_r G_j'^0 + R \cdot T \cdot \sum_{i \in I_{path}} S_{ij}^T x_i, \quad \forall j \in J_{path} \quad (19)$$

$$\Delta_r G_j' \leq 0 - \varepsilon, \quad \forall j \in J_{path} \quad (20)$$

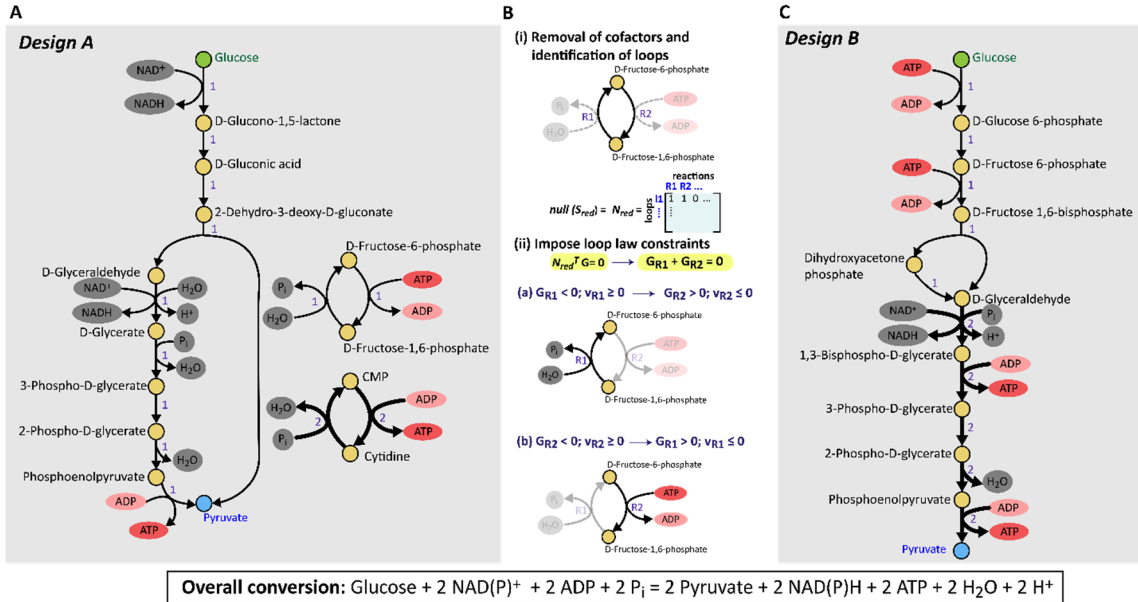
$$\ln c_i^{min} \leq x_i \leq \ln c_i^{max}, \quad \forall i \in I_{path} \quad (21)$$

$$\ln r_p^{min} \leq \ln r_p \leq \ln r_p^{max}, \quad \forall p \in P \quad (22)$$

143 Using log-concentration simplifies the formulation as the product term in equations 12 and
 144 13 can be replaced by the summation term in equations 18 and 19, respectively. Note that
 145 we have set ε to a very small number (i.e., $1e^{-6}$) to ensure that the denominator of $\lambda_{e,j}$ does
 146 not become zero. We increase ε stepwise by 10-fold up to 0.1 if the optimization failed to
 147 converge at a lower ε value. If the optimization still fails to terminate successfully at $\varepsilon =$
 148 0.1, we exclude the pathway from the final solution.
 149

150 The optimal concentrations of metabolites obtained from the MDF problem are used as the
 151 initial condition for the ECM problem, which is then solved using the sequential least
 152 squares quadratic programming method (Python SciPy package). Due to the lack of
 153 experimentally measured kinetic parameters, we assumed generic values ($M_E =$
 154 40 kDa , $k_{cat} = 79 \text{ s}^{-1}$ and $K_M = 200 \text{ } \mu\text{M}$) (12) for all kinetic parameters as was carried
 155 out in the original study (10). This implies that all enzymes were treated as equally fast in
 156 every pathway. The allowable metabolite concentration ranges are identical to that of the
 157 MDF analysis.

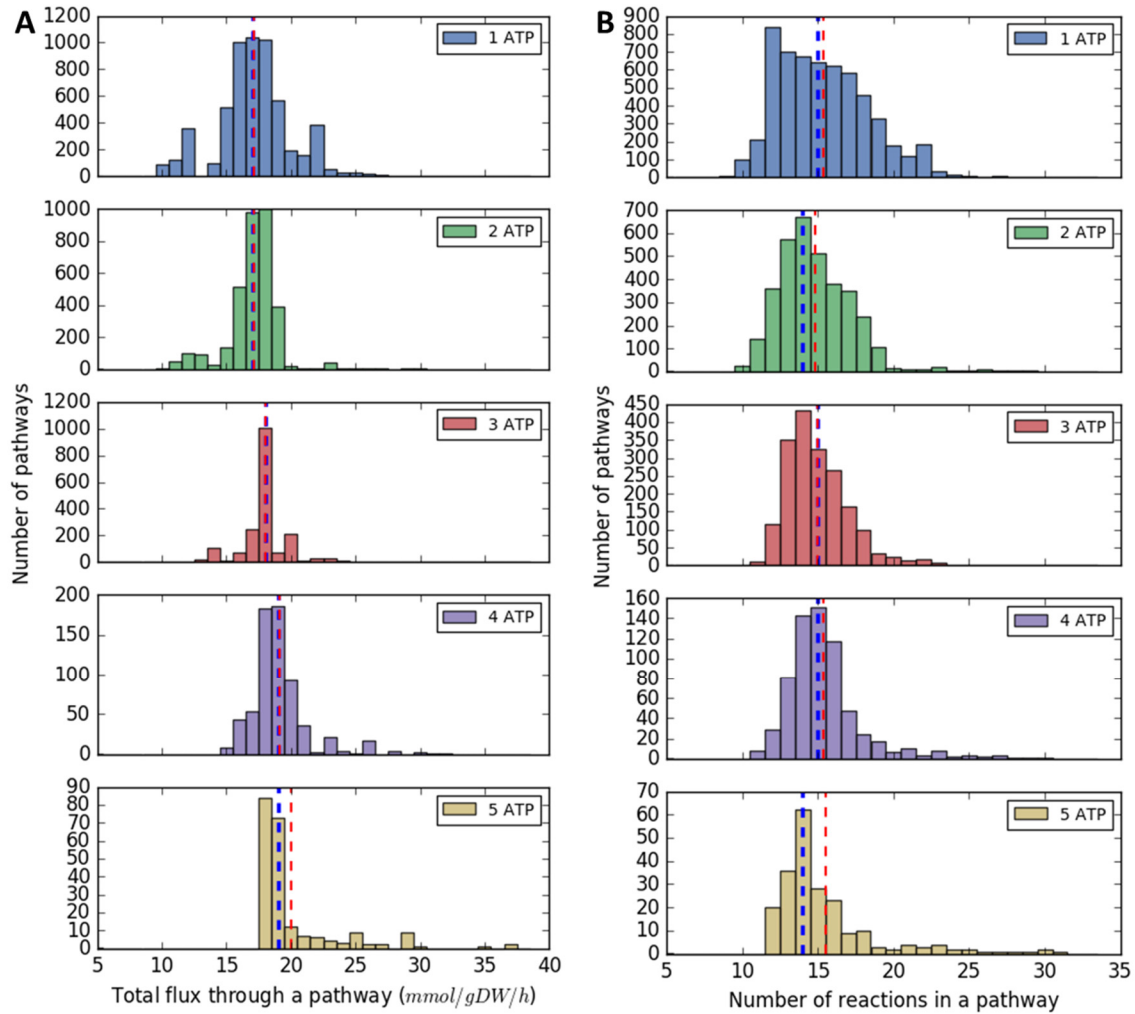
158
 159



160

161 **Fig. S1.** Pathways shown here perform the overall conversion defined below the panels.
 162 (A) Design A is a pathway with disjoint subnetworks that generate cofactors such as ATP.
 163 This design is obtained using the previous optStoic formulation. (B) The \mathbf{S}_{int} matrix, which
 164 contains only internal reactions, was processed by removing rows containing cofactors.
 165 The basis of the null space of the resulting \mathbf{S}_{red} matrix is then obtained ($\text{null}(\mathbf{S}_{red}) =$
 166 \mathbf{N}_{red}). Each row of the \mathbf{N}_{red} matrix is an internal cycle that results in no net non-cofactor
 167 metabolite production. The loop law is imposed as $\mathbf{N}_{red}^T \mathbf{G} = \mathbf{0}$, which implies that flux
 168 could traverse only through one of the directions in a loop. Two cases are shown here for
 169 the loop involving reaction R1 (D-Fructose-1,6-phosphate + H₂O → D-Fructose-6-
 170 phosphate + P_i) and R2 (D-Fructose-6-phosphate + ATP → D-Fructose-1,6-phosphate +
 171 ADP). In case (ii) (a), when reaction R1 is active ($v_{R1} > 0$), then reaction R2 can carry
 172 only zero flux or flux in the same direction with R1. (C) After adding the loop law
 173 constraints, we found that ATP and redox generation occurs only on the main carbon
 174 transfer pathway.

175



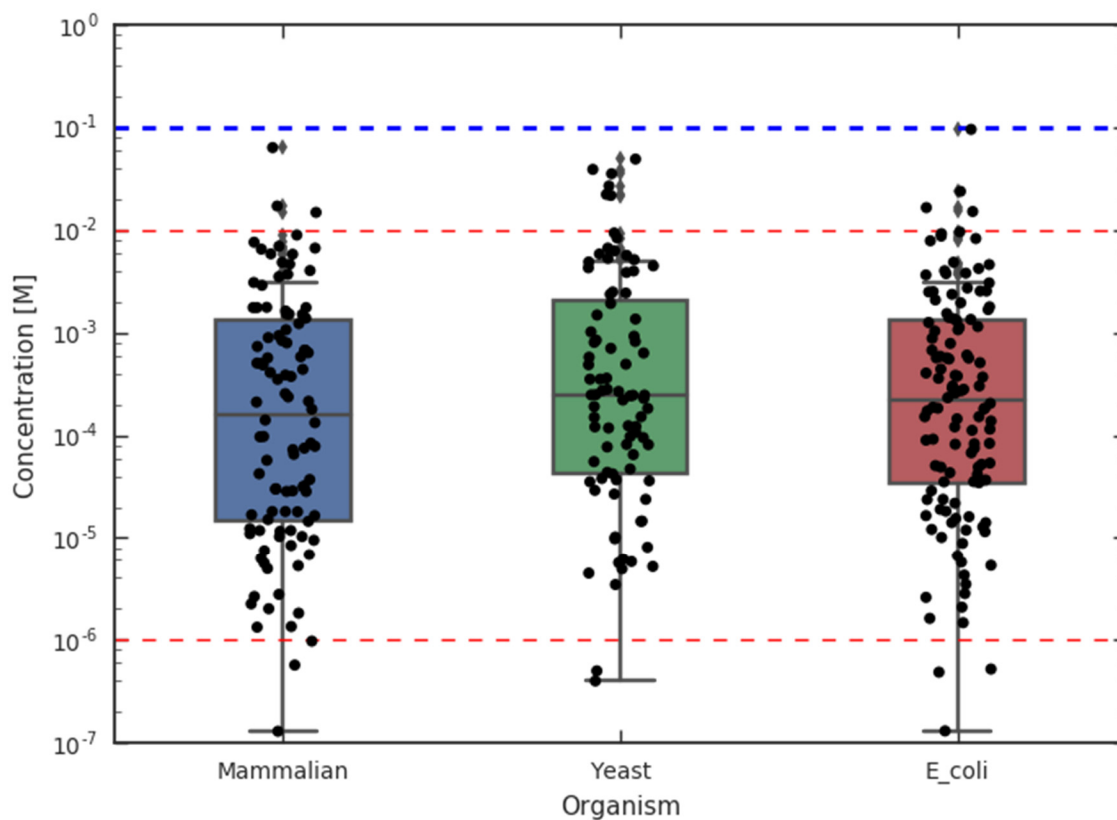
176

177 **Fig. S2.** The distribution of the glycolytic pathway alternatives based on (A) total flux
 178 through a pathway, and (B) the number of reactions in a pathway. Note that the total flux
 179 through a pathway and the number of reactions are calculated without accounting for the
 180 exchange reactions. The colors represent the ATP yield per glucose (mol ATP/mol glucose)
 181 generated by a pathway at a fixed glucose uptake flux. Red dashed lines indicate the mean
 182 values, whereas blue dashed lines denote the median values.

183

184

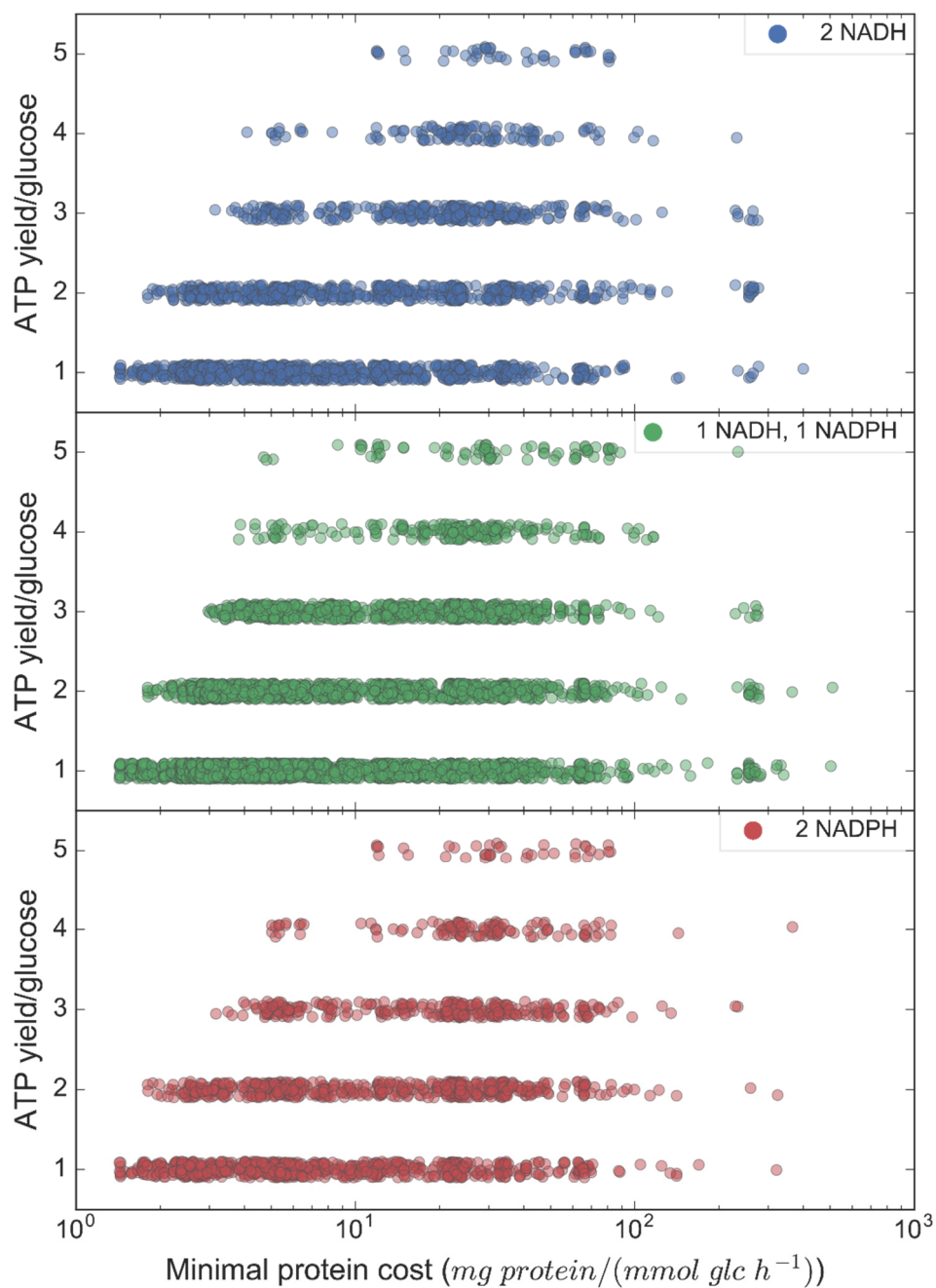
185



186

187 **Fig. S3.** Distribution of absolute metabolite concentrations across different organisms (8).
 188 The fraction of metabolites that are within 1 μ M and 100 mM are 97.1%, 97.7% and 97.5%
 189 for mammalian cells, yeast and *E. coli*, respectively. The fraction of metabolites that fall
 190 within 1 μ M and 10 mM are 94.1%, 90.9% and 94.2% for mammalian cells, yeast and *E.*
 191 *coli*, respectively.

192

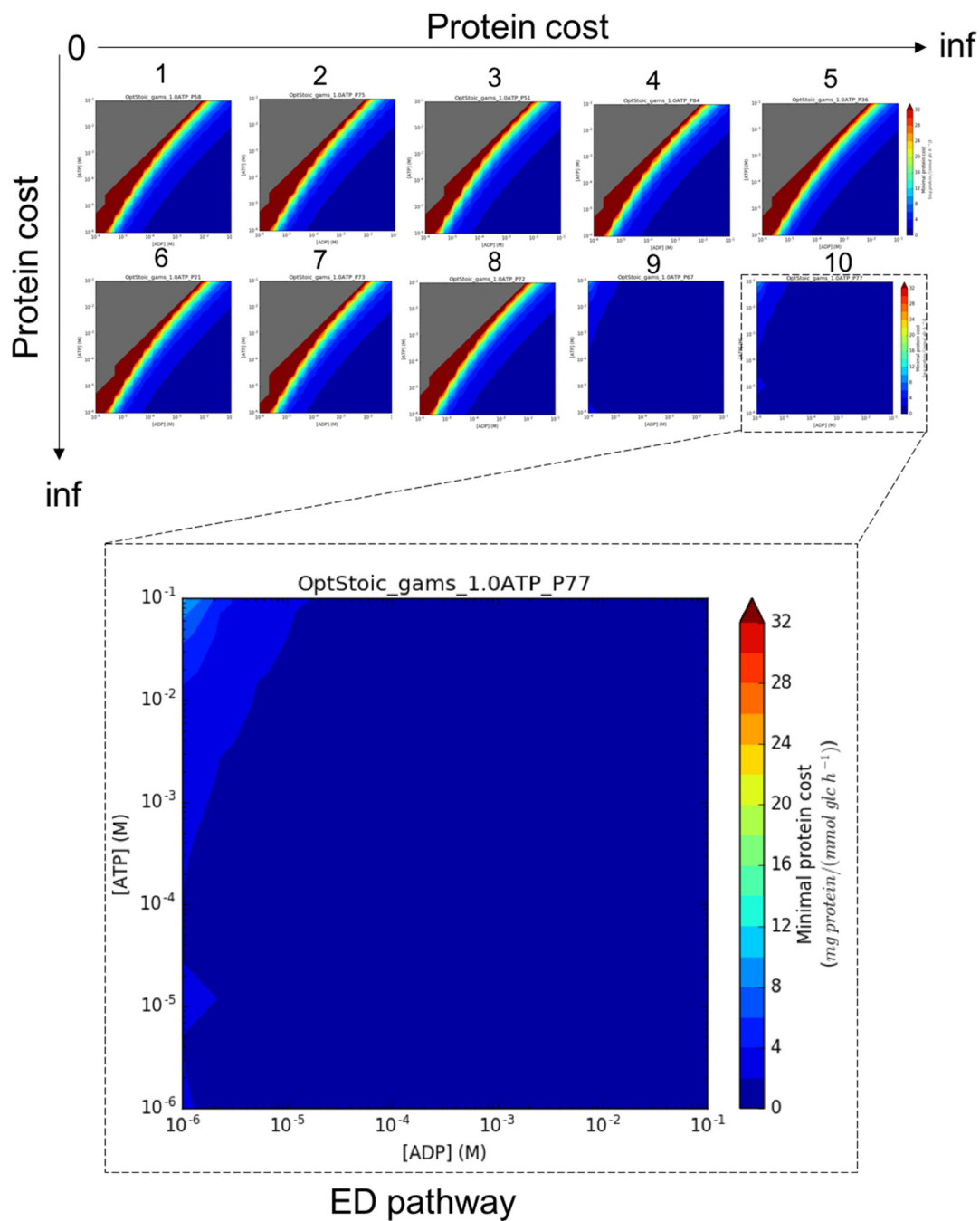


194

195 **Fig. S4.** The ATP yield versus minimal protein cost scatter plot. Note that the Y-axis is
 196 categorical. Jittering effect was applied to the plot to show the distribution more clearly.
 197 Pathways are color-coded based on the type of redox cofactors produced: (Blue) 2 NADH,
 198 (Green) 1 NADH and 1 NADPH, and (Red) 2 NADPH.

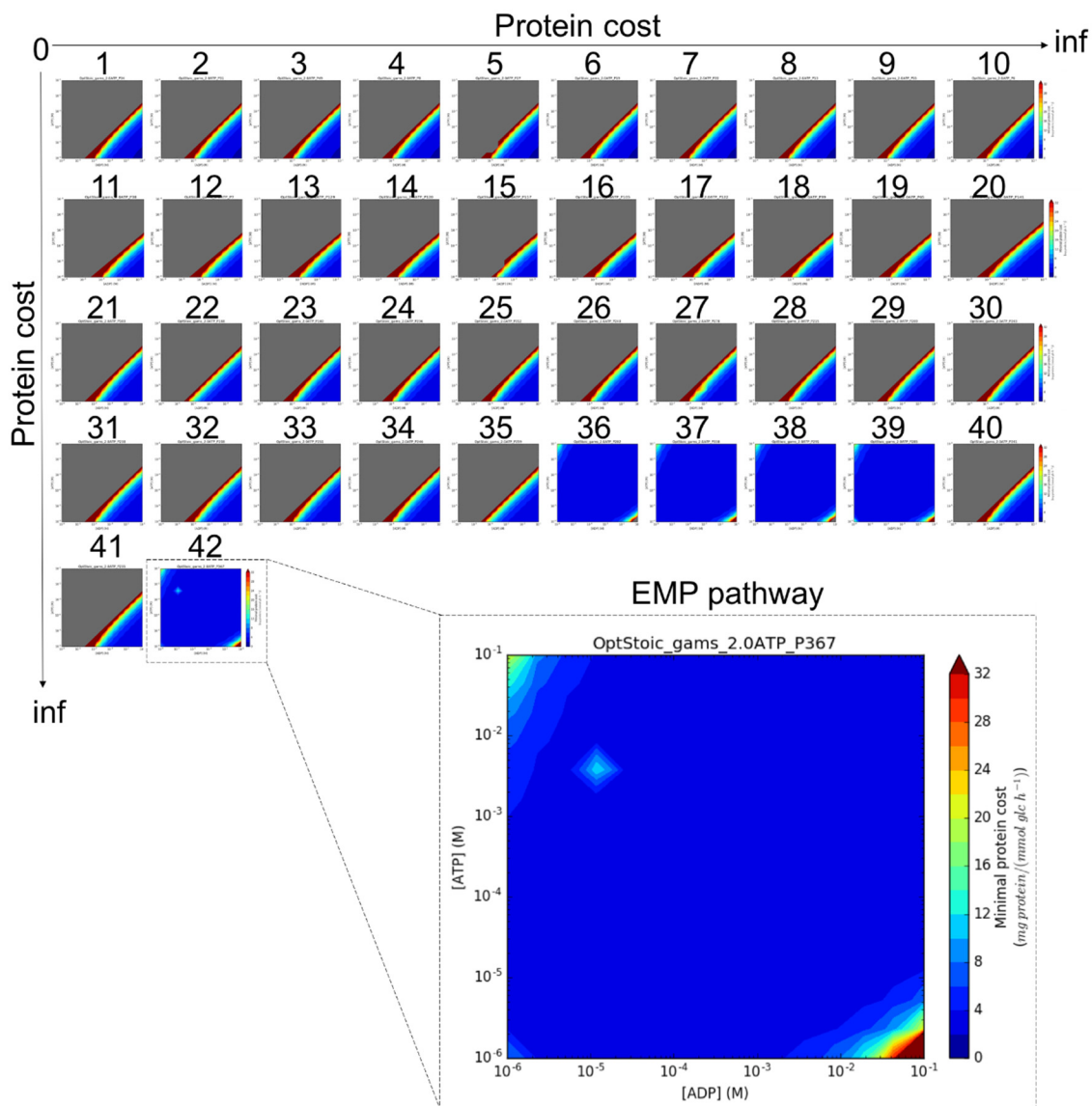
199

200
201



202
203
204
205

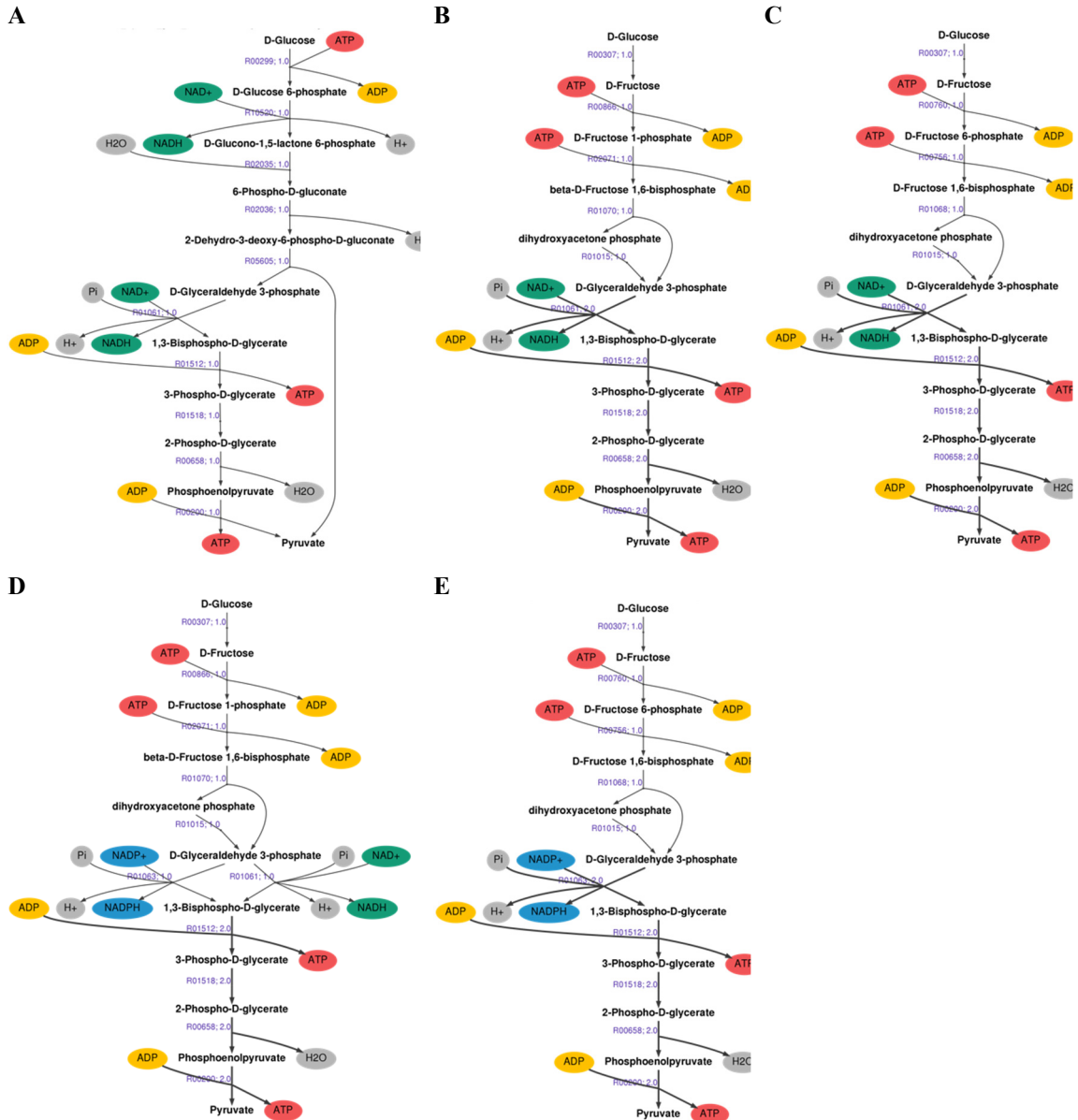
Fig. S5. Robustness analysis of the effect of ATP and ADP concentrations on the top ten 1-ATP generating pathways. Note that the legends for the x and y-axes are the same for all the ten pathways.



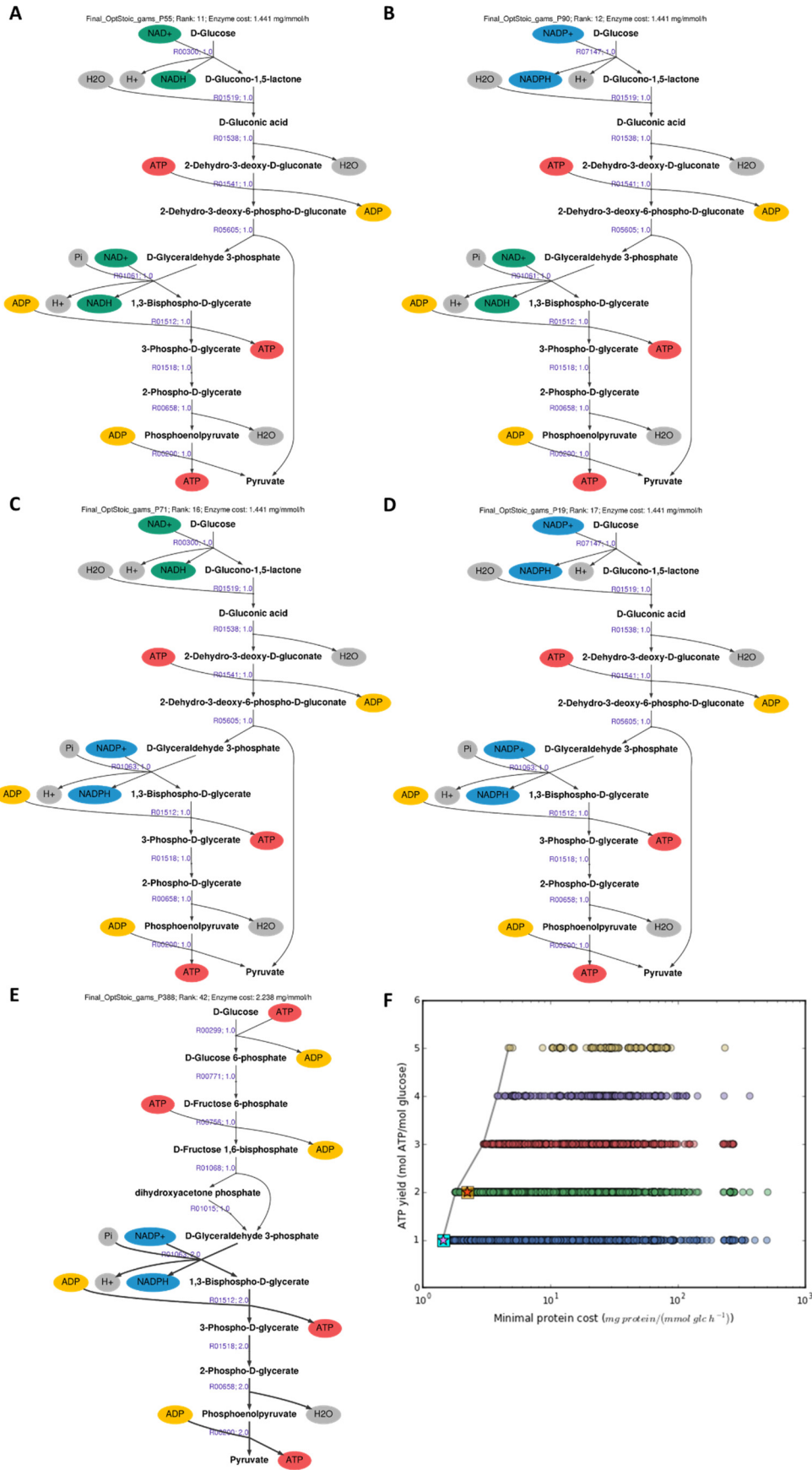
207

208 **Fig. S6.** Robustness analysis of the effect of ATP and ADP concentrations on the top forty-
 209 two 2-ATP generating pathways. Note that the legends for the x and y-axes are the same
 210 for all the 42 pathways. In addition, the ranges of the axes and heat map scales are the same
 211 for each panel.

212



213 **Fig. S7.** Robust 1-ATP and 2-ATP generating glycolytic pathways within a broad range of
 214 ATP/ADP ratio. The pathway diagrams were generated using the pathway visualization
 215 tool described in the Method section. Pathway (A) is the 9th 1-ATP generating pathways
 216 ranked by protein cost in Fig. S5. Pathways (B), (C), (D) and (E) are 36th to 39th 2-ATP
 217 generating pathways ranked by protein cost in Fig. S6.
 218



220 **Fig. S8.** (A-D) All four variants of the semi-phosphorylative ED pathway and (E) the
221 NADPH-dependent EMP pathway described in the text. The pathway diagrams were
222 generated using the pathway visualization tool described in the Method section. (F) The
223 tradeoff plot of the minimal protein cost and the ATP yield of all glycolytic pathway
224 variants as shown in Figure 3 (B). In addition to the ED (pink star) and the EMP pathways
225 (red star), the semi-phosphorylative ED pathway variants (A-D) are represented as light-
226 blue squares, whereas the NADPH-dependent EMP pathway is shown in the yellow square.
227

228 **Table S1.** Cofactors that were removed from the S matrix when generating the internal
 229 stoichiometric matrix (S^*).

KEGG ID	Description	KEGG ID	Description
C00001	H2O	C00112	CDP
C00002	ATP	C00131	dATP
C00003	NAD+	C00138	Reduced ferredoxin
C00004	NADH	C00139	Oxidized ferredoxin
C00005	NADPH	C00144	GMP
C00006	NADP+	C00206	dADP
C00007	Oxygen	C00286	dGTP
C00008	ADP	C00360	dAMP
C00009	Orthophosphate	C00361	dGDP
C00010	CoA	C00362	dGMP
C00011	CO2	C00363	dTDP
C00013	Diphosphate	C00364	dTMP
C00015	UDP	C00365	dUMP
C00016	FAD	C00390	Ubiquinol
C00020	AMP	C00399	Ubiquinone
C00035	GDP	C00458	dCTP
C00044	GTP	C00459	dTTP
C00055	CMP	C00460	dUTP
C00063	CTP	C01352	FADH2
C00075	UTP		
C00080	H+		
C00081	ITP		
C00104	IDP		
C00105	UMP		

230
 231

232 **Table S2A.** The number of pathways that are thermodynamically feasible (MDF < 0) at
 233 physiological concentration ranges and ratio.

Conditions	# of 1-ATP Pathways	# of 2-ATP Pathways	# of 3-ATP Pathways	# of 4-ATP Pathways	# of 5-ATP Pathways
optStoic	5,739	3,430	1,873	659	215
condition (i)	4,550	2,891	1,542	466	165
condition (ii)	2,549	1,099	281	4	0
condition (iii)	2,525	1,098	281	4	0
condition (iv)	1,824	778	173	2	0
condition (v)	538	105	0	0	0
condition (vi)	2,558	1,099	281	4	0

- 234 i. All metabolites are allowed to vary between 1 μ M and 100 mM.
 235 ii. Same with (i), except that ATP and ADP concentrations are bounded based on
 236 Park *et al.* (8) (i.e., $1.66 \text{ mM} \leq C_{ATP} \leq 11.4 \text{ mM}$; $0.429 \text{ mM} \leq C_{ADP} \leq$
 237 0.715 mM).
 238 iii. Same with (ii), except that the concentration range of CO₂ was bounded based on
 239 Park *et al.* (8) (i.e., $50 \mu\text{M} \leq C_{CO_2} \leq 10 \text{ mM}$).
 240 iv. All metabolites are allowed to vary between 1 μ M and 100 mM except CO₂. The
 241 range of CO₂ was obtained from Park *et al.* (8) (i.e., $50 \mu\text{M} \leq C_{CO_2} \leq 10 \text{ mM}$).
 242 The ratio ranges for different cofactor pairs were imposed as followed: $0.2 \leq$
 243 $\frac{C_{ATP}}{C_{ADP}} \leq 20$, $0.2 \leq \frac{C_{NADPH}}{C_{NADP}} \leq 100$, $0.0005 \leq \frac{C_{NADH}}{C_{NAD}} \leq 0.5$ based on data collected
 244 from Bionumbers (9) and literature (6-8).
 245 v. All metabolites other than CO₂ are allowed to vary between 1 μ M and 10 mM.
 246 The range of CO₂ was obtained from Park *et al.* (8) (i.e., $50 \mu\text{M} \leq C_{CO_2} \leq 10$
 247 mM). The ratio ranges for different cofactor pairs were imposed as followed:
 248 $0.2 \leq \frac{C_{ATP}}{C_{ADP}} \leq 20$, $0.2 \leq \frac{C_{NADPH}}{C_{NADP}} \leq 100$, $0.0005 \leq \frac{C_{NADH}}{C_{NAD}} \leq 0.5$.
 249 vi. All metabolites are allowed to vary between 1 μ M and 100 mM. The ratio ranges
 250 for different cofactor pairs were imposed as followed: $1 \leq \frac{C_{ATP}}{C_{ADP}} \leq 10,000$.

251

252
253

Table S2B. All the constraints used for the simulation of Table S2A.

Conditions	(i)	(ii)	(iii)	(iv)	(v)	(vi)
(A) $1 \mu\text{M} \leq C_i \leq 100 \text{ mM}$ for all metabolite i	+	+	+	+		+
(B) $1.66 \text{ mM} \leq C_{ATP} \leq 11.4 \text{ mM};$ $0.429 \text{ mM} \leq C_{ADP} \leq 0.715 \text{ mM}$		+	+			
(C) $50 \mu\text{M} \leq C_{CO_2} \leq 10 \text{ mM}$			+	+	+	
(D) $0.2 \leq \frac{C_{ATP}}{C_{ADP}} \leq 20;$ $0.2 \leq \frac{C_{NADPH}}{C_{NADP}} \leq 100;$ $0.0005 \leq \frac{C_{NADH}}{C_{NAD}} \leq 0.5$				+	+	
(E) $1 \mu\text{M} \leq C_i \leq 10 \text{ mM}$, for all metabolite i					+	
(F) $1 \leq \frac{C_{ATP}}{C_{ADP}} \leq 10,000$						+

254

255
256
257
258
259
260
261
262
263
264
265
266
267
268
269
270
271
272
273
274
275
276
277
278
279
280
281
282
283
284
285
286
287
288
289

References

1. Noor E, *et al.* (2014) Pathway thermodynamics highlights kinetic obstacles in central metabolism. *PLoS computational biology* 10(2):e1003483.
2. Noor E, Haraldsdottir HS, Milo R, & Fleming RM (2013) Consistent estimation of Gibbs energy using component contributions. *PLoS computational biology* 9(7):e1003098.
3. Biemans-Oldehinkel E, Mahmood NA, & Poolman B (2006) A sensor for intracellular ionic strength. *Proceedings of the National Academy of Sciences of the United States of America* 103(28):10624-10629.
4. Storey KB (2004) *Functional Metabolism: Regulation and Adaptation* (John Wiley & Sons, Inc., Hoboken, NJ, USA) p 616.
5. Zhang J, *et al.* (2015) Determination of the Cytosolic NADPH/NADP Ratio in *Saccharomyces cerevisiae* using Shikimate Dehydrogenase as Sensor Reaction. *Scientific Reports* 5:12846.
6. Bennett BD, *et al.* (2009) Absolute metabolite concentrations and implied enzyme active site occupancy in *Escherichia coli*. *Nature chemical biology* 5(8):593-599.
7. Gerosa L, *et al.* (2015) Pseudo-transition Analysis Identifies the Key Regulators of Dynamic Metabolic Adaptations from Steady-State Data. *Cell Syst* 1(4):270-282.
8. Park JO, *et al.* (2016) Metabolite concentrations, fluxes and free energies imply efficient enzyme usage. *Nature chemical biology*.
9. Milo R, Jorgensen P, Moran U, Weber G, & Springer M (2010) BioNumbers--the database of key numbers in molecular and cell biology. *Nucleic acids research* 38(Database issue):D750-753.
10. Flamholz A, Noor E, Bar-Even A, Liebermeister W, & Milo R (2013) Glycolytic strategy as a tradeoff between energy yield and protein cost. *Proceedings of the National Academy of Sciences of the United States of America* 110(24):10039-10044.
11. Noor E, *et al.* (2016) The Protein Cost of Metabolic Fluxes: Prediction from Enzymatic Rate Laws and Cost Minimization. *PLoS computational biology* 12(11):e1005167.
12. Bar-Even A, *et al.* (2011) The moderately efficient enzyme: evolutionary and physicochemical trends shaping enzyme parameters. *Biochemistry* 50(21):4402-4410.