Supporting information: High-Frequency Stochastic Switching of Graphene Resonators Near Room Temperature

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S1: Equations of motion

The dimensionless equation that governs the dynamics of the drum is

$$\ddot{x} + 2\zeta\dot{x} + x + \alpha x^3 = \lambda \cos \omega_F t. \tag{1}$$

In our formulation the displacement of the membrane's center q is normalized with respect to the membrane radius R, i.e. $x = q/R$. The time variable τ is made dimensionless by making

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use of the resonant frequency ω. The overdot in eq. (1) means differentiation with respect to the dimensionless time $t = \omega\tau$. The amplitude and frequency of the excitation are f and Ω_F , related to their dimensionless counterparts $\lambda = f/\left(R\omega^2 m_{\text{eff}}\right)$ and $\omega_F = \Omega_F/\omega$, respectively. The symbol m_{eff} indicates the effective mass of the drum. The membrane damping c is scaled to the dimensionless damping ratio $2\zeta = c/\left(\omega m_{\text{eff}}\right)$. Finally, $\alpha = R^2 k_3/\left(\omega^2 m_{\text{eff}}\right)$ is the dimensionless conjugate of the cubic stiffness coeficient k_3.

In order to analyze the slow dynamical evolution of the system, the solution is assumed to have the form

$$x(t) = P\left(t\right)\cos\omega_F t + Q\left(t\right)\sin\omega_F t, \tag{2}$$

in which $P\left(t\right)$ and $Q\left(t\right)$ are slowly varying functions of time representing the real and imaginary part coefficients of the solution. This can be realized by expressing the dimensionless displacement of the membrane $x(t)$ in complex functions $u_1(t)$, $u_2(t)$ with $u_2(t)$ complex conjugate of $u_1(t)$:[1]

$$x = u_1 \exp^{i\omega_F t} + u_2 \exp^{-i\omega_F t}. \tag{3}$$

By exploiting Euler's formula and dividing real and imaginary parts we recast eq. 2 to $P = (u_1 + u_2)$ and $Q = i(u_1 - u_2)$ which are the real and imaginary part coefficients, respectively.

Following the method of variation of parameters, the solution is subject to the condition:[2]

$$\dot{P}\left(t\right)\cos\omega_F t + \dot{Q}\left(t\right)\sin\omega_F t = 0. \tag{4}$$

By substituting eq. 2 with its corresponding time derivatives into eq. 1, and making use of eq. 4, we obtain:

$$\begin{cases} \dot{P} = \dfrac{\omega^2 - {\omega_F}^2}{2\omega_F} Q - \zeta P + \dfrac{3}{8}\dfrac{\alpha}{\omega_F} Q\left(P^2 + Q^2\right) \\ \dot{Q} = -\dfrac{\omega^2 - {\omega_F}^2}{2\omega_F} P - \zeta Q - \dfrac{3}{8}\dfrac{\alpha}{\omega_F} P\left(P^2 + Q^2\right) + \dfrac{\lambda}{2\omega_F}. \end{cases} \tag{5}$$

The deterministic system of eqs. (5) is then perturbed by two independent Gaussian white noise processes with equal intensity σ in the equations for $\dot{P}$ and $\dot{Q}$, respectively. The system of stochastic differential equations (SDE) with additive noise is:

$$\begin{cases} dP = \left(\dfrac{\omega^2 - {\omega_F}^2}{2\omega_F} Q - \zeta P + \dfrac{3}{8}\dfrac{\alpha}{\omega_F} Q \left(P^2 + Q^2\right)\right) dt + \sigma dW_1 \\ dQ = \left(-\dfrac{\omega^2 - {\omega_F}^2}{2\omega_F} P - \zeta Q - \dfrac{3}{8}\dfrac{\alpha}{\omega_F} P \left(P^2 + Q^2\right) + \dfrac{\lambda}{2\omega_F}\right) dt + \sigma dW_2 \end{cases} \tag{6}$$

in which $W_1(t)$ and $W_2(t)$ are independent Wiener processes, normally distributed random variables with mean zero and variance dt. Note that neither W nor the state variables P and Q are anywhere differentiable now that the system is converted to a set of stochastic differential equations. For the integration of eq. 6, the Itô scheme will be employed.[3]

References

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