Supplementary Material for "A Bayesian Design for Phase I Cancer Therapeutic Vaccine Trials"

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1 Simulation scenarios

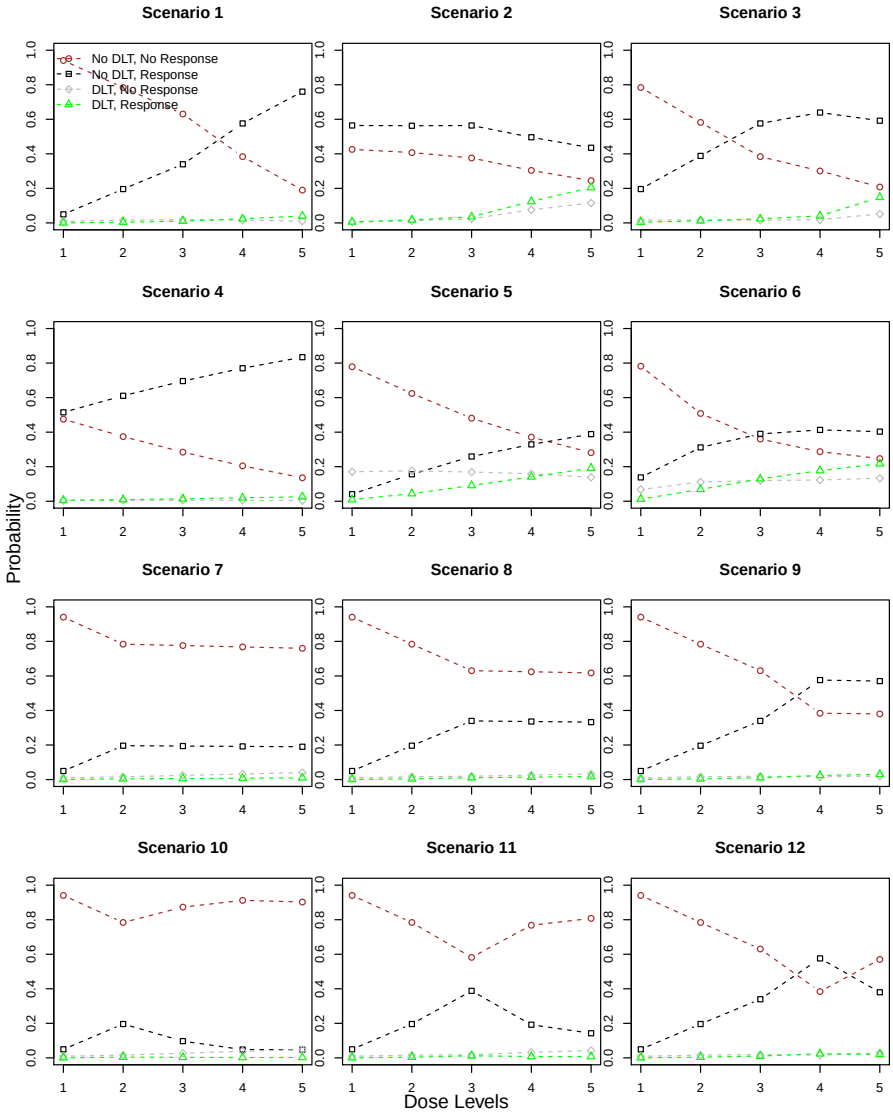

Figure 1: Plot of $\boldsymbol{\theta}^{(l)}$ for simulation scenarios 1-15. $r^{(l)} = 10$ for all l.

2 Simulation results

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	2.1	6.2	3.7	4.3	83.7	100
	N	7.47	8.48	8.16	8.05	8.01	40.17
	(D,R)	(0.08,0.38)	(0.14,1.65)	(0.24,2.89)	(0.34,4.81)	(0.43,6.39)	(1.23,16.12)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	21.7	20.3	27.7	20	10.3	100
	N	7.5	8.3	7.16	5.98	2.93	31.88
	(D,R)	(0.08,4.3)	(0.28,4.84)	(0.46,4.34)	(1.23,3.66)	(0.99,1.92)	(3.04,19.06)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	4.1	5	12.9	24.9	53.1	100
	N	7.86	8.34	8.3	8.36	8.46	41.32
	(D,R)	(0.14,1.55)	(0.24,3.29)	(0.32,5.03)	(0.49,5.67)	(1.71,6.27)	(2.91,21.81)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	12.4	10.9	7.9	6.4	62.4	100
	N	7.46	7.58	6.96	6.1	5.59	33.68
	(D,R)	(0.07,3.85)	(0.11,4.68)	(0.16,4.9)	(0.14,4.87)	(0.16,4.8)	(0.63,23.09)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	15.6	24.1	24	15.6	9.9	89.2
	N	11.65	10.57	8.35	5.26	2.58	38.4
	(D,R)	(2.09,0.6)	(2.22,2.03)	(2.2,3.04)	(1.6,2.5)	(0.89,1.53)	(9,9.7)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	14.9	26.7	29.1	19.7	8.2	98.6
	N	10	11.32	9.5	6	2.62	39.44
	(D,R)	(0.8,1.49)	(2.03,4.25)	(2.39,4.91)	(1.82,3.5)	(0.96,1.71)	(7.99,15.85)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	1.8	20	20	15.4	42.8	100
	N	7.38	8.44	8.38	6.58	5.33	36.11
	(D,R)	(0.06,0.4)	(0.18,2.12)	(0.24,2.08)	(0.23,1.57)	(0.29,1.35)	(1,7.53)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	1.5	7.5	21.5	17.6	51.9	100
	N	7.46	8.23	8.13	7.75	6.17	37.74
	(D,R)	(0.07,0.38)	(0.15,2.07)	(0.25,3.17)	(0.31,3.04)	(0.33,2.51)	(1.11,11.17)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	1.4	7.3	7.1	19.4	64.8	100
	N	7.54	8.06	8.25	7.79	7.15	38.79
	(D,R)	(0.08,0.38)	(0.14,2.03)	(0.25,3.36)	(0.3,4.62)	(0.33,4.3)	(1.1,14.69)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	1.2	65.7	10.3	5.2	17.6	100
	N	7.57	8.29	9.22	3.81	2.6	31.49
	(D,R)	(0.09,0.39)	(0.17,2.03)	(0.26,0.44)	(0.13,0.18)	(0.12,0.1)	(0.77,3.14)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	2.3	4.7	61.2	10.7	21.1	100
	N	7.51	8.45	8.21	7.35	2.99	34.51
	(D,R)	(0.08,0.37)	(0.17,1.66)	(0.25,3.32)	(0.3,1.12)	(0.13,0.42)	(0.93,6.89)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	1.7	8.3	2.2	62.6	25.2	100
	N	7.52	8.41	8.37	7.9	6.89	39.09
	(D,R)	(0.08,0.38)	(0.15,1.66)	(0.24,2.5)	(0.33,4.81)	(0.32,2.09)	(1.12,11.45)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	15.1	0.8	64.2	6.2	13.7	100
	N	7.42	9.52	7.34	7.33	2.37	33.98
	(D,R)	(0.07,0.35)	(0.2,0.48)	(0.23,2.21)	(0.27,0.36)	(0.14,0.14)	(0.9,3.53)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	13.3	14.8	0.6	55.1	16.2	100
	N	7.47	9.53	8.93	6.69	6.19	38.8
	(D,R)	(0.07,0.35)	(0.18,0.5)	(0.29,0.47)	(0.31,2)	(0.33,0.33)	(1.18,3.65)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	15	13.2	11.9	0.9	59	100
	N	7.46	9.46	8.57	7.73	5.42	38.64
	(D,R)	(0.07,0.39)	(0.17,0.42)	(0.23,0.39)	(0.29,0.35)	(0.26,1.59)	(1.02,3.14)

Table 1: Simulation results using the non-parametric model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	8	8.2	2.7	3.7	77.4	100
	N	7.49	7.9	7.6	7.13	7.46	37.58
	(D,R)	(0.08,0.4)	(0.16,1.62)	(0.23,2.53)	(0.26,4.24)	(0.36,5.94)	(1.09,14.72)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	21	18.9	19.8	20	20.3	100
	N	7.59	8.09	7.21	6.78	4.61	34.27
	(D,R)	(0.1,4.38)	(0.24,4.77)	(0.46,4.34)	(1.37,4.14)	(1.46,2.92)	(3.62,20.55)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	4.5	5.8	12.3	19.4	58	100
	N	7.8	8.32	8.19	8.36	8.94	41.61
	(D,R)	(0.14,1.53)	(0.25,3.33)	(0.31,4.82)	(0.53,5.66)	(1.77,6.58)	(3,21.91)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	12.6	12.4	11.8	11.1	52.1	100
	N	7.47	7.51	6.92	6.01	5.1	33.01
	(D,R)	(0.08,3.91)	(0.1,4.67)	(0.16,4.91)	(0.15,4.73)	(0.16,4.41)	(0.64,22.64)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	11.8	18.3	21.5	18	26.9	96.5
	N	12.28	11.94	10.35	7.92	5.38	47.87
	(D,R)	(2.26,0.64)	(2.6,2.39)	(2.68,3.6)	(2.38,3.75)	(1.79,3.11)	(11.71,13.49)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	7.9	21.4	28.3	21.4	20.8	99.8
	N	10.08	11.87	11.14	8.22	4.86	46.18
	(D,R)	(0.79,1.5)	(2.13,4.53)	(2.77,5.83)	(2.43,4.84)	(1.69,3.02)	(9.81,19.71)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	4.6	20.5	20.5	15.6	38.8	100
	N	7.55	7.85	7.66	6.37	4.68	34.11
	(D,R)	(0.09,0.4)	(0.14,1.98)	(0.2,1.96)	(0.27,1.57)	(0.21,1.11)	(0.91,7.01)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	4.9	6.4	19.4	17	52.3	100
	N	7.4	8.04	7.94	7.57	6.06	37.02
	(D,R)	(0.06,0.36)	(0.18,1.97)	(0.24,3.18)	(0.31,3.06)	(0.29,2.5)	(1.07,11.07)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	3.7	9.3	4.8	20.5	61.7	100
	N	7.53	7.91	7.8	7.58	7.21	38.04
	(D,R)	(0.08,0.37)	(0.16,2.03)	(0.21,3.06)	(0.32,4.57)	(0.36,4.34)	(1.14,14.37)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	3.5	69.6	12.7	5.5	8.7	100
	N	7.46	7.83	7.27	2.23	1.2	25.98
	(D,R)	(0.07,0.38)	(0.14,1.95)	(0.21,0.37)	(0.09,0.12)	(0.07,0.07)	(0.58,2.89)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	7.6	6	60.4	11.5	14.5	100
	N	7.55	7.88	7.62	6.75	2.25	32.05
	(D,R)	(0.08,0.4)	(0.14,1.58)	(0.22,3.11)	(0.26,0.98)	(0.12,0.34)	(0.82,6.4)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	6.8	12.3	1.8	56.7	22.4	100
	N	7.47	7.76	7.61	7.03	6.2	36.06
	(D,R)	(0.07,0.37)	(0.13,1.52)	(0.21,2.21)	(0.28,4.26)	(0.28,1.87)	(0.97,10.23)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	20	2.7	62	9	6.3	100
	N	7.47	7.76	6.65	5.88	1.29	29.05
	(D,R)	(0.07,0.38)	(0.16,0.39)	(0.19,2.05)	(0.22,0.29)	(0.08,0.07)	(0.72,3.18)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	21.1	21.3	2.5	43.4	11.7	100
	N	7.44	7.85	6.24	5.1	4.2	30.83
	(D,R)	(0.07,0.38)	(0.16,0.42)	(0.16,0.3)	(0.23,1.5)	(0.22,0.21)	(0.84,2.81)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	20	19.3	15.7	1.5	43.5	100
	N	7.54	7.75	6.47	4.98	4.17	30.91
	(D,R)	(0.09,0.42)	(0.16,0.42)	(0.2,0.33)	(0.17,0.26)	(0.22,1.23)	(0.84,2.66)

Table 2: Simulation results using the non-parametric+ model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	5.6	7.5	4.1	4.4	78.4	100
	N	7.55	7.16	6.94	6.34	6.24	34.22
	(D,R)	(0.08,0.39)	(0.15,1.46)	(0.2,2.45)	(0.24,3.81)	(0.32,4.99)	(0.99,13.1)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	20.2	18.4	19	21.3	21.1	100
	N	7.44	7.2	6.4	6.71	4.66	32.4
	(D,R)	(0.07,4.21)	(0.21,4.13)	(0.39,3.9)	(1.36,4.14)	(1.5,2.94)	(3.53,19.32)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	5.1	5.4	13.1	14.6	61.8	100
	N	7.97	7.43	7.26	6.97	8.96	38.59
	(D,R)	(0.17,1.57)	(0.22,2.95)	(0.28,4.31)	(0.42,4.73)	(1.83,6.68)	(2.92,20.24)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	11.6	10	9.9	11	57.5	100
	N	7.43	7.08	6.27	5.53	4.85	31.16
	(D,R)	(0.06,3.95)	(0.1,4.34)	(0.13,4.47)	(0.13,4.41)	(0.12,4.19)	(0.54,21.36)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	9.3	12.6	19.7	19.1	33.7	94.4
	N	12.11	11.73	11.06	9.11	6.38	50.39
	(D,R)	(2.24,0.65)	(2.59,2.28)	(2.88,3.89)	(2.89,4.39)	(2.08,3.65)	(12.69,14.86)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	4.8	15.6	25.2	31	23.3	99.9
	N	9.99	11.45	11.97	9.71	5.85	48.97
	(D,R)	(0.76,1.5)	(2.1,4.43)	(3.06,6.17)	(2.9,5.73)	(2.02,3.6)	(10.84,21.43)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	3.5	23.2	17.6	16.7	39	100
	N	7.46	7.08	7.05	5.32	4.12	31.03
	(D,R)	(0.07,0.36)	(0.14,1.76)	(0.2,1.72)	(0.19,1.37)	(0.23,1.01)	(0.82,6.22)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	2.7	8.1	19.3	20	49.9	100
	N	7.47	7.16	7.15	6.5	5.19	33.46
	(D,R)	(0.07,0.34)	(0.16,1.79)	(0.23,2.84)	(0.25,2.63)	(0.26,2.03)	(0.97,9.64)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	3.3	7.3	5.8	20.3	63.3	100
	N	7.46	7.11	7.09	6.45	6.24	34.36
	(D,R)	(0.07,0.38)	(0.14,1.74)	(0.22,2.8)	(0.26,3.88)	(0.3,3.72)	(0.99,12.51)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	2.6	66.5	14.7	5.9	10.3	100
	N	7.56	7.2	7.02	2.25	1.2	25.23
	(D,R)	(0.08,0.37)	(0.17,1.77)	(0.2,0.34)	(0.1,0.12)	(0.07,0.05)	(0.61,2.66)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	5.6	4.7	56.8	12.3	20.6	100
	N	7.5	7.17	6.87	6.4	2.43	30.38
	(D,R)	(0.08,0.38)	(0.15,1.44)	(0.19,2.71)	(0.24,0.93)	(0.13,0.38)	(0.79,5.83)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	5.8	11.1	2.8	52.7	27.6	100
	N	7.4	7.11	6.89	6.07	5.85	33.31
	(D,R)	(0.06,0.36)	(0.13,1.47)	(0.21,2.07)	(0.25,3.64)	(0.29,1.82)	(0.94,9.35)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	20.4	2	58.9	9.8	8.9	100
	N	7.38	7.1	5.92	5.52	1.36	27.28
	(D,R)	(0.06,0.4)	(0.15,0.37)	(0.18,1.85)	(0.21,0.28)	(0.07,0.07)	(0.67,2.98)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	18.6	20.8	2.4	44.8	13.4	100
	N	7.46	7.13	6	4.56	4.16	29.32
	(D,R)	(0.07,0.39)	(0.16,0.4)	(0.19,0.31)	(0.18,1.35)	(0.2,0.2)	(0.79,2.66)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	20.2	20.6	13.6	1.2	44.4	100
	N	7.52	7.14	5.85	4.35	3.44	28.3
	(D,R)	(0.08,0.37)	(0.16,0.38)	(0.16,0.27)	(0.17,0.21)	(0.18,1.06)	(0.75,2.29)

Table 3: Simulation results using the parametric model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	4.8	8.7	4.2	3.5	78.8	100
	N	7.41	7.12	7.04	6.31	6.2	34.09
	(D,R)	(0.07,0.37)	(0.13,1.45)	(0.21,2.48)	(0.26,3.77)	(0.32,4.97)	(0.99,13.04)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	18.4	18.7	18.9	23	21	100
	N	7.36	7.2	6.42	6.94	4.79	32.71
	(D,R)	(0.06,4.14)	(0.22,4.18)	(0.42,3.91)	(1.43,4.33)	(1.52,3.08)	(3.65,19.64)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	4.9	5	12.7	16.1	61.3	100
	N	7.86	7.39	7.27	7.19	9.1	38.81
	(D,R)	(0.15,1.58)	(0.23,2.96)	(0.3,4.39)	(0.45,4.86)	(1.83,6.74)	(2.95,20.53)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	11.6	12.7	12.9	11.7	51.1	100
	N	7.44	7.1	6.34	5.36	4.48	30.71
	(D,R)	(0.07,3.82)	(0.1,4.47)	(0.14,4.5)	(0.12,4.23)	(0.13,3.84)	(0.57,20.86)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	8.4	12.8	19	19.5	36.9	96.6
	N	12.03	12.14	11.42	9.43	6.87	51.89
	(D,R)	(2.12,0.56)	(2.69,2.34)	(2.98,4.02)	(2.87,4.49)	(2.3,4.01)	(12.96,15.42)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	5.1	14	27	28.8	24.8	99.7
	N	10.04	11.16	11.85	9.65	5.8	48.5
	(D,R)	(0.8,1.48)	(2,4.14)	(2.99,6.09)	(2.83,5.65)	(2.08,3.6)	(10.71,20.97)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	3.6	20.5	19.2	14.7	42	100
	N	7.55	7.13	7.08	5.46	4.27	31.49
	(D,R)	(0.08,0.37)	(0.15,1.79)	(0.22,1.82)	(0.22,1.39)	(0.22,1.06)	(0.89,6.42)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	3.5	7.7	20	17	51.8	100
	N	7.41	7.11	7.01	6.38	5.15	33.06
	(D,R)	(0.06,0.34)	(0.12,1.81)	(0.21,2.81)	(0.25,2.56)	(0.26,2.04)	(0.9,9.55)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	3.9	8.1	5.1	19.3	63.6	100
	N	7.41	7.11	7.02	6.34	6.15	34.02
	(D,R)	(0.06,0.36)	(0.14,1.82)	(0.2,2.78)	(0.25,3.85)	(0.28,3.68)	(0.93,12.5)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	5.5	66.4	14.7	6.6	6.8	100
	N	7.46	7.11	6.69	2	0.98	24.24
	(D,R)	(0.07,0.38)	(0.14,1.71)	(0.21,0.34)	(0.08,0.1)	(0.05,0.05)	(0.54,2.59)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	6	5.1	56.2	12.7	20	100
	N	7.48	7.14	6.87	6.34	2.44	30.27
	(D,R)	(0.08,0.37)	(0.14,1.42)	(0.19,2.7)	(0.24,0.94)	(0.13,0.37)	(0.77,5.81)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	6.4	12.2	2	53.5	25.9	100
	N	7.59	7.2	6.94	6.01	5.68	33.43
	(D,R)	(0.1,0.36)	(0.14,1.4)	(0.21,2.01)	(0.22,3.64)	(0.27,1.66)	(0.94,9.08)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	22.1	2.6	57.7	10	7.6	100
	N	7.55	7.13	5.8	5.35	1.27	27.08
	(D,R)	(0.09,0.41)	(0.13,0.35)	(0.19,1.74)	(0.21,0.29)	(0.05,0.07)	(0.67,2.85)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	19.7	20.7	1.8	47.3	10.5	100
	N	7.46	7.13	5.77	4.3	4.09	28.75
	(D,R)	(0.07,0.34)	(0.15,0.34)	(0.15,0.28)	(0.17,1.31)	(0.17,0.16)	(0.71,2.44)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	22.2	20.3	16.9	1.1	39.5	100
	N	7.39	7.11	5.73	4.16	3.04	27.43
	(D,R)	(0.06,0.38)	(0.16,0.34)	(0.17,0.3)	(0.18,0.2)	(0.16,0.86)	(0.74,2.08)

Table 4: Simulation results using the parametric+ model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$.

3 Simulation results for robustness analysis

3.1 10+10

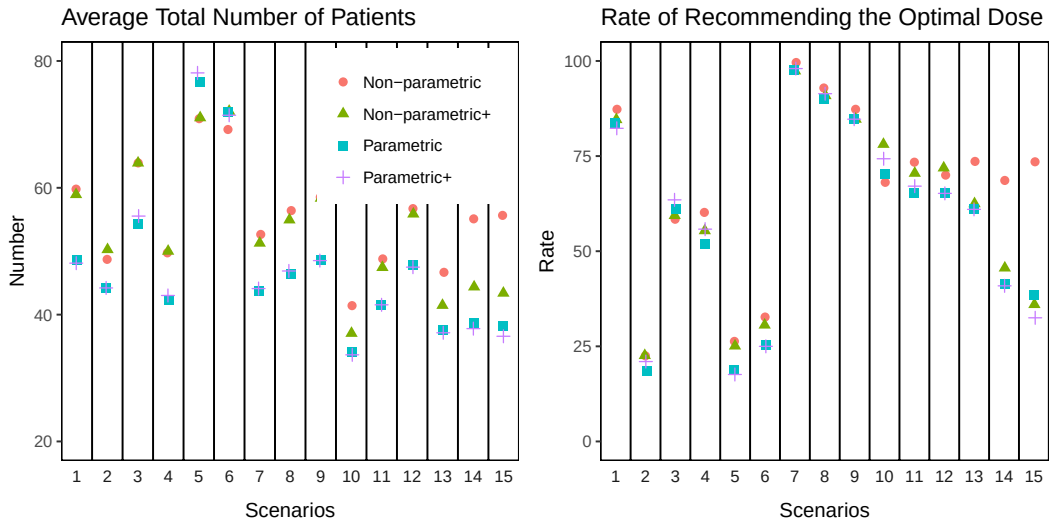

Figure 2: Average total number of patients and the average rate of reporting the optimal dose levels(s) for Scenarios 1-15.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	1.5	5.7	2.2	3.3	87.3	100
	N	10.9	11.72	12.13	12.32	12.72	59.79
	(D,R)	(0.11,0.55)	(0.22,2.32)	(0.36,4.23)	(0.5,7.51)	(0.65,10.16)	(1.83,24.76)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	24	19	19.3	22.5	15.2	100
	N	10.96	12.15	10.41	9.33	5.86	48.71
	(D,R)	(0.1,6.25)	(0.36,7.02)	(0.62,6.25)	(1.76,5.76)	(1.85,3.74)	(4.7,29.03)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	3	4.5	12.9	21.2	58.4	100
	N	11.91	12.47	12.78	13.28	13.48	63.92
	(D,R)	(0.23,2.36)	(0.38,5.03)	(0.5,7.78)	(0.83,9.11)	(2.73,9.91)	(4.68,34.19)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	13.5	10.6	8.9	6.8	60.2	100
	N	10.83	11.19	10.09	9.28	8.35	49.74
	(D,R)	(0.1,5.72)	(0.16,6.89)	(0.2,7.18)	(0.25,7.33)	(0.26,7.19)	(0.97,34.31)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	10.1	18.2	26.3	20.6	21.9	97.1
	N	18.33	17.97	15.82	11.77	7.01	70.9
	(D,R)	(3.22,0.85)	(3.94,3.5)	(4,5.59)	(3.5,5.49)	(2.36,4.1)	(17.02,19.52)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	5.4	21.4	32.7	26.5	13.9	99.9
	N	15.59	18.31	16.98	12.17	6.14	69.19
	(D,R)	(1.23,2.35)	(3.29,6.95)	(4.29,8.92)	(3.61,7.2)	(2.17,3.8)	(14.59,29.22)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	0.4	20.8	22.3	16	40.5	100
	N	10.93	11.91	12.32	9.98	7.51	52.65
	(D,R)	(0.11,0.54)	(0.24,3.05)	(0.4,3.12)	(0.41,2.48)	(0.39,1.82)	(1.55,11.01)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	0.9	6.2	22	22.3	48.6	100
	N	11	11.84	12.37	11.91	9.28	56.4
	(D,R)	(0.11,0.54)	(0.24,2.91)	(0.37,4.97)	(0.48,4.86)	(0.46,3.64)	(1.65,16.94)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	0.6	7.3	4.8	23.6	63.7	100
	N	10.91	11.75	12.25	12.21	11.4	58.52
	(D,R)	(0.11,0.56)	(0.22,2.98)	(0.34,4.97)	(0.5,7.35)	(0.59,6.73)	(1.75,22.59)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	0.6	68.1	6.3	2.5	22.5	100
	N	11.01	11.71	11.03	4.1	3.57	41.42
	(D,R)	(0.12,0.55)	(0.22,2.98)	(0.32,0.56)	(0.16,0.2)	(0.19,0.18)	(1.02,4.47)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	1.4	3.2	73.4	7.3	14.7	100
	N	10.96	11.92	12.39	10.62	2.9	48.79
	(D,R)	(0.11,0.57)	(0.24,2.37)	(0.39,4.94)	(0.4,1.57)	(0.14,0.46)	(1.28,9.91)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	1.9	8.6	1.5	70	18	100
	N	10.91	11.49	12.38	12.06	9.88	56.72
	(D,R)	(0.1,0.54)	(0.19,2.26)	(0.38,3.72)	(0.51,7.17)	(0.46,2.96)	(1.64,16.66)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	9.2	0	73.6	3.5	13.7	100
	N	10.99	11.89	11.42	9.92	2.45	46.67
	(D,R)	(0.13,0.61)	(0.24,0.57)	(0.33,3.4)	(0.39,0.47)	(0.12,0.12)	(1.21,5.17)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	7.8	8.1	0.2	68.6	15.3	100
	N	10.88	11.85	11.73	11.23	9.41	55.1
	(D,R)	(0.1,0.56)	(0.23,0.61)	(0.38,0.62)	(0.47,3.43)	(0.46,0.49)	(1.63,5.72)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	7.8	10.1	8.2	0.4	73.5	100
	N	10.97	11.83	11.65	10.82	10.38	55.65
	(D,R)	(0.11,0.53)	(0.23,0.6)	(0.35,0.56)	(0.42,0.54)	(0.51,3.08)	(1.62,5.33)

Table 5: Simulation results using the non-parametric model. Cohort size 10 and maximum size 20 for each dose level. $r^{(l)} = 10$.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	4.2	5	2.8	3.4	84.6	100
	N	10.84	11.8	11.83	12.35	12.12	58.94
	(D,R)	(0.1,0.54)	(0.23,2.31)	(0.34,4.09)	(0.52,7.5)	(0.6,9.71)	(1.8,24.14)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	21.6	20.1	19.1	22.6	16.6	100
	N	10.85	12.4	10.87	9.76	6.38	50.26
	(D,R)	(0.1,6.06)	(0.39,7.17)	(0.66,6.55)	(1.96,6.06)	(2.03,4.07)	(5.13,29.92)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	3	4.5	14.5	18.6	59.4	100
	N	11.73	12.49	12.83	13.31	13.55	63.91
	(D,R)	(0.22,2.36)	(0.36,4.99)	(0.52,7.71)	(0.84,9.08)	(2.71,10.04)	(4.64,34.19)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	11.8	11.2	11.4	10.2	55.4	100
	N	11.04	11.28	10.29	9.29	8.11	50.01
	(D,R)	(0.13,5.71)	(0.16,6.98)	(0.21,7.33)	(0.26,7.39)	(0.26,6.94)	(1.01,34.35)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	10.8	17.9	25.1	19	23.4	96.2
	N	18.45	17.82	15.56	11.9	7.32	71.05
	(D,R)	(3.3,0.88)	(3.98,3.66)	(3.98,5.44)	(3.66,5.63)	(2.43,4.2)	(17.35,19.81)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	4.9	19	30.6	27.3	18.2	100
	N	15.94	18.32	17.46	13.04	7.32	72.08
	(D,R)	(1.3,2.37)	(3.31,7.01)	(4.39,9.17)	(3.88,7.67)	(2.59,4.57)	(15.48,30.78)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	2.6	21.4	21.3	16.1	38.6	100
	N	11.16	11.81	11.57	9.61	7.13	51.28
	(D,R)	(0.13,0.62)	(0.23,2.98)	(0.31,2.9)	(0.38,2.4)	(0.34,1.79)	(1.39,10.68)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	3	6.1	23.1	19.1	48.7	100
	N	10.75	11.72	12.15	11.39	8.93	54.94
	(D,R)	(0.08,0.5)	(0.23,2.85)	(0.37,4.95)	(0.44,4.54)	(0.47,3.64)	(1.6,16.48)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	2	7.1	6.2	21.2	63.5	100
	N	10.92	11.82	12.22	12.05	11.35	58.36
	(D,R)	(0.1,0.55)	(0.22,2.95)	(0.36,4.86)	(0.48,7.24)	(0.6,6.77)	(1.76,22.38)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	2.1	78.1	10.4	5.2	4.2	100
	N	11.12	11.82	10.6	2.38	1.14	37.06
	(D,R)	(0.12,0.55)	(0.24,3.04)	(0.32,0.54)	(0.1,0.13)	(0.06,0.05)	(0.84,4.32)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	5.7	2.3	70.5	10.7	10.8	100
	N	10.84	11.86	11.75	10.3	2.69	47.44
	(D,R)	(0.1,0.58)	(0.25,2.28)	(0.35,4.63)	(0.42,1.54)	(0.13,0.42)	(1.25,9.45)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	3.9	8.3	1.6	71.9	14.3	100
	N	10.88	11.85	12.03	11.7	9.41	55.87
	(D,R)	(0.1,0.53)	(0.25,2.38)	(0.35,3.65)	(0.48,7.07)	(0.45,2.75)	(1.63,16.38)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	23.5	0.9	62.5	7.1	6	100
	N	10.99	11.22	9.49	8.21	1.56	41.47
	(D,R)	(0.11,0.54)	(0.18,0.56)	(0.27,2.86)	(0.33,0.44)	(0.07,0.07)	(0.97,4.48)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	23.5	24.7	0.6	45.6	5.6	100
	N	10.96	11.72	9.14	7.05	5.5	44.37
	(D,R)	(0.12,0.54)	(0.26,0.6)	(0.26,0.41)	(0.31,2.14)	(0.26,0.24)	(1.19,3.93)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	23.2	24.6	15.6	0.6	36	100
	N	10.95	11.52	9.11	6.62	5.19	43.39
	(D,R)	(0.11,0.53)	(0.23,0.62)	(0.28,0.43)	(0.27,0.32)	(0.26,1.53)	(1.15,3.43)

Table 6: Simulation results using the non-parametric+ model. Cohort size 10 and maximum size 20 for each dose level. $r^{(l)} = 10$.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	4.3	6.2	3	2.7	83.8	100
	N	10.07	10.28	9.83	9.33	9.12	48.63
	(D,R)	(0.1,0.48)	(0.2,2.03)	(0.27,3.45)	(0.38,5.54)	(0.47,7.32)	(1.42,18.81)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	25.6	19	17.4	18.5	19.5	100
	N	10.06	10.47	8.55	9.15	5.95	44.18
	(D,R)	(0.09,5.84)	(0.3,5.97)	(0.56,5.14)	(1.87,5.61)	(1.9,3.84)	(4.71,26.4)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	4.2	4.8	14.5	15.4	61.1	100
	N	10.09	10.81	9.94	10.2	13.33	54.37
	(D,R)	(0.2,2.02)	(0.32,4.19)	(0.37,5.9)	(0.61,6.86)	(2.69,9.89)	(4.2,28.87)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	15.3	12.4	9.5	11	51.8	100
	N	10.04	10.22	8.53	7.26	6.32	42.37
	(D,R)	(0.11,5.25)	(0.12,6.32)	(0.17,6.06)	(0.18,5.77)	(0.2,5.37)	(0.78,28.78)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	5.9	11.5	18.8	24	35.8	96
	N	17.52	18.02	16.91	14.27	9.94	76.66
	(D,R)	(3.18,0.85)	(3.95,3.65)	(4.42,5.97)	(4.32,6.8)	(3.29,5.69)	(19.15,22.96)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	2.3	12.4	25.3	35.2	24.7	99.9
	N	11.76	17.49	18.39	15.15	9.17	71.96
	(D,R)	(0.94,1.81)	(3.08,6.59)	(4.57,9.54)	(4.54,8.87)	(3.13,5.69)	(16.27,32.49)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	2.5	21.9	20.4	14.4	40.8	100
	N	10.03	10.24	9.88	7.87	5.74	43.76
	(D,R)	(0.08,0.53)	(0.19,2.54)	(0.3,2.52)	(0.31,1.99)	(0.27,1.45)	(1.16,9.03)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	2	7.9	25.9	19.6	44.6	100
	N	10.04	10.35	9.96	9.36	6.68	46.39
	(D,R)	(0.11,0.52)	(0.21,2.6)	(0.29,4.03)	(0.38,3.76)	(0.35,2.62)	(1.34,13.52)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	2.1	8.6	4.6	21.4	63.3	100
	N	10.03	10.3	10.03	9.39	8.83	48.58
	(D,R)	(0.1,0.47)	(0.2,2.59)	(0.35,3.94)	(0.35,5.6)	(0.45,5.36)	(1.45,17.96)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	2.6	70.3	14.7	5.2	7.2	100
	N	10.04	10.32	9.77	2.78	1.27	34.18
	(D,R)	(0.1,0.48)	(0.2,2.54)	(0.27,0.52)	(0.1,0.13)	(0.06,0.08)	(0.73,3.74)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	5.8	4.5	65.2	9.9	14.6	100
	N	10.03	10.34	9.52	9.1	2.54	41.53
	(D,R)	(0.1,0.51)	(0.19,1.97)	(0.28,3.77)	(0.34,1.38)	(0.12,0.39)	(1.04,8.01)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	3.7	10	1.8	65.4	19.1	100
	N	10.06	10.32	9.86	9.05	8.61	47.9
	(D,R)	(0.11,0.5)	(0.2,2.02)	(0.31,2.95)	(0.37,5.41)	(0.46,2.58)	(1.44,13.47)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	22.3	1.3	61	7.1	8.3	100
	N	10.03	10.29	7.91	7.7	1.6	37.53
	(D,R)	(0.09,0.47)	(0.19,0.51)	(0.25,2.36)	(0.29,0.4)	(0.08,0.09)	(0.9,3.82)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	24.1	23.8	1.1	41.3	9.7	100
	N	10.03	10.23	7.73	5.55	5.18	38.72
	(D,R)	(0.1,0.49)	(0.19,0.52)	(0.25,0.36)	(0.23,1.68)	(0.25,0.28)	(1.02,3.34)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	22.2	22.2	16.7	0.5	38.4	100
	N	10.03	10.3	7.92	5.81	4.11	38.17
	(D,R)	(0.1,0.47)	(0.22,0.49)	(0.24,0.36)	(0.23,0.3)	(0.2,1.16)	(0.99,2.78)

Table 7: Simulation results using the parametric model. Cohort size 10 and maximum size 20 for each dose level. $r^{(l)} = 10$.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	4.9	6.5	3.4	2.9	82.3	100
	N	10	10.27	9.67	9.21	8.97	48.12
	(D,R)	(0.08,0.48)	(0.2,2.08)	(0.31,3.42)	(0.36,5.52)	(0.45,7.15)	(1.38,18.65)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	26.4	17.3	18.9	21	16.4	100
	N	10.08	10.54	8.64	9.21	5.74	44.21
	(D,R)	(0.1,5.82)	(0.34,5.97)	(0.55,5.24)	(1.85,5.67)	(1.88,3.69)	(4.71,26.4)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	3.6	4.7	14.1	14.1	63.5	100
	N	10.23	10.94	10.14	10.46	13.77	55.54
	(D,R)	(0.23,2.02)	(0.37,4.39)	(0.4,6.03)	(0.62,7.06)	(2.78,10.3)	(4.41,29.8)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	13.1	12.3	9.4	9.4	55.8	100
	N	10.05	10.17	8.79	7.46	6.55	43.02
	(D,R)	(0.1,5.29)	(0.16,6.36)	(0.17,6.23)	(0.18,5.87)	(0.18,5.6)	(0.8,29.35)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	4.5	10.4	17.6	27.3	37.3	97.1
	N	16.24	18.31	17.54	15.12	10.91	78.12
	(D,R)	(2.9,0.81)	(4.07,3.76)	(4.62,6.21)	(4.46,7.13)	(3.65,6.34)	(19.71,24.24)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	2.8	12.2	25	36.7	23.3	100
	N	11.78	17.4	18.28	15.07	8.91	71.44
	(D,R)	(0.92,1.75)	(3.19,6.64)	(4.53,9.47)	(4.53,8.97)	(3.11,5.54)	(16.28,32.38)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	2	21.3	20.8	15.8	40.1	100
	N	10.06	10.34	9.93	7.89	5.89	44.11
	(D,R)	(0.08,0.49)	(0.21,2.64)	(0.29,2.48)	(0.32,2.02)	(0.31,1.52)	(1.22,9.15)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	1.7	6.9	23.4	20.8	47.2	100
	N	10.05	10.31	9.99	9.5	7.03	46.88
	(D,R)	(0.11,0.46)	(0.19,2.57)	(0.28,3.99)	(0.36,3.83)	(0.35,2.85)	(1.3,13.71)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	2.4	6.6	6.3	21.2	63.5	100
	N	10.03	10.31	9.94	9.49	8.75	48.52
	(D,R)	(0.09,0.48)	(0.21,2.55)	(0.31,3.99)	(0.36,5.66)	(0.44,5.28)	(1.42,17.96)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	2.8	74.3	12.1	5.9	4.9	100
	N	10.04	10.43	9.79	2.31	1.09	33.66
	(D,R)	(0.11,0.52)	(0.22,2.65)	(0.28,0.48)	(0.1,0.12)	(0.05,0.05)	(0.76,3.82)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	5.9	3.6	67.1	9.9	13.5	100
	N	10.04	10.36	9.56	9.18	2.41	41.55
	(D,R)	(0.11,0.53)	(0.21,2.03)	(0.3,3.88)	(0.38,1.36)	(0.12,0.39)	(1.1,8.2)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	5.3	8.8	1.2	65.2	19.5	100
	N	10.02	10.33	9.67	8.89	8.58	47.49
	(D,R)	(0.1,0.5)	(0.19,2.04)	(0.3,2.91)	(0.35,5.34)	(0.42,2.52)	(1.35,13.31)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	23.2	1.4	61	8.1	6.3	100
	N	10.06	10.23	7.76	7.59	1.5	37.14
	(D,R)	(0.11,0.5)	(0.19,0.51)	(0.24,2.33)	(0.3,0.4)	(0.08,0.08)	(0.91,3.82)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	26.4	24.1	0.6	40.9	8	100
	N	10.05	10.2	7.46	5.16	4.92	37.79
	(D,R)	(0.09,0.54)	(0.21,0.49)	(0.21,0.35)	(0.21,1.51)	(0.24,0.24)	(0.96,3.13)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	23.9	27	16.4	0.2	32.5	100
	N	10.05	10.3	7.75	5.04	3.45	36.59
	(D,R)	(0.1,0.51)	(0.21,0.54)	(0.24,0.37)	(0.2,0.25)	(0.16,1.04)	(0.91,2.7)

Table 8: Simulation results using the parametric+ model. Cohort size 10 and maximum size 20 for each dose level. $r^{(l)} = 10$.

3.2 $r = 1$

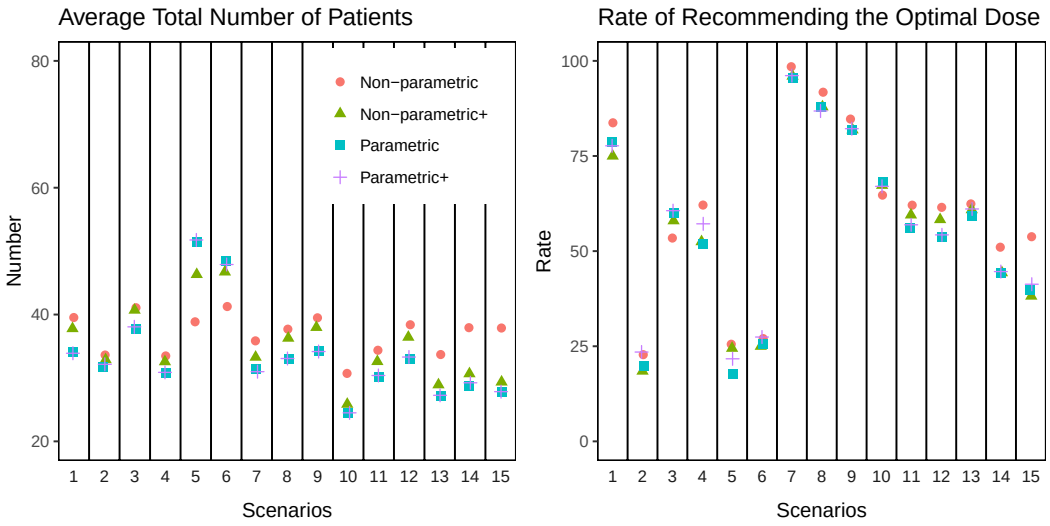

Figure 3: Average total number of patients and the average rate of reporting the optimal dose levels(s) for Scenarios 1-15.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	2.1	6.3	3.8	4.1	83.7	100
	N	7.38	8.26	8.13	7.88	7.88	39.53
	(D,R)	(0.06,0.34)	(0.16,1.64)	(0.23,2.86)	(0.31,4.75)	(0.39,6.28)	(1.15,15.86)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	18.8	18.6	23.7	22.8	16.1	100
	N	7.46	8.15	7.42	6.6	3.98	33.61
	(D,R)	(0.07,4.23)	(0.26,4.75)	(0.47,4.5)	(1.28,4.07)	(1.31,2.51)	(3.39,20.06)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	4.9	5.1	14	22.6	53.4	100
	N	7.89	8.41	8.28	8.25	8.25	41.08
	(D,R)	(0.15,1.6)	(0.28,3.34)	(0.34,5.02)	(0.51,5.62)	(1.66,6.1)	(2.94,21.67)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	11.7	12	9.1	5.1	62.1	100
	N	7.46	7.58	6.75	6.15	5.53	33.47
	(D,R)	(0.07,3.91)	(0.1,4.76)	(0.11,4.81)	(0.16,4.89)	(0.17,4.79)	(0.61,23.17)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	16.9	22.7	25.5	12.2	13.7	91
	N	11.67	10.69	8.36	5.37	2.76	38.85
	(D,R)	(2.06,0.57)	(2.41,2.14)	(2.16,2.94)	(1.7,2.45)	(0.87,1.59)	(9.2,9.7)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	12.2	26.5	27	21.7	11.9	99.3
	N	10.02	11.58	9.77	6.73	3.18	41.27
	(D,R)	(0.77,1.42)	(2.16,4.43)	(2.41,5.09)	(2.02,4.01)	(1.12,1.98)	(8.48,16.93)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	1.5	20.2	19.2	16.2	42.9	100
	N	7.46	8.17	8.31	6.8	5.12	35.85
	(D,R)	(0.07,0.38)	(0.16,2.07)	(0.27,2.08)	(0.29,1.7)	(0.25,1.24)	(1.04,7.48)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	1.4	6.8	22.3	19.4	50.1	100
	N	7.57	8.22	8.18	7.81	5.91	37.7
	(D,R)	(0.09,0.38)	(0.17,2.02)	(0.23,3.31)	(0.35,3.13)	(0.3,2.31)	(1.14,11.15)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	1.2	7.7	6.4	19.6	65.1	100
	N	7.56	8.29	8.36	7.9	7.38	39.49
	(D,R)	(0.09,0.39)	(0.18,2.12)	(0.26,3.35)	(0.32,4.7)	(0.36,4.47)	(1.21,15.03)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	1.7	64.7	10.4	6.1	17.1	100
	N	7.5	8.09	8.88	3.68	2.58	30.72
	(D,R)	(0.08,0.38)	(0.14,2.05)	(0.27,0.46)	(0.14,0.2)	(0.12,0.12)	(0.76,3.22)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	1.8	4.2	62.1	12.9	19	100
	N	7.48	8.44	8.04	7.44	2.96	34.37
	(D,R)	(0.07,0.39)	(0.17,1.66)	(0.22,3.26)	(0.29,1.12)	(0.16,0.4)	(0.92,6.82)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	2.2	10	2.6	61.5	23.7	100
	N	7.51	8.4	8.2	7.64	6.64	38.39
	(D,R)	(0.08,0.36)	(0.16,1.72)	(0.24,2.4)	(0.31,4.57)	(0.36,1.95)	(1.14,11.01)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	15.4	0.5	62.4	7.4	14.3	100
	N	7.38	9.43	7.23	7.2	2.46	33.69
	(D,R)	(0.06,0.4)	(0.17,0.47)	(0.23,2.13)	(0.28,0.36)	(0.13,0.12)	(0.88,3.48)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	14.9	16.2	0.5	51	17.4	100
	N	7.48	9.48	8.74	6.12	6.1	37.93
	(D,R)	(0.08,0.37)	(0.18,0.47)	(0.24,0.46)	(0.24,1.79)	(0.33,0.31)	(1.07,3.4)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	15.6	16.7	13.2	0.7	53.8	100
	N	7.35	9.53	8.64	7.22	5.14	37.88
	(D,R)	(0.05,0.37)	(0.21,0.5)	(0.26,0.43)	(0.29,0.35)	(0.27,1.53)	(1.07,3.18)

Table 9: Simulation results using the non-parametric model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 1$.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	6.5	9	4	5.5	75	100
	N	7.46	7.88	7.76	7.34	7.34	37.79
	(D,R)	(0.07,0.4)	(0.16,1.65)	(0.25,2.75)	(0.3,4.41)	(0.38,5.81)	(1.15,15.02)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	21.6	21.5	19.1	18.5	19.3	100
	N	7.45	7.97	6.97	6.2	4.3	32.89
	(D,R)	(0.07,4.33)	(0.22,4.65)	(0.44,4.16)	(1.26,3.81)	(1.36,2.79)	(3.36,19.73)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	6.4	5	13.7	16.9	58	100
	N	7.92	8.31	8.06	7.94	8.47	40.7
	(D,R)	(0.15,1.57)	(0.26,3.33)	(0.31,4.88)	(0.46,5.41)	(1.66,6.32)	(2.83,21.52)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	12.3	14.2	11.4	9.6	52.5	100
	N	7.33	7.62	6.81	5.82	4.98	32.56
	(D,R)	(0.05,3.78)	(0.11,4.79)	(0.13,4.82)	(0.14,4.63)	(0.14,4.27)	(0.57,22.28)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	12	18.4	24.5	17	23.4	95.3
	N	12.17	11.61	10.12	7.66	4.77	46.32
	(D,R)	(2.24,0.58)	(2.5,2.25)	(2.64,3.51)	(2.37,3.55)	(1.56,2.78)	(11.31,12.67)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	8.5	18.8	25	27.9	19.8	100
	N	10.05	11.81	11.06	8.61	5.19	46.72
	(D,R)	(0.76,1.54)	(2.06,4.43)	(2.7,5.74)	(2.53,5.09)	(1.83,3.2)	(9.88,20)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	4	22.4	23.3	15.2	35.1	100
	N	7.39	7.8	7.73	6.05	4.3	33.28
	(D,R)	(0.07,0.36)	(0.14,1.92)	(0.25,1.91)	(0.26,1.43)	(0.21,1.05)	(0.92,6.68)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	4	8.1	20.8	17.4	49.7	100
	N	7.47	7.91	7.93	7.31	5.66	36.28
	(D,R)	(0.07,0.34)	(0.17,1.95)	(0.24,3.09)	(0.3,2.87)	(0.26,2.28)	(1.05,10.53)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	3.9	8.9	5.3	21	60.9	100
	N	7.46	7.9	7.88	7.74	7	37.98
	(D,R)	(0.07,0.34)	(0.16,2)	(0.24,3.15)	(0.34,4.69)	(0.34,4.23)	(1.14,14.41)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	3.5	67.3	16	5.1	8.1	100
	N	7.47	7.76	7.18	2.35	1.11	25.86
	(D,R)	(0.07,0.34)	(0.15,1.95)	(0.21,0.39)	(0.12,0.11)	(0.07,0.05)	(0.61,2.84)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	6.2	4.7	59.5	14.5	15.1	100
	N	7.58	7.9	7.8	6.84	2.48	32.6
	(D,R)	(0.09,0.35)	(0.17,1.56)	(0.25,3.12)	(0.3,1.02)	(0.13,0.33)	(0.94,6.39)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	4.8	13.2	2	58.3	21.7	100
	N	7.5	7.97	7.84	7.01	6.11	36.43
	(D,R)	(0.08,0.37)	(0.17,1.65)	(0.24,2.39)	(0.26,4.28)	(0.29,1.81)	(1.04,10.49)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	19.5	2.8	60.9	9.5	7.3	100
	N	7.52	7.71	6.61	5.73	1.36	28.92
	(D,R)	(0.08,0.35)	(0.16,0.41)	(0.2,1.97)	(0.21,0.3)	(0.08,0.07)	(0.72,3.1)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	22.2	19.7	1.3	44.3	12.5	100
	N	7.43	7.63	6.24	5.05	4.31	30.67
	(D,R)	(0.07,0.38)	(0.13,0.36)	(0.19,0.28)	(0.21,1.49)	(0.21,0.23)	(0.81,2.73)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	23.4	22.3	14.3	1.8	38.2	100
	N	7.42	7.72	6.12	4.48	3.63	29.37
	(D,R)	(0.07,0.39)	(0.15,0.43)	(0.18,0.32)	(0.18,0.25)	(0.19,1.13)	(0.78,2.52)

Table 10: Simulation results using the non-parametric+ model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 1$.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	6.3	7.1	3.2	4.7	78.7	100
	N	7.5	7.19	6.85	6.29	6.29	34.12
	(D,R)	(0.08,0.36)	(0.15,1.39)	(0.21,2.4)	(0.25,3.83)	(0.33,4.99)	(1.02,12.96)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	19.6	22.9	16.6	19.9	21	100
	N	7.48	7.18	6.26	6.19	4.58	31.69
	(D,R)	(0.07,4.21)	(0.21,4.22)	(0.36,3.73)	(1.22,3.9)	(1.47,2.93)	(3.33,18.99)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	5.5	6.5	12.8	15.1	60.1	100
	N	7.79	7.41	7.11	6.77	8.69	37.77
	(D,R)	(0.14,1.54)	(0.22,3.02)	(0.29,4.25)	(0.41,4.65)	(1.74,6.47)	(2.8,19.93)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	12.4	12.9	12	10.9	51.8	100
	N	7.56	7.15	6.24	5.3	4.47	30.72
	(D,R)	(0.09,3.96)	(0.12,4.43)	(0.12,4.42)	(0.12,4.16)	(0.16,3.83)	(0.6,20.8)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	8.6	13.9	17.8	20.7	35.4	96.4
	N	12.06	12.03	11.36	9.25	6.69	51.39
	(D,R)	(2.13,0.63)	(2.68,2.42)	(2.92,3.9)	(2.87,4.44)	(2.16,3.85)	(12.77,15.24)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	4.3	16.7	25.6	29.6	23.6	99.8
	N	10.07	11.14	11.83	9.58	5.78	48.4
	(D,R)	(0.8,1.45)	(2.03,4.29)	(3.01,6.07)	(2.88,5.68)	(2.02,3.62)	(10.74,21.12)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	4.5	20.4	18.6	16.4	40.1	100
	N	7.55	7.17	6.94	5.45	4.23	31.34
	(D,R)	(0.08,0.37)	(0.14,1.72)	(0.21,1.72)	(0.2,1.37)	(0.22,1.03)	(0.86,6.21)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	4.3	7.7	19.8	20.2	48	100
	N	7.48	7.2	6.99	6.34	5.02	33.03
	(D,R)	(0.08,0.41)	(0.16,1.82)	(0.2,2.85)	(0.26,2.6)	(0.25,1.99)	(0.95,9.68)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	4.1	8.8	5.3	17.2	64.6	100
	N	7.41	7.17	7.15	6.38	6.08	34.19
	(D,R)	(0.07,0.4)	(0.16,1.83)	(0.22,2.83)	(0.26,3.83)	(0.28,3.68)	(0.98,12.58)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	3.1	68	16.6	5.5	6.8	100
	N	7.39	7.18	6.87	2.1	0.92	24.47
	(D,R)	(0.06,0.33)	(0.17,1.83)	(0.2,0.36)	(0.08,0.11)	(0.05,0.05)	(0.56,2.68)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	6.2	4.5	56	12.5	20.8	100
	N	7.43	7.1	6.82	6.33	2.4	30.08
	(D,R)	(0.06,0.37)	(0.14,1.36)	(0.2,2.72)	(0.25,0.96)	(0.11,0.36)	(0.77,5.77)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	6.2	13	1.9	53.7	25.2	100
	N	7.52	7.11	6.88	5.87	5.63	33
	(D,R)	(0.08,0.39)	(0.13,1.45)	(0.23,2.06)	(0.25,3.48)	(0.28,1.67)	(0.97,9.05)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	20.5	2.1	59.3	10.1	8	100
	N	7.41	7.13	5.91	5.47	1.29	27.22
	(D,R)	(0.07,0.34)	(0.16,0.33)	(0.18,1.77)	(0.22,0.25)	(0.07,0.06)	(0.7,2.75)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	20.9	20.2	1.6	44.3	13	100
	N	7.43	7.13	5.7	4.28	4.11	28.66
	(D,R)	(0.07,0.38)	(0.15,0.38)	(0.16,0.29)	(0.18,1.26)	(0.23,0.18)	(0.78,2.5)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	19.1	20.9	18.9	1.2	39.9	100
	N	7.4	7.08	5.89	4.35	3.13	27.85
	(D,R)	(0.06,0.38)	(0.12,0.39)	(0.17,0.34)	(0.18,0.22)	(0.16,0.94)	(0.7,2.27)

Table 11: Simulation results using the parametric model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 1$.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	7.2	7.8	3.4	3.9	77.7	100
	N	7.53	7.14	6.85	6.24	6.14	33.89
	(D,R)	(0.08,0.39)	(0.14,1.4)	(0.2,2.39)	(0.26,3.77)	(0.31,4.92)	(1,12.86)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	20.5	18.6	18.9	23.5	18.5	100
	N	7.43	7.17	6.38	6.69	4.49	32.15
	(D,R)	(0.07,4.21)	(0.2,4.1)	(0.45,3.84)	(1.36,4.16)	(1.43,2.83)	(3.52,19.13)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	4.7	6.7	13.5	14.5	60.6	100
	N	7.79	7.39	7.2	6.96	8.71	38.06
	(D,R)	(0.14,1.55)	(0.22,2.9)	(0.28,4.3)	(0.4,4.71)	(1.7,6.48)	(2.75,19.95)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	11.8	12.2	10.7	8.1	57.2	100
	N	7.48	7.07	6.33	5.4	4.61	30.88
	(D,R)	(0.07,3.86)	(0.1,4.37)	(0.14,4.47)	(0.12,4.16)	(0.15,4)	(0.58,20.86)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	8	11.8	21.7	19.6	34.7	95.8
	N	12.17	12.17	11.47	9.38	6.57	51.76
	(D,R)	(2.24,0.63)	(2.67,2.42)	(2.97,4.11)	(2.84,4.34)	(2.19,3.77)	(12.9,15.27)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	6.1	15.3	27.4	29	22.2	100
	N	10.07	11.21	11.73	9.4	5.47	47.89
	(D,R)	(0.81,1.54)	(1.99,4.28)	(2.91,6.05)	(2.84,5.56)	(1.92,3.33)	(10.46,20.76)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	3.9	20.8	20.2	15.4	39.7	100
	N	7.41	7.13	7	5.37	4.07	30.99
	(D,R)	(0.07,0.38)	(0.14,1.75)	(0.21,1.78)	(0.23,1.33)	(0.21,0.99)	(0.86,6.23)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	4.3	8.9	19.9	18	48.9	100
	N	7.46	7.16	7.13	6.26	5.06	33.07
	(D,R)	(0.07,0.38)	(0.17,1.8)	(0.23,2.83)	(0.27,2.5)	(0.27,2.03)	(1.01,9.54)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	4.6	7.1	6.1	19.3	62.9	100
	N	7.51	7.11	7.01	6.42	6.12	34.17
	(D,R)	(0.08,0.4)	(0.11,1.72)	(0.22,2.76)	(0.26,3.8)	(0.29,3.66)	(0.95,12.34)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	5.1	67	13.7	7.2	7	100
	N	7.55	7.13	6.76	2.01	1.06	24.5
	(D,R)	(0.08,0.4)	(0.12,1.8)	(0.18,0.34)	(0.09,0.12)	(0.06,0.05)	(0.54,2.72)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	5.5	4.9	56.9	14.5	18.2	100
	N	7.53	7.15	6.99	6.33	2.39	30.39
	(D,R)	(0.08,0.38)	(0.15,1.45)	(0.21,2.78)	(0.25,0.97)	(0.11,0.33)	(0.8,5.92)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	6.1	12.1	1.7	54.3	25.8	100
	N	7.54	7.2	6.86	5.96	5.75	33.3
	(D,R)	(0.09,0.41)	(0.15,1.43)	(0.21,2.04)	(0.25,3.67)	(0.28,1.69)	(0.97,9.25)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	18.8	2.6	61.1	10.9	6.6	100
	N	7.39	7.12	5.97	5.52	1.26	27.27
	(D,R)	(0.06,0.35)	(0.14,0.39)	(0.2,1.79)	(0.22,0.27)	(0.06,0.06)	(0.68,2.86)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	18.3	20.6	3.2	44.6	13.3	100
	N	7.51	7.13	5.99	4.45	4.17	29.24
	(D,R)	(0.08,0.34)	(0.14,0.36)	(0.2,0.33)	(0.18,1.36)	(0.21,0.24)	(0.81,2.63)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	21.5	19.2	17.2	0.8	41.3	100
	N	7.47	7.13	5.74	4.3	3.2	27.84
	(D,R)	(0.07,0.38)	(0.14,0.34)	(0.18,0.3)	(0.17,0.2)	(0.16,0.97)	(0.71,2.19)

Table 12: Simulation results using the parametric+ model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 1$.

3.3 τ misspecified

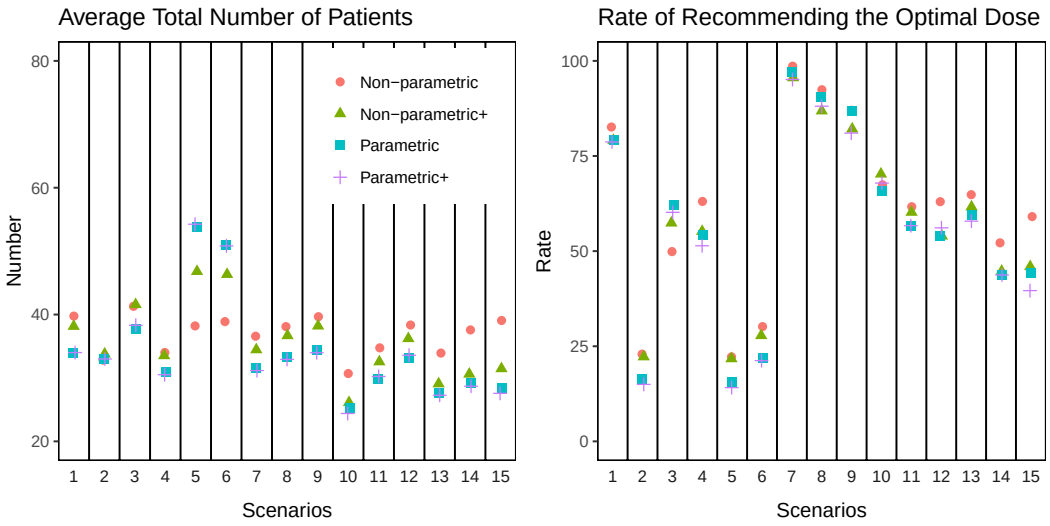

Figure 4: Average total number of patients and the average rate of reporting the optimal dose levels(s) for Scenarios 1-15.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	3.4	5.6	4	4.4	82.6	100
	N	7.45	8.48	8.05	8.04	7.74	39.75
	(D,R)	(0.07,0.38)	(0.2,1.69)	(0.19,2.81)	(0.34,4.84)	(0.37,6.21)	(1.17,15.93)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	20	21.1	21.9	22.9	14.1	100
	N	7.43	8.23	7.15	6.31	3.69	32.82
	(D,R)	(0.07,4.17)	(0.25,4.81)	(0.41,4.23)	(1.3,3.95)	(1.13,2.34)	(3.16,19.49)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	3.9	5.5	14.2	26.5	49.9	100
	N	7.85	8.38	8.45	8.33	8.29	41.3
	(D,R)	(0.14,1.54)	(0.25,3.4)	(0.34,5.18)	(0.53,5.68)	(1.7,6.07)	(2.96,21.86)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	11.3	9.9	8.7	7	63.1	100
	N	7.47	7.64	6.97	6.22	5.72	34.01
	(D,R)	(0.07,3.88)	(0.11,4.68)	(0.15,4.95)	(0.15,4.94)	(0.17,4.92)	(0.65,23.36)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	13.9	26.2	22.2	16.1	9.3	87.7
	N	11.44	10.68	8.37	5.14	2.58	38.21
	(D,R)	(2.13,0.58)	(2.3,2.13)	(2.18,2.93)	(1.55,2.42)	(0.88,1.53)	(9.04,9.58)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	13	30.6	30.2	16.2	8.8	98.8
	N	9.82	11.52	9.46	5.68	2.41	38.88
	(D,R)	(0.79,1.5)	(2.11,4.36)	(2.41,4.95)	(1.75,3.41)	(0.83,1.53)	(7.89,15.75)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	1.4	18.3	19.7	16	44.6	100
	N	7.49	8.14	8.51	7.07	5.37	36.58
	(D,R)	(0.08,0.34)	(0.15,1.98)	(0.28,2.14)	(0.29,1.75)	(0.25,1.33)	(1.03,7.54)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	1.4	6.2	22.5	18.6	51.3	100
	N	7.53	8.3	8.28	7.92	6.07	38.1
	(D,R)	(0.09,0.37)	(0.17,2.06)	(0.25,3.27)	(0.32,3.16)	(0.3,2.48)	(1.13,11.34)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	1.5	6.7	4.8	18.6	68.4	100
	N	7.59	8.31	8.25	7.96	7.55	39.66
	(D,R)	(0.1,0.37)	(0.18,2.04)	(0.26,3.3)	(0.32,4.77)	(0.35,4.59)	(1.2,15.07)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	1.1	67.4	9.9	4.1	17.5	100
	N	7.52	8.14	8.97	3.58	2.48	30.7
	(D,R)	(0.08,0.37)	(0.15,2.1)	(0.28,0.44)	(0.16,0.17)	(0.12,0.12)	(0.8,3.2)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	2.2	4.2	61.7	9.7	22.1	99.9
	N	7.43	8.53	8.24	7.45	3.09	34.75
	(D,R)	(0.07,0.35)	(0.18,1.75)	(0.25,3.31)	(0.3,1.14)	(0.16,0.48)	(0.97,7.03)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	2.1	9.9	2.7	63	22.3	100
	N	7.42	8.45	8.2	7.61	6.67	38.35
	(D,R)	(0.06,0.34)	(0.16,1.76)	(0.23,2.46)	(0.3,4.61)	(0.32,1.97)	(1.07,11.15)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	13.9	1.6	64.8	7.9	11.8	100
	N	7.48	9.57	7.3	7.25	2.32	33.92
	(D,R)	(0.07,0.36)	(0.19,0.48)	(0.2,2.25)	(0.29,0.35)	(0.1,0.1)	(0.85,3.55)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	15.6	16.3	0.5	52.2	15.4	100
	N	7.41	9.59	8.68	6.15	5.75	37.58
	(D,R)	(0.06,0.37)	(0.18,0.5)	(0.25,0.41)	(0.26,1.95)	(0.26,0.31)	(1.02,3.54)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	13.3	14.5	12.5	0.6	59.1	100
	N	7.41	9.65	8.86	7.61	5.54	39.06
	(D,R)	(0.06,0.35)	(0.21,0.45)	(0.27,0.43)	(0.29,0.38)	(0.27,1.6)	(1.1,3.21)

Table 13: Simulation results using the non-parametric model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\tau^{(l)}$s are misspecified as half of the true DLT risk.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	5.6	7.6	3.6	4.1	79.1	100
	N	7.44	7.94	7.69	7.62	7.46	38.15
	(D,R)	(0.07,0.4)	(0.17,1.61)	(0.22,2.68)	(0.32,4.56)	(0.37,5.97)	(1.15,15.22)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	20.2	19.1	21.1	22.2	17.4	100
	N	7.52	7.97	7.28	6.72	4.3	33.78
	(D,R)	(0.08,4.28)	(0.21,4.68)	(0.46,4.4)	(1.35,4.19)	(1.38,2.73)	(3.48,20.29)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	5.3	5.7	13.2	18.4	57.4	100
	N	8.02	8.25	8.22	8.3	8.79	41.58
	(D,R)	(0.18,1.6)	(0.23,3.35)	(0.31,4.99)	(0.53,5.66)	(1.78,6.48)	(3.03,22.08)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	11.8	11.9	10	11.1	55.2	100
	N	7.62	7.63	6.73	6.1	5.44	33.52
	(D,R)	(0.09,3.97)	(0.1,4.75)	(0.11,4.76)	(0.15,4.82)	(0.16,4.66)	(0.61,22.96)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	12.3	21.6	21.7	18.4	22.2	96.2
	N	12.14	11.92	10.26	7.66	4.82	46.8
	(D,R)	(2.18,0.61)	(2.64,2.39)	(2.7,3.61)	(2.29,3.55)	(1.63,2.8)	(11.45,12.97)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	6.9	22	27.8	22.2	20.9	99.8
	N	9.84	11.94	11.42	8.18	4.96	46.33
	(D,R)	(0.75,1.46)	(2.16,4.6)	(2.87,5.98)	(2.41,4.8)	(1.69,3.07)	(9.87,19.9)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	4.4	20.2	19	14.5	41.9	100
	N	7.53	7.78	7.88	6.32	4.94	34.45
	(D,R)	(0.08,0.36)	(0.14,1.91)	(0.25,1.99)	(0.23,1.55)	(0.24,1.23)	(0.94,7.04)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	4.2	8.9	22	19	45.9	100
	N	7.43	7.97	8.09	7.46	5.73	36.68
	(D,R)	(0.07,0.42)	(0.17,2.04)	(0.26,3.35)	(0.3,3.03)	(0.27,2.3)	(1.08,11.14)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	4.3	7.7	5.9	19.6	62.5	100
	N	7.39	7.87	8.01	7.7	7.22	38.19
	(D,R)	(0.06,0.35)	(0.15,1.94)	(0.26,3.27)	(0.31,4.61)	(0.38,4.36)	(1.16,14.54)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	3.4	70.3	11.7	4.8	9.8	100
	N	7.48	7.93	7.25	2.17	1.3	26.13
	(D,R)	(0.08,0.37)	(0.16,2)	(0.2,0.35)	(0.1,0.11)	(0.08,0.06)	(0.61,2.88)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	6.7	4.7	60.2	13	15.4	100
	N	7.43	7.92	7.75	7.04	2.41	32.55
	(D,R)	(0.07,0.38)	(0.16,1.59)	(0.24,3.1)	(0.31,1.04)	(0.11,0.32)	(0.89,6.43)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	6.5	12.8	2.1	54	24.6	100
	N	7.44	7.87	7.74	6.96	6.19	36.21
	(D,R)	(0.07,0.37)	(0.15,1.52)	(0.24,2.31)	(0.26,4.17)	(0.3,1.88)	(1.03,10.24)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	19.6	2.6	61.6	8	8.2	100
	N	7.38	7.78	6.64	5.89	1.39	29.09
	(D,R)	(0.06,0.35)	(0.15,0.4)	(0.2,1.95)	(0.24,0.27)	(0.08,0.08)	(0.72,3.05)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	22.8	19.3	2.1	44.7	11.1	100
	N	7.41	7.65	6.2	4.98	4.38	30.61
	(D,R)	(0.06,0.38)	(0.14,0.35)	(0.17,0.33)	(0.18,1.53)	(0.2,0.21)	(0.77,2.8)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	19	18.5	14.9	1.7	45.9	100
	N	7.56	7.75	6.49	5.3	4.38	31.47
	(D,R)	(0.09,0.38)	(0.16,0.38)	(0.19,0.33)	(0.21,0.25)	(0.22,1.25)	(0.87,2.59)

Table 14: Simulation results using the non-parametric+ model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\tau^{(l)}$s are misspecified as half of the true DLT risk.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	5.6	7.4	3.6	4.2	79.2	100
	N	7.48	7.15	6.8	6.27	6.22	33.92
	(D,R)	(0.07,0.35)	(0.14,1.44)	(0.18,2.37)	(0.28,3.78)	(0.33,5)	(1,12.94)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	20.2	18.7	17.1	16.3	27.7	100
	N	7.46	7.21	6.42	6.64	5.2	32.94
	(D,R)	(0.07,4.26)	(0.22,4.2)	(0.41,3.83)	(1.31,4.12)	(1.59,3.32)	(3.6,19.72)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	5.7	5	13.2	13.9	62.2	100
	N	7.83	7.44	7.08	6.69	8.64	37.69
	(D,R)	(0.15,1.52)	(0.23,2.94)	(0.27,4.25)	(0.37,4.52)	(1.78,6.42)	(2.8,19.65)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	11.2	11.5	11.5	11.5	54.3	100
	N	7.47	7.12	6.3	5.47	4.66	31.02
	(D,R)	(0.08,3.92)	(0.12,4.45)	(0.13,4.51)	(0.12,4.29)	(0.15,3.97)	(0.6,21.14)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	8.3	11.2	15.5	14.9	46.6	96.5
	N	12.25	12.22	11.57	9.93	7.83	53.8
	(D,R)	(2.23,0.59)	(2.66,2.36)	(3.02,4.1)	(2.94,4.65)	(2.57,4.54)	(13.41,16.24)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	4.6	11.5	21.9	29.3	32.5	99.8
	N	10.03	11.19	12.26	10.46	7.1	51.04
	(D,R)	(0.8,1.5)	(2.05,4.24)	(3.14,6.43)	(3.25,6.22)	(2.53,4.39)	(11.77,22.78)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	3	18.9	19.7	15.6	42.8	100
	N	7.5	7.12	7.08	5.63	4.3	31.63
	(D,R)	(0.08,0.38)	(0.14,1.75)	(0.21,1.79)	(0.22,1.43)	(0.19,1.08)	(0.83,6.43)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	3.1	6.4	22.2	19.4	48.9	100
	N	7.54	7.13	7.05	6.51	5.08	33.31
	(D,R)	(0.08,0.37)	(0.13,1.78)	(0.23,2.83)	(0.26,2.56)	(0.27,2.02)	(0.97,9.57)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	3	6.4	3.7	19.7	67.2	100
	N	7.37	7.11	7.06	6.51	6.38	34.43
	(D,R)	(0.06,0.34)	(0.15,1.74)	(0.21,2.77)	(0.26,3.94)	(0.31,3.85)	(0.98,12.64)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	3.8	65.9	12.7	6.7	10.9	100
	N	7.59	7.24	6.87	2.18	1.31	25.2
	(D,R)	(0.1,0.38)	(0.15,1.79)	(0.19,0.34)	(0.09,0.12)	(0.06,0.06)	(0.6,2.68)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	8	5.1	56.6	11.9	18.4	100
	N	7.48	7.13	6.8	6.14	2.23	29.78
	(D,R)	(0.07,0.42)	(0.15,1.39)	(0.2,2.76)	(0.23,0.9)	(0.09,0.32)	(0.75,5.79)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	6.3	10.6	2.1	54	27	100
	N	7.41	7.11	6.85	6.03	5.8	33.21
	(D,R)	(0.06,0.4)	(0.16,1.42)	(0.22,2.08)	(0.25,3.64)	(0.26,1.75)	(0.95,9.28)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	19.3	1.5	59.5	10.3	9.4	100
	N	7.46	7.15	5.92	5.62	1.45	27.59
	(D,R)	(0.07,0.35)	(0.15,0.35)	(0.17,1.76)	(0.22,0.28)	(0.07,0.07)	(0.69,2.8)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	19.6	19.7	1.5	43.8	15.3	99.9
	N	7.44	7.11	5.92	4.43	4.23	29.13
	(D,R)	(0.07,0.35)	(0.16,0.36)	(0.18,0.3)	(0.18,1.34)	(0.22,0.24)	(0.8,2.6)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	19.3	19.4	15.4	1.6	44.3	100
	N	7.45	7.15	5.87	4.42	3.45	28.35
	(D,R)	(0.07,0.36)	(0.16,0.34)	(0.17,0.28)	(0.17,0.22)	(0.18,1.05)	(0.74,2.25)

Table 15: Simulation results using the parametric model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\tau^{(l)}$s are misspecified as half of the true DLT risk.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	6.4	7.3	2.7	4.9	78.7	100
	N	7.57	7.19	6.83	6.22	6.2	34.01
	(D,R)	(0.09,0.36)	(0.15,1.42)	(0.21,2.36)	(0.24,3.76)	(0.3,5.01)	(0.98,12.91)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	20.6	19	16.2	15	29.2	100
	N	7.5	7.24	6.29	6.66	5.31	32.99
	(D,R)	(0.08,4.21)	(0.23,4.13)	(0.39,3.76)	(1.32,4.1)	(1.7,3.42)	(3.71,19.62)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	5.1	4.5	13.1	17.1	60.2	100
	N	7.86	7.43	7.2	6.84	8.97	38.3
	(D,R)	(0.15,1.6)	(0.24,2.94)	(0.29,4.38)	(0.42,4.68)	(1.86,6.61)	(2.95,20.21)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	12.3	11.9	13.1	11.3	51.4	100
	N	7.41	7.1	6.21	5.36	4.44	30.51
	(D,R)	(0.07,3.89)	(0.11,4.38)	(0.14,4.46)	(0.14,4.22)	(0.13,3.81)	(0.58,20.76)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	7.3	10.6	14.1	16.5	47.9	96.4
	N	12.14	12.1	11.66	10.18	8.16	54.24
	(D,R)	(2.17,0.61)	(2.63,2.45)	(3.04,4.08)	(3.03,4.75)	(2.66,4.75)	(13.54,16.64)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	4.5	12.2	21.2	25.2	36.7	99.8
	N	9.98	11	12.2	10.37	7.27	50.83
	(D,R)	(0.77,1.44)	(2.02,4.14)	(3.08,6.33)	(3.08,6.12)	(2.52,4.51)	(11.48,22.54)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	4.9	20.3	18.9	14.8	41.1	100
	N	7.47	7.2	6.89	5.42	4.18	31.16
	(D,R)	(0.07,0.39)	(0.14,1.7)	(0.21,1.69)	(0.24,1.36)	(0.22,1.04)	(0.88,6.18)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	4	7.9	19.9	16.9	51.3	100
	N	7.41	7.15	7.04	6.31	5	32.92
	(D,R)	(0.06,0.38)	(0.16,1.75)	(0.2,2.82)	(0.26,2.49)	(0.24,1.99)	(0.92,9.43)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	3.6	9.4	6	15.5	65.5	100
	N	7.48	7.15	7.07	6.28	5.99	33.98
	(D,R)	(0.08,0.37)	(0.15,1.79)	(0.19,2.8)	(0.24,3.75)	(0.29,3.61)	(0.95,12.32)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	4.2	67.9	15.9	4.9	7.1	100
	N	7.53	7.19	6.81	1.97	0.9	24.4
	(D,R)	(0.08,0.39)	(0.15,1.8)	(0.2,0.33)	(0.07,0.09)	(0.05,0.06)	(0.55,2.67)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	5.8	5.5	56.7	13.3	18.7	100
	N	7.56	7.18	6.87	6.3	2.32	30.24
	(D,R)	(0.09,0.39)	(0.16,1.46)	(0.22,2.8)	(0.26,0.95)	(0.1,0.34)	(0.82,5.94)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	6.5	9.1	2.4	56.1	25.9	100
	N	7.53	7.17	6.86	6.15	5.87	33.59
	(D,R)	(0.08,0.37)	(0.16,1.44)	(0.2,2.17)	(0.24,3.74)	(0.31,1.79)	(0.99,9.5)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	21.2	1.6	57.9	12	7.3	100
	N	7.53	7.14	5.73	5.46	1.42	27.27
	(D,R)	(0.08,0.38)	(0.14,0.33)	(0.18,1.76)	(0.22,0.28)	(0.07,0.07)	(0.7,2.82)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	20.2	21.8	1.6	43.7	12.7	100
	N	7.53	7.15	5.78	4.19	4.06	28.7
	(D,R)	(0.08,0.38)	(0.15,0.38)	(0.17,0.3)	(0.17,1.24)	(0.2,0.2)	(0.77,2.51)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	20.4	21.6	16.5	1.9	39.6	100
	N	7.38	7.17	5.78	4.16	3.08	27.57
	(D,R)	(0.06,0.35)	(0.15,0.38)	(0.19,0.28)	(0.17,0.22)	(0.16,0.9)	(0.72,2.13)

Table 16: Simulation results using the parametric+ model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\tau^{(l)}$s are misspecified as half of the true DLT risk.

3.4 $\mathscr{C}_1 = 0.75$

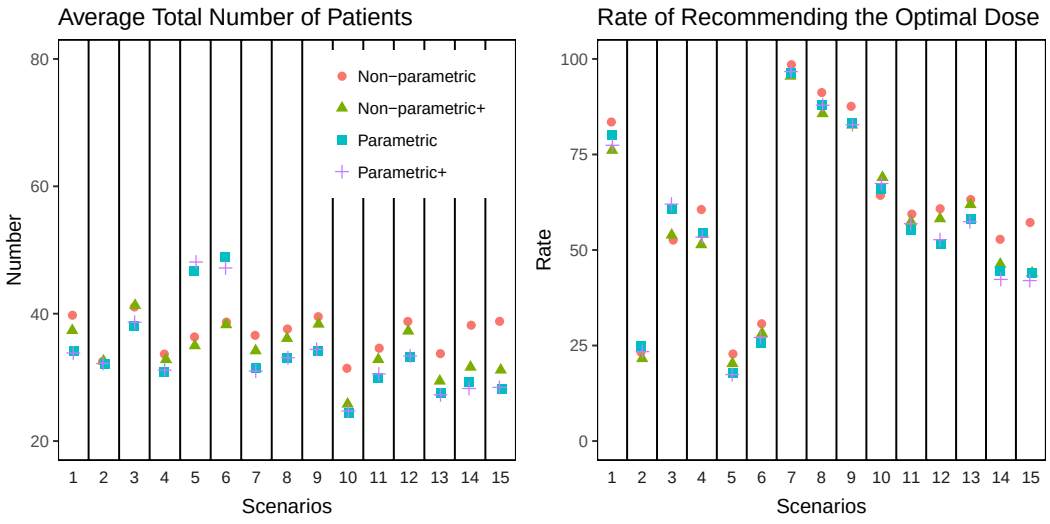

Figure 5: Average total number of patients and the average rate of reporting the optimal dose levels(s) for Scenarios 1-15.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	2	7.2	3.4	3.9	83.5	100
	N	7.46	8.34	8.19	7.92	7.85	39.76
	(D,R)	(0.07,0.41)	(0.14,1.68)	(0.21,2.83)	(0.31,4.72)	(0.39,6.33)	(1.12,15.97)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	20.6	21.4	21.6	23.3	13.1	100
	N	7.55	8.08	7.06	6.22	3.58	32.49
	(D,R)	(0.08,4.33)	(0.22,4.74)	(0.4,4.16)	(1.25,3.89)	(1.14,2.34)	(3.1,19.47)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	3.8	5.6	13.5	24.5	52.6	100
	N	7.79	8.34	8.29	8.47	8.17	41.05
	(D,R)	(0.14,1.58)	(0.25,3.36)	(0.31,4.98)	(0.52,5.73)	(1.58,6.11)	(2.8,21.76)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	12.1	12.5	8.8	6	60.6	100
	N	7.48	7.72	6.84	6.01	5.61	33.66
	(D,R)	(0.08,3.88)	(0.13,4.78)	(0.13,4.81)	(0.15,4.76)	(0.19,4.81)	(0.67,23.05)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	17.1	26.8	22.8	11.8	9.6	88.1
	N	11.37	10.46	7.66	4.6	2.27	36.36
	(D,R)	(2,0.53)	(2.3,2.1)	(1.96,2.68)	(1.38,2.18)	(0.75,1.32)	(8.39,8.82)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	13.1	29.9	30.7	16.2	8.5	98.4
	N	9.92	11.28	9.38	5.7	2.39	38.67
	(D,R)	(0.76,1.51)	(2.06,4.37)	(2.33,4.93)	(1.74,3.32)	(0.85,1.49)	(7.74,15.62)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	1.5	19.9	17.3	16.1	45.2	100
	N	7.55	8.25	8.37	6.9	5.52	36.6
	(D,R)	(0.09,0.37)	(0.17,2.05)	(0.26,2.07)	(0.28,1.81)	(0.28,1.36)	(1.06,7.65)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	1.4	7.4	21.6	19.4	50.2	100
	N	7.53	8.25	8.29	7.6	5.94	37.6
	(D,R)	(0.08,0.37)	(0.17,2.1)	(0.26,3.29)	(0.27,3.01)	(0.29,2.37)	(1.08,11.14)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	0.3	7	5.1	20.9	66.7	100
	N	7.47	8.16	8.32	7.94	7.64	39.53
	(D,R)	(0.08,0.33)	(0.16,2.04)	(0.25,3.31)	(0.31,4.77)	(0.41,4.6)	(1.2,15.05)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	1	64.3	11.5	4.6	18.6	100
	N	7.38	8.37	9.25	3.82	2.6	31.42
	(D,R)	(0.06,0.37)	(0.17,2.06)	(0.28,0.44)	(0.15,0.18)	(0.13,0.14)	(0.79,3.19)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	2.6	5.3	59.4	10.7	22	100
	N	7.47	8.58	8.21	7.36	2.96	34.58
	(D,R)	(0.08,0.38)	(0.2,1.74)	(0.25,3.22)	(0.28,1.06)	(0.11,0.42)	(0.9,6.81)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	1.9	10.2	1.9	60.8	25.2	100
	N	7.5	8.42	8.39	7.68	6.79	38.79
	(D,R)	(0.08,0.38)	(0.16,1.68)	(0.26,2.51)	(0.3,4.57)	(0.35,2.03)	(1.15,11.18)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	16.3	0.6	63.2	7.5	12.4	100
	N	7.43	9.57	7.04	7.26	2.43	33.73
	(D,R)	(0.07,0.39)	(0.17,0.48)	(0.18,2.15)	(0.3,0.34)	(0.11,0.11)	(0.84,3.47)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	16.2	14.8	0.5	52.8	15.7	100
	N	7.5	9.79	8.59	6.14	6.17	38.19
	(D,R)	(0.08,0.37)	(0.2,0.45)	(0.26,0.44)	(0.22,1.82)	(0.29,0.27)	(1.05,3.35)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	15.4	14.7	12.2	0.5	57.2	100
	N	7.41	9.66	8.81	7.49	5.43	38.8
	(D,R)	(0.07,0.36)	(0.21,0.49)	(0.25,0.46)	(0.3,0.35)	(0.27,1.61)	(1.1,3.28)

Table 17: Simulation results using the non-parametric model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\mathscr{C}_1 = 0.75$

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	6.7	8.3	4.2	4.7	76.1	100
	N	7.38	7.92	7.69	7.29	7.1	37.37
	(D,R)	(0.06,0.38)	(0.16,1.61)	(0.22,2.62)	(0.3,4.36)	(0.35,5.65)	(1.09,14.62)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	21.5	18.1	25	21.6	13.8	100
	N	7.41	8.13	7.13	6.42	3.48	32.58
	(D,R)	(0.06,4.28)	(0.24,4.74)	(0.41,4.34)	(1.3,3.98)	(1.11,2.22)	(3.13,19.55)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	4	5.3	14.7	22	53.9	99.9
	N	7.91	8.33	8.42	8.17	8.48	41.31
	(D,R)	(0.15,1.57)	(0.27,3.35)	(0.34,5.14)	(0.49,5.54)	(1.64,6.26)	(2.89,21.85)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	13.9	13.7	11	10	51.4	100
	N	7.59	7.64	6.79	5.84	4.94	32.8
	(D,R)	(0.09,4.06)	(0.12,4.67)	(0.13,4.74)	(0.15,4.57)	(0.13,4.27)	(0.63,22.3)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	20.7	25.4	20.3	12.6	8.5	87.5
	N	11.27	9.86	7.39	4.32	2.14	34.98
	(D,R)	(2,0.55)	(2.18,1.95)	(1.98,2.57)	(1.3,2.04)	(0.7,1.25)	(8.16,8.36)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	17.2	27.3	28.2	18.3	7.7	98.7
	N	9.97	11.25	8.97	5.66	2.43	38.28
	(D,R)	(0.76,1.45)	(2.1,4.32)	(2.29,4.71)	(1.68,3.38)	(0.89,1.52)	(7.72,15.38)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	4.5	21	20	16.1	38.4	100
	N	7.43	7.98	7.75	6.24	4.78	34.18
	(D,R)	(0.07,0.36)	(0.17,1.9)	(0.23,1.95)	(0.24,1.55)	(0.26,1.19)	(0.96,6.95)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	3.8	10.5	22	19.6	44.1	100
	N	7.55	7.95	8.14	7.15	5.34	36.12
	(D,R)	(0.08,0.39)	(0.16,2.06)	(0.27,3.22)	(0.26,2.85)	(0.24,2.1)	(1.02,10.62)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	3.3	6.8	7.2	20.2	62.5	100
	N	7.55	7.92	7.92	7.76	7.22	38.37
	(D,R)	(0.08,0.37)	(0.16,2)	(0.22,3.19)	(0.33,4.61)	(0.38,4.36)	(1.17,14.51)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	4.4	69	13.5	4.9	8.2	100
	N	7.41	7.92	7.16	2.13	1.2	25.82
	(D,R)	(0.06,0.34)	(0.16,1.94)	(0.23,0.34)	(0.09,0.09)	(0.07,0.05)	(0.61,2.76)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	6.2	5.3	57.4	12.9	18.2	100
	N	7.58	7.88	7.73	6.93	2.69	32.81
	(D,R)	(0.09,0.41)	(0.16,1.6)	(0.21,3.06)	(0.28,1.05)	(0.13,0.41)	(0.87,6.54)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	5.6	8.7	2.5	58.2	25	100
	N	7.5	7.87	7.81	7.45	6.64	37.27
	(D,R)	(0.08,0.39)	(0.15,1.59)	(0.24,2.4)	(0.29,4.44)	(0.33,1.96)	(1.08,10.79)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	17.9	3.2	61.9	9.8	7.2	100
	N	7.48	7.78	6.71	6.03	1.41	29.43
	(D,R)	(0.08,0.36)	(0.16,0.4)	(0.19,2.05)	(0.26,0.33)	(0.08,0.06)	(0.77,3.2)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	19	20	2.2	46.3	12.5	100
	N	7.4	7.74	6.59	5.31	4.58	31.6
	(D,R)	(0.06,0.36)	(0.15,0.4)	(0.2,0.33)	(0.22,1.58)	(0.23,0.24)	(0.87,2.91)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	20.8	17.7	16.5	1	44	100
	N	7.5	7.79	6.47	5.31	4.08	31.16
	(D,R)	(0.08,0.38)	(0.16,0.37)	(0.19,0.36)	(0.24,0.26)	(0.2,1.23)	(0.87,2.6)

Table 18: Simulation results using the non-parametric+ model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\mathscr{C}_1 = 0.75$

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	6.1	7	3	3.9	80	100
	N	7.47	7.14	6.86	6.33	6.36	34.16
	(D,R)	(0.08,0.39)	(0.13,1.37)	(0.2,2.42)	(0.24,3.83)	(0.33,5.06)	(0.98,13.07)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	20.4	19.5	18.1	24.9	17.1	100
	N	7.54	7.27	6.32	6.57	4.36	32.06
	(D,R)	(0.08,4.26)	(0.24,4.17)	(0.37,3.78)	(1.34,4.09)	(1.37,2.8)	(3.4,19.1)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	6.2	5.8	12.2	15	60.8	100
	N	7.92	7.36	7.03	6.87	8.85	38.02
	(D,R)	(0.16,1.57)	(0.22,2.85)	(0.26,4.18)	(0.41,4.69)	(1.84,6.59)	(2.89,19.88)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	12	12.3	10.1	11.2	54.4	100
	N	7.41	7.13	6.32	5.33	4.64	30.83
	(D,R)	(0.07,3.84)	(0.12,4.39)	(0.1,4.46)	(0.13,4.23)	(0.14,3.97)	(0.57,20.89)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	9.3	10.5	17.7	18.3	31.3	87.1
	N	11.49	10.6	10.2	8.43	5.94	46.66
	(D,R)	(2.1,0.59)	(2.3,2.18)	(2.62,3.52)	(2.54,3.97)	(1.97,3.44)	(11.53,13.71)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	4.4	15.3	25.6	30.1	24	99.4
	N	10.15	11.32	11.91	9.64	5.94	48.96
	(D,R)	(0.81,1.53)	(1.98,4.32)	(2.95,6.18)	(2.9,5.7)	(2.1,3.69)	(10.74,21.42)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	3.6	21.7	17.7	17.2	39.8	100
	N	7.48	7.19	7.03	5.48	4.29	31.46
	(D,R)	(0.08,0.34)	(0.15,1.81)	(0.19,1.73)	(0.23,1.37)	(0.21,1.05)	(0.87,6.3)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	4	8.1	20.6	18	49.3	100
	N	7.57	7.13	7.03	6.32	5	33.05
	(D,R)	(0.09,0.4)	(0.1,1.73)	(0.21,2.77)	(0.24,2.49)	(0.24,1.98)	(0.88,9.38)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	3.5	8.3	4.9	21.7	61.6	100
	N	7.43	7.1	7.08	6.43	6.17	34.21
	(D,R)	(0.07,0.37)	(0.14,1.84)	(0.2,2.85)	(0.24,3.96)	(0.29,3.65)	(0.94,12.68)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	5.8	66	14.9	4.7	8.6	100
	N	7.51	7.16	6.75	2.04	1.01	24.48
	(D,R)	(0.08,0.38)	(0.13,1.79)	(0.2,0.33)	(0.09,0.1)	(0.05,0.04)	(0.54,2.64)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	7.2	6.3	55.2	12.9	18.4	100
	N	7.51	7.13	6.87	6.15	2.26	29.93
	(D,R)	(0.08,0.36)	(0.15,1.45)	(0.22,2.68)	(0.24,0.94)	(0.1,0.35)	(0.79,5.77)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	7	11.7	1.7	51.7	27.9	100
	N	7.52	7.17	6.83	5.92	5.7	33.14
	(D,R)	(0.08,0.38)	(0.13,1.37)	(0.2,1.97)	(0.24,3.55)	(0.27,1.76)	(0.94,9.03)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	18.7	2.4	58.2	9.9	10.8	100
	N	7.42	7.11	5.95	5.63	1.49	27.6
	(D,R)	(0.06,0.36)	(0.13,0.38)	(0.18,1.75)	(0.21,0.3)	(0.08,0.08)	(0.66,2.88)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	19.9	19.5	2.2	44.4	14	100
	N	7.53	7.15	5.87	4.54	4.16	29.24
	(D,R)	(0.08,0.38)	(0.14,0.38)	(0.17,0.31)	(0.17,1.38)	(0.2,0.23)	(0.77,2.67)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	19.8	20.4	14.6	1.3	43.9	100
	N	7.48	7.04	5.76	4.38	3.47	28.12
	(D,R)	(0.07,0.39)	(0.12,0.36)	(0.17,0.29)	(0.18,0.23)	(0.2,1.05)	(0.74,2.32)

Table 19: Simulation results using the parametric model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\mathscr{C}_1 = 0.75$

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	6.9	7.5	3.4	4.8	77.4	100
	N	7.48	7.17	6.8	6.2	6.17	33.82
	(D,R)	(0.07,0.4)	(0.15,1.45)	(0.2,2.38)	(0.25,3.77)	(0.31,4.94)	(0.97,12.93)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	20.2	20.7	16.2	23.4	19.5	100
	N	7.52	7.22	6.34	6.45	4.67	32.19
	(D,R)	(0.08,4.26)	(0.23,4.18)	(0.39,3.72)	(1.29,3.97)	(1.53,2.98)	(3.53,19.11)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	4.6	5	13.1	15.3	62	100
	N	7.85	7.41	7.23	7.06	9.11	38.65
	(D,R)	(0.15,1.5)	(0.21,2.95)	(0.32,4.37)	(0.42,4.81)	(1.79,6.72)	(2.89,20.35)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	10.5	11.1	12.8	12.2	53.4	100
	N	7.44	7.08	6.41	5.52	4.68	31.13
	(D,R)	(0.07,3.91)	(0.12,4.4)	(0.13,4.62)	(0.13,4.37)	(0.16,4.01)	(0.6,21.3)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	7.8	12.3	17.4	17.2	33.9	88.6
	N	11.59	11.14	10.54	8.69	6.13	48.1
	(D,R)	(2.12,0.59)	(2.46,2.27)	(2.72,3.7)	(2.57,4.12)	(2.04,3.59)	(11.9,14.27)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	5.7	16.8	27.1	27.4	22.5	99.5
	N	10.02	11.13	11.51	9.16	5.37	47.18
	(D,R)	(0.79,1.48)	(1.98,4.26)	(2.84,6.04)	(2.78,5.38)	(1.84,3.33)	(10.23,20.5)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	3.3	23.3	19.9	14.8	38.7	100
	N	7.59	7.15	7.04	5.26	3.95	30.99
	(D,R)	(0.09,0.39)	(0.11,1.78)	(0.21,1.73)	(0.2,1.27)	(0.2,0.98)	(0.81,6.15)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	4.4	7.7	21.2	18.6	48.1	100
	N	7.44	7.16	7.01	6.47	5	33.08
	(D,R)	(0.07,0.35)	(0.16,1.75)	(0.18,2.86)	(0.29,2.61)	(0.24,1.96)	(0.94,9.54)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	3.7	7.6	5.9	19.3	63.5	100
	N	7.54	7.13	7.13	6.39	6.2	34.38
	(D,R)	(0.08,0.36)	(0.14,1.8)	(0.22,2.85)	(0.26,3.82)	(0.33,3.68)	(1.03,12.51)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	4	67.4	13.8	5.6	9.2	100
	N	7.53	7.14	6.92	2.04	1.11	24.72
	(D,R)	(0.08,0.38)	(0.15,1.77)	(0.22,0.36)	(0.07,0.1)	(0.05,0.06)	(0.57,2.67)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	5	5.2	56.9	14.9	18	100
	N	7.55	7.11	7.01	6.39	2.48	30.53
	(D,R)	(0.08,0.39)	(0.14,1.49)	(0.2,2.84)	(0.26,0.99)	(0.13,0.34)	(0.81,6.05)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	6.2	11.8	2	52.7	27.3	100
	N	7.43	7.17	6.86	6.06	5.83	33.36
	(D,R)	(0.07,0.36)	(0.14,1.39)	(0.2,2.08)	(0.26,3.66)	(0.3,1.79)	(0.98,9.28)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	20.1	3.4	57.4	11.6	7.5	100
	N	7.46	7.14	5.84	5.43	1.39	27.26
	(D,R)	(0.08,0.4)	(0.14,0.38)	(0.16,1.71)	(0.22,0.28)	(0.08,0.06)	(0.67,2.82)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	22.1	21.2	1.8	42.3	12.6	100
	N	7.45	7.1	5.66	4.11	3.95	28.26
	(D,R)	(0.07,0.38)	(0.12,0.33)	(0.18,0.28)	(0.16,1.23)	(0.2,0.2)	(0.74,2.41)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	18.9	20.4	16.7	2	42	100
	N	7.55	7.18	5.96	4.4	3.34	28.43
	(D,R)	(0.09,0.35)	(0.16,0.35)	(0.17,0.32)	(0.17,0.22)	(0.19,1.03)	(0.77,2.26)

Table 20: Simulation results using the parametric+ model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\mathscr{C}_1 = 0.75$

3.5 $\mathscr{C}_2 = 0.75$

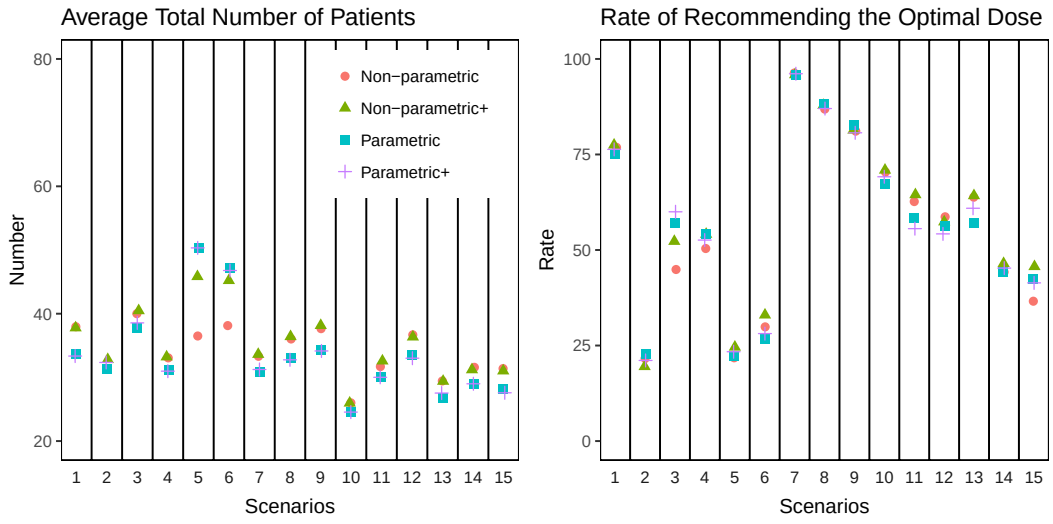

Figure 6: Average total number of patients and the average rate of reporting the optimal dose levels(s) for Scenarios 1-15.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	6.9	7.9	3.6	4.7	76.9	100
	N	7.5	8.06	7.71	7.32	7.38	37.95
	(D,R)	(0.08,0.38)	(0.15,1.56)	(0.22,2.69)	(0.29,4.33)	(0.37,5.88)	(1.12,14.85)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	22.9	18.7	27.9	21.6	8.9	100
	N	7.48	7.95	6.94	6	2.87	31.25
	(D,R)	(0.07,4.23)	(0.22,4.58)	(0.41,4.22)	(1.23,3.73)	(0.96,1.86)	(2.9,18.63)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	5.4	6.2	16.4	27.1	44.9	100
	N	8.05	8.32	8.18	7.99	7.47	40.01
	(D,R)	(0.19,1.64)	(0.25,3.32)	(0.35,4.95)	(0.49,5.43)	(1.46,5.54)	(2.74,20.88)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	12.3	12.3	13.5	11.5	50.4	100
	N	7.5	7.74	6.92	5.89	5.01	33.06
	(D,R)	(0.08,3.93)	(0.14,4.79)	(0.14,4.94)	(0.14,4.64)	(0.14,4.29)	(0.63,22.59)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	21.1	25.5	21.8	12	9.6	90
	N	11.54	10.49	7.67	4.59	2.2	36.5
	(D,R)	(2.1,0.53)	(2.34,2.08)	(1.97,2.65)	(1.36,2.14)	(0.7,1.24)	(8.47,8.64)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	15.4	30.4	29.9	17.3	6	99
	N	10.03	11.43	9.21	5.36	2.09	38.13
	(D,R)	(0.81,1.55)	(2.13,4.42)	(2.24,4.73)	(1.64,3.16)	(0.72,1.29)	(7.55,15.15)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	3.7	25.6	22.3	14.5	33.9	100
	N	7.53	8.02	7.69	5.89	4.18	33.31
	(D,R)	(0.09,0.41)	(0.15,2.05)	(0.21,1.86)	(0.22,1.44)	(0.21,1.08)	(0.87,6.84)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	4.3	8.8	23.5	18.8	44.6	100
	N	7.49	8.12	7.82	7.17	5.43	36.03
	(D,R)	(0.08,0.4)	(0.17,2.03)	(0.23,3.14)	(0.28,2.81)	(0.28,2.2)	(1.04,10.58)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	5.2	8	5.7	23.5	57.6	100
	N	7.42	8.1	7.85	7.38	6.89	37.65
	(D,R)	(0.07,0.41)	(0.17,1.99)	(0.24,3.2)	(0.28,4.51)	(0.34,4.1)	(1.1,14.2)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	4.1	70.4	14.9	5.2	5.4	100
	N	7.53	8	7.27	2.19	1	26
	(D,R)	(0.09,0.4)	(0.15,2.01)	(0.23,0.37)	(0.08,0.1)	(0.06,0.06)	(0.59,2.93)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	6	7.2	62.7	10.8	13.3	100
	N	7.41	8.02	7.73	6.54	2	31.7
	(D,R)	(0.06,0.36)	(0.15,1.57)	(0.23,3.12)	(0.26,0.95)	(0.08,0.28)	(0.79,6.27)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	4.9	11.9	3.5	58.7	20.9	99.9
	N	7.46	8.06	7.86	7.22	6.08	36.68
	(D,R)	(0.08,0.38)	(0.15,1.57)	(0.24,2.41)	(0.3,4.34)	(0.29,1.76)	(1.05,10.47)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	21.3	2.7	63.8	7.7	4.5	100
	N	7.48	8.54	6.64	5.73	1.04	29.43
	(D,R)	(0.07,0.38)	(0.16,0.44)	(0.2,2.03)	(0.22,0.28)	(0.05,0.06)	(0.71,3.19)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	22.7	22.6	2.3	44.2	8.2	100
	N	7.49	8.57	6.74	4.77	4	31.58
	(D,R)	(0.08,0.38)	(0.22,0.44)	(0.21,0.37)	(0.19,1.48)	(0.21,0.2)	(0.91,2.88)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	23	20.4	19	1	36.6	100
	N	7.45	8.48	6.81	5.26	3.38	31.38
	(D,R)	(0.07,0.4)	(0.17,0.4)	(0.19,0.35)	(0.21,0.25)	(0.14,1.02)	(0.78,2.42)

Table 21: Simulation results using the non-parametric model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\mathscr{C}_2 = 0.75$

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	7.1	7.3	3.2	4.9	77.5	100
	N	7.38	7.92	7.74	7.38	7.36	37.79
	(D,R)	(0.06,0.37)	(0.17,1.54)	(0.24,2.71)	(0.28,4.42)	(0.36,5.89)	(1.11,14.94)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	22.2	22.1	19.9	19.5	16.3	100
	N	7.41	8.18	7.01	6.27	3.93	32.81
	(D,R)	(0.06,4.24)	(0.25,4.76)	(0.45,4.18)	(1.19,3.86)	(1.21,2.51)	(3.16,19.54)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	7	6.2	14.5	20	52.3	100
	N	7.95	8.29	8.03	8.01	8.2	40.48
	(D,R)	(0.16,1.63)	(0.25,3.31)	(0.3,4.85)	(0.49,5.45)	(1.61,6.06)	(2.81,21.3)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	11.6	12.2	12.2	9.9	54.1	100
	N	7.42	7.61	6.89	6.08	5.22	33.22
	(D,R)	(0.06,3.88)	(0.12,4.7)	(0.14,4.87)	(0.14,4.77)	(0.15,4.51)	(0.61,22.72)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	12.5	21.7	24.6	17.5	19.7	96
	N	12.29	11.71	10.12	7.47	4.25	45.84
	(D,R)	(2.22,0.61)	(2.61,2.41)	(2.61,3.53)	(2.28,3.54)	(1.38,2.45)	(11.1,12.54)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	9.7	21	33	23.8	12.4	99.9
	N	10.12	11.92	11.19	8	3.98	45.21
	(D,R)	(0.82,1.5)	(2.19,4.49)	(2.81,5.89)	(2.47,4.74)	(1.38,2.41)	(9.67,19.03)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	4.1	22.2	24.2	15.4	34.1	100
	N	7.41	7.84	7.78	6.2	4.4	33.62
	(D,R)	(0.07,0.38)	(0.15,1.96)	(0.22,1.99)	(0.24,1.54)	(0.23,1.09)	(0.9,6.97)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	3.8	8.3	22.5	17.6	47.8	100
	N	7.41	7.88	8.04	7.34	5.74	36.41
	(D,R)	(0.06,0.37)	(0.15,1.93)	(0.24,3.22)	(0.29,2.89)	(0.28,2.29)	(1.03,10.7)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	4.3	8.2	6.1	21.8	59.6	100
	N	7.5	7.87	8.08	7.57	7.14	38.16
	(D,R)	(0.08,0.36)	(0.15,1.91)	(0.27,3.23)	(0.29,4.52)	(0.36,4.29)	(1.14,14.3)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	3.5	70.9	11.6	4.2	9.8	100
	N	7.42	7.9	7.31	2.16	1.19	25.97
	(D,R)	(0.06,0.38)	(0.15,2.02)	(0.24,0.36)	(0.1,0.12)	(0.06,0.06)	(0.61,2.94)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	5	5.6	64.5	11.1	13.8	100
	N	7.45	7.97	7.95	7.07	2.15	32.59
	(D,R)	(0.07,0.36)	(0.17,1.66)	(0.25,3.17)	(0.29,1.06)	(0.11,0.33)	(0.88,6.58)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	5.4	13.7	3	57.4	20.5	100
	N	7.51	7.85	7.71	7.15	6.14	36.35
	(D,R)	(0.08,0.36)	(0.14,1.58)	(0.22,2.27)	(0.3,4.34)	(0.35,1.9)	(1.08,10.44)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	19.2	1.7	64.2	7	7.9	100
	N	7.6	7.81	6.72	6.03	1.24	29.4
	(D,R)	(0.09,0.38)	(0.16,0.4)	(0.2,2.05)	(0.25,0.3)	(0.06,0.07)	(0.76,3.19)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	21	19.6	1.5	46.5	11.4	100
	N	7.43	7.83	6.36	5.15	4.43	31.21
	(D,R)	(0.06,0.37)	(0.17,0.37)	(0.19,0.28)	(0.2,1.56)	(0.2,0.21)	(0.83,2.8)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	20.5	18.5	14	1.3	45.7	100
	N	7.55	7.8	6.41	5.02	4.25	31.02
	(D,R)	(0.09,0.39)	(0.15,0.4)	(0.18,0.31)	(0.18,0.25)	(0.21,1.31)	(0.81,2.65)

Table 22: Simulation results using the non-parametric+ model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\mathscr{C}_2 = 0.75$

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	6.5	9.2	4	5.3	75	100
	N	7.37	7.16	6.87	6.18	6.06	33.64
	(D,R)	(0.06,0.34)	(0.15,1.4)	(0.21,2.41)	(0.26,3.71)	(0.3,4.89)	(0.98,12.74)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	21.9	20.4	17	22.7	18	100
	N	7.41	7.18	6.08	6.33	4.3	31.31
	(D,R)	(0.06,4.29)	(0.22,4.13)	(0.34,3.6)	(1.33,3.98)	(1.37,2.77)	(3.31,18.76)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	5.8	6	13.5	17.6	57.1	100
	N	7.78	7.37	7.12	6.86	8.66	37.79
	(D,R)	(0.13,1.59)	(0.22,2.95)	(0.28,4.22)	(0.44,4.65)	(1.76,6.37)	(2.83,19.79)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	10.9	12.3	10.6	11.9	54.3	100
	N	7.52	7.11	6.38	5.43	4.7	31.13
	(D,R)	(0.08,3.92)	(0.1,4.41)	(0.13,4.48)	(0.14,4.27)	(0.15,4.08)	(0.59,21.16)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	8.2	15.8	22.3	18.9	30.7	95.9
	N	12.17	12.11	11.29	8.86	5.92	50.35
	(D,R)	(2.19,0.61)	(2.65,2.45)	(2.86,3.97)	(2.58,4.11)	(1.95,3.43)	(12.22,14.57)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	5.4	18.2	26.6	28.4	21.1	99.7
	N	9.74	11.24	11.69	9.17	5.28	47.12
	(D,R)	(0.72,1.45)	(2.01,4.27)	(2.94,6.06)	(2.74,5.43)	(1.85,3.27)	(10.26,20.47)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	4.2	21.3	20	17.6	36.9	100
	N	7.45	7.11	6.94	5.35	4	30.86
	(D,R)	(0.07,0.37)	(0.14,1.77)	(0.19,1.69)	(0.22,1.34)	(0.21,0.99)	(0.84,6.15)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	3.7	8.2	21	19.1	48	100
	N	7.46	7.17	7.06	6.38	4.96	33.03
	(D,R)	(0.07,0.34)	(0.16,1.84)	(0.22,2.84)	(0.25,2.53)	(0.26,2)	(0.95,9.54)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	2.8	8.2	6.2	20.3	62.5	100
	N	7.34	7.16	7.18	6.48	6.07	34.23
	(D,R)	(0.05,0.35)	(0.15,1.82)	(0.23,2.91)	(0.25,3.8)	(0.3,3.68)	(0.99,12.57)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	4	67.4	15	6.9	6.7	100
	N	7.46	7.14	6.85	2.08	0.97	24.51
	(D,R)	(0.07,0.4)	(0.14,1.81)	(0.18,0.35)	(0.08,0.11)	(0.04,0.04)	(0.51,2.7)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	6.4	4.3	58.5	14	16.8	100
	N	7.44	7.15	6.89	6.34	2.28	30.09
	(D,R)	(0.07,0.38)	(0.14,1.42)	(0.2,2.78)	(0.26,0.95)	(0.13,0.34)	(0.8,5.87)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	5.4	11.5	1.9	56.4	24.8	100
	N	7.47	7.17	6.94	6.05	5.87	33.5
	(D,R)	(0.07,0.34)	(0.14,1.43)	(0.2,2.1)	(0.26,3.64)	(0.28,1.69)	(0.94,9.2)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	23.8	2.4	57	8.9	7.9	100
	N	7.5	7.15	5.66	5.26	1.25	26.83
	(D,R)	(0.08,0.41)	(0.15,0.33)	(0.2,1.65)	(0.22,0.26)	(0.06,0.06)	(0.72,2.72)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	21.1	20.2	1.9	44.2	12.6	100
	N	7.47	7.15	5.87	4.4	4.05	28.94
	(D,R)	(0.08,0.34)	(0.14,0.34)	(0.19,0.29)	(0.17,1.32)	(0.22,0.18)	(0.8,2.48)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	18.7	22.1	15.3	1.5	42.4	100
	N	7.54	7.11	5.94	4.28	3.31	28.19
	(D,R)	(0.08,0.36)	(0.15,0.37)	(0.18,0.27)	(0.17,0.22)	(0.18,1.03)	(0.76,2.25)

Table 23: Simulation results using the parametric model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\mathscr{C}_2 = 0.75$

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	7	9.3	3.3	4.1	76.3	100
	N	7.46	7.11	6.75	6.04	5.99	33.35
	(D,R)	(0.07,0.39)	(0.14,1.38)	(0.18,2.29)	(0.25,3.62)	(0.3,4.8)	(0.93,12.48)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	20.3	17.7	18.8	21.1	22.1	100
	N	7.47	7.19	6.15	6.71	4.84	32.35
	(D,R)	(0.07,4.22)	(0.19,4.12)	(0.37,3.7)	(1.29,4.13)	(1.51,3.12)	(3.43,19.29)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	4.5	5.6	14.2	15.7	60	100
	N	7.91	7.41	7.28	7.08	8.88	38.56
	(D,R)	(0.16,1.57)	(0.22,3)	(0.29,4.38)	(0.42,4.74)	(1.81,6.65)	(2.9,20.33)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	11.7	13	11.2	11.5	52.6	100
	N	7.58	7.14	6.34	5.36	4.56	30.98
	(D,R)	(0.09,3.91)	(0.11,4.44)	(0.14,4.5)	(0.15,4.21)	(0.13,3.9)	(0.62,20.97)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	7	14.9	23.4	22.5	28.1	95.9
	N	11.91	12.03	11.43	9.01	5.95	50.33
	(D,R)	(2.12,0.56)	(2.63,2.39)	(3.04,4.03)	(2.77,4.29)	(1.99,3.45)	(12.54,14.72)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	7.6	17.1	28.1	28.1	18.8	99.7
	N	10.32	11.17	11.49	8.9	4.91	46.78
	(D,R)	(0.87,1.54)	(1.97,4.17)	(2.84,5.9)	(2.71,5.27)	(1.71,3.06)	(10.1,19.95)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	3.9	22	16.2	16.9	41	100
	N	7.46	7.15	7.02	5.33	4.28	31.24
	(D,R)	(0.07,0.38)	(0.14,1.81)	(0.2,1.7)	(0.21,1.39)	(0.2,1.05)	(0.82,6.35)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	3.8	9.2	18.7	16.7	51.6	100
	N	7.38	7.1	6.97	6.26	5.05	32.76
	(D,R)	(0.06,0.34)	(0.14,1.78)	(0.18,2.75)	(0.27,2.46)	(0.25,2.01)	(0.9,9.34)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	4.3	8.2	6.8	18.5	62.2	100
	N	7.5	7.13	7.07	6.39	6.05	34.15
	(D,R)	(0.08,0.37)	(0.16,1.79)	(0.21,2.86)	(0.23,3.84)	(0.31,3.66)	(0.98,12.53)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	3.1	69.2	15.8	4.6	7.3	100
	N	7.51	7.21	6.95	1.97	0.91	24.55
	(D,R)	(0.08,0.35)	(0.16,1.79)	(0.19,0.34)	(0.08,0.08)	(0.06,0.04)	(0.56,2.61)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	6.9	5.8	55.6	14.4	17.3	100
	N	7.47	7.11	6.87	6.21	2.35	30.01
	(D,R)	(0.08,0.39)	(0.14,1.46)	(0.22,2.71)	(0.27,0.97)	(0.11,0.35)	(0.81,5.89)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	6.5	12.5	2.4	54.2	24.4	100
	N	7.45	7.13	6.8	5.93	5.7	33.01
	(D,R)	(0.07,0.4)	(0.13,1.38)	(0.2,2.03)	(0.28,3.54)	(0.28,1.64)	(0.96,9)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	18.4	2.8	60.9	11.3	6.6	100
	N	7.48	7.16	6.01	5.58	1.31	27.54
	(D,R)	(0.08,0.37)	(0.16,0.37)	(0.19,1.8)	(0.21,0.28)	(0.06,0.05)	(0.71,2.87)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	18.6	22.9	1.8	45.3	11.4	100
	N	7.55	7.15	5.93	4.3	4.06	28.99
	(D,R)	(0.09,0.35)	(0.16,0.39)	(0.18,0.32)	(0.17,1.34)	(0.2,0.19)	(0.79,2.58)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	21.5	19.3	16.2	1.6	41.4	100
	N	7.41	7.1	5.68	4.23	3.16	27.58
	(D,R)	(0.06,0.39)	(0.13,0.34)	(0.16,0.29)	(0.15,0.22)	(0.17,0.98)	(0.68,2.22)

Table 24: Simulation results using the parametric+ model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\mathscr{C}_2 = 0.75$

3.6 $\mathscr{C}_3 = 0.55$

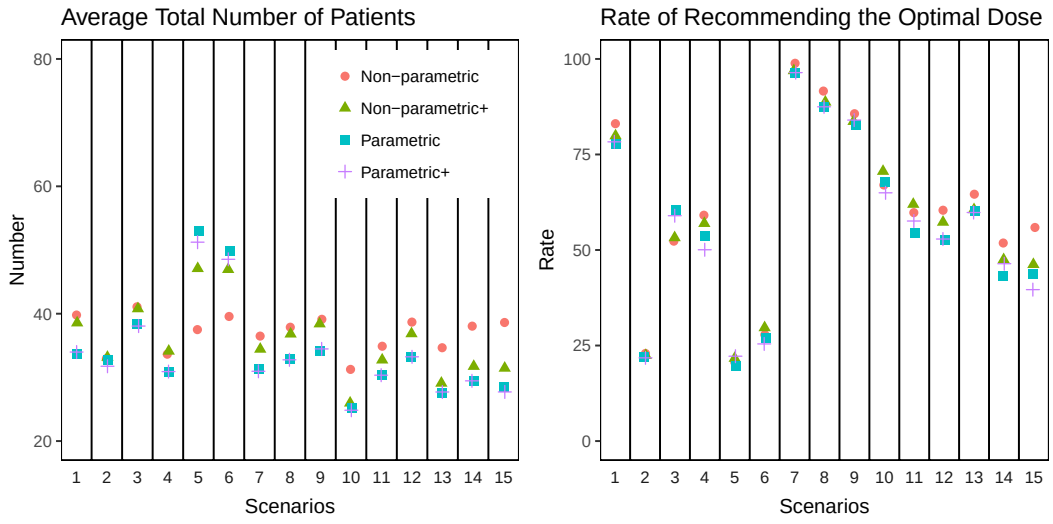

Figure 7: Average total number of patients and the average rate of reporting the optimal dose levels(s) for Scenarios 1-15.

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	1.7	8.1	3.5	3.6	83.1	100
	N	7.5	8.42	8.18	7.69	7.99	39.79
	(D,R)	(0.08,0.37)	(0.16,1.75)	(0.23,2.78)	(0.29,4.53)	(0.42,6.41)	(1.18,15.83)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	18.6	21.2	24.4	22.9	12.9	100
	N	7.53	8.15	7.35	6.35	3.56	32.94
	(D,R)	(0.08,4.23)	(0.23,4.77)	(0.47,4.34)	(1.31,3.91)	(1.16,2.27)	(3.25,19.53)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	3.9	6.3	13.4	24.1	52.3	100
	N	7.86	8.28	8.27	8.25	8.41	41.08
	(D,R)	(0.15,1.52)	(0.24,3.25)	(0.33,4.97)	(0.49,5.63)	(1.73,6.26)	(2.94,21.63)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	13.1	11.5	8.6	7.7	59.1	100
	N	7.57	7.66	6.9	6.09	5.42	33.64
	(D,R)	(0.09,4.01)	(0.11,4.78)	(0.16,4.87)	(0.16,4.84)	(0.15,4.66)	(0.66,23.17)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	18.7	25.8	20.6	12.5	11.8	89.4
	N	11.58	10.64	7.92	4.78	2.58	37.5
	(D,R)	(2.1,0.57)	(2.37,2.13)	(2.03,2.7)	(1.43,2.21)	(0.82,1.48)	(8.76,9.08)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	13.1	28.2	27.8	19.4	9.2	97.7
	N	9.93	11.4	9.45	5.93	2.85	39.56
	(D,R)	(0.86,1.56)	(2.03,4.28)	(2.3,4.89)	(1.72,3.5)	(1.01,1.78)	(7.92,16.02)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	1.1	17.5	20.3	15.8	45.3	100
	N	7.43	8.21	8.35	7.04	5.45	36.48
	(D,R)	(0.07,0.37)	(0.15,2.04)	(0.26,2.06)	(0.26,1.75)	(0.26,1.37)	(1,7.59)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	1.2	7.2	21.4	19.4	50.8	100
	N	7.46	8.35	8.15	7.78	6.14	37.88
	(D,R)	(0.07,0.37)	(0.16,2.06)	(0.23,3.22)	(0.33,3.07)	(0.31,2.46)	(1.11,11.18)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	1.2	7.4	5.7	19.2	66.5	100
	N	7.41	8.04	8.2	7.92	7.54	39.12
	(D,R)	(0.06,0.38)	(0.14,2.04)	(0.24,3.27)	(0.32,4.71)	(0.38,4.5)	(1.14,14.91)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	0.8	67	10.1	4.1	18	100
	N	7.42	8.44	9.19	3.75	2.46	31.25
	(D,R)	(0.06,0.4)	(0.18,2.14)	(0.27,0.42)	(0.14,0.17)	(0.12,0.12)	(0.77,3.26)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	2.1	3.5	59.8	12	22.6	100
	N	7.43	8.55	8.09	7.61	3.21	34.89
	(D,R)	(0.07,0.38)	(0.2,1.68)	(0.22,3.24)	(0.31,1.2)	(0.15,0.46)	(0.94,6.96)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	1.8	10.1	2	60.4	25.7	100
	N	7.47	8.46	8.27	7.73	6.74	38.68
	(D,R)	(0.08,0.36)	(0.17,1.73)	(0.24,2.45)	(0.32,4.66)	(0.33,2.04)	(1.14,11.24)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	13.1	0.6	64.6	8.2	13.5	100
	N	7.41	9.64	7.48	7.63	2.49	34.66
	(D,R)	(0.07,0.34)	(0.22,0.49)	(0.22,2.19)	(0.35,0.38)	(0.13,0.12)	(0.98,3.53)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	16	15.2	0.6	51.8	16.4	100
	N	7.55	9.8	8.62	6.05	6.03	38.05
	(D,R)	(0.09,0.39)	(0.2,0.49)	(0.26,0.4)	(0.22,1.81)	(0.3,0.31)	(1.08,3.39)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	15.7	14.3	13.3	0.8	55.9	100
	N	7.43	9.71	8.76	7.52	5.2	38.62
	(D,R)	(0.07,0.39)	(0.21,0.48)	(0.27,0.46)	(0.28,0.37)	(0.26,1.59)	(1.09,3.3)

Table 25: Simulation results using the non-parametric model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\mathscr{C}_3 = 0.55$

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	5.5	7.1	3.5	4	79.9	100
	N	7.55	7.99	7.78	7.6	7.65	38.56
	(D,R)	(0.08,0.34)	(0.17,1.61)	(0.23,2.75)	(0.31,4.53)	(0.4,6.13)	(1.2,15.37)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	22.7	17.7	22.3	22.4	14.9	100
	N	7.5	8.02	6.92	6.68	4.05	33.17
	(D,R)	(0.08,4.28)	(0.22,4.62)	(0.41,4.25)	(1.34,4.08)	(1.35,2.62)	(3.4,19.86)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	6.1	6.9	14.8	19	53.2	100
	N	7.99	8.3	8.2	7.94	8.34	40.78
	(D,R)	(0.18,1.62)	(0.25,3.31)	(0.34,4.91)	(0.48,5.41)	(1.68,6.22)	(2.93,21.47)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	10.2	11.2	10.8	10.8	57	100
	N	7.5	7.71	7.07	6.28	5.57	34.12
	(D,R)	(0.08,3.77)	(0.13,4.79)	(0.14,5.02)	(0.16,4.91)	(0.18,4.76)	(0.7,23.25)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	12.3	19.6	21.7	18.2	23.8	95.6
	N	12.26	11.77	10.33	7.74	4.99	47.1
	(D,R)	(2.25,0.64)	(2.63,2.31)	(2.7,3.62)	(2.39,3.7)	(1.68,2.96)	(11.65,13.23)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	5.8	21	29.7	22.9	20.4	99.8
	N	10.12	11.86	11.42	8.53	5.01	46.95
	(D,R)	(0.8,1.51)	(2.05,4.47)	(2.87,6.04)	(2.54,4.96)	(1.78,3.15)	(10.05,20.13)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	3.1	22	19.8	16.8	38.3	100
	N	7.5	7.98	7.88	6.37	4.71	34.43
	(D,R)	(0.08,0.39)	(0.16,2.03)	(0.23,1.99)	(0.27,1.58)	(0.22,1.15)	(0.96,7.15)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	3.8	7.5	20.9	20.6	47.2	100
	N	7.41	7.97	8	7.62	5.82	36.81
	(D,R)	(0.06,0.33)	(0.18,1.98)	(0.26,3.18)	(0.32,3.07)	(0.31,2.32)	(1.12,10.89)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	3.8	7.7	4.8	21.7	62	100
	N	7.54	7.87	8.11	7.6	7.29	38.41
	(D,R)	(0.09,0.37)	(0.15,1.96)	(0.27,3.24)	(0.29,4.57)	(0.38,4.34)	(1.18,14.47)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	3.7	70.6	12.8	4	8.9	100
	N	7.42	8.06	7.25	2.13	1.11	25.98
	(D,R)	(0.07,0.34)	(0.18,2.05)	(0.21,0.34)	(0.1,0.11)	(0.07,0.06)	(0.62,2.9)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	6	4.4	62	12	15.6	100
	N	7.52	7.95	7.85	6.94	2.48	32.74
	(D,R)	(0.09,0.38)	(0.17,1.61)	(0.24,3.13)	(0.25,1.01)	(0.14,0.37)	(0.88,6.5)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	6.7	11.7	2	57.3	22.3	100
	N	7.64	8.02	7.73	7.19	6.29	36.86
	(D,R)	(0.1,0.39)	(0.18,1.55)	(0.22,2.32)	(0.3,4.32)	(0.32,1.94)	(1.11,10.51)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	20.5	1.7	60.6	7.9	9.3	100
	N	7.41	7.82	6.52	5.96	1.39	29.1
	(D,R)	(0.06,0.34)	(0.16,0.37)	(0.19,1.95)	(0.24,0.29)	(0.06,0.07)	(0.72,3.03)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	20.3	18.2	2	47.5	12	100
	N	7.46	7.8	6.57	5.35	4.55	31.74
	(D,R)	(0.07,0.37)	(0.16,0.41)	(0.22,0.35)	(0.21,1.61)	(0.2,0.23)	(0.86,2.97)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	20.2	18.9	13.9	0.8	46.2	100
	N	7.45	7.76	6.63	5.2	4.42	31.45
	(D,R)	(0.08,0.39)	(0.15,0.39)	(0.22,0.34)	(0.22,0.27)	(0.24,1.33)	(0.9,2.72)

Table 26: Simulation results using the non-parametric+ model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\mathscr{C}_3 = 0.55$

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	7.2	8	3	4.1	77.7	100
	N	7.49	7.15	6.8	6.17	6.12	33.75
	(D,R)	(0.08,0.41)	(0.15,1.46)	(0.17,2.29)	(0.24,3.71)	(0.3,4.92)	(0.95,12.78)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	19.6	18.3	17.9	22.1	22.1	100
	N	7.51	7.24	6.24	6.78	4.9	32.67
	(D,R)	(0.08,4.33)	(0.24,4.26)	(0.33,3.74)	(1.4,4.27)	(1.56,3.12)	(3.61,19.72)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	5.2	6.3	13	15.1	60.4	100
	N	8.06	7.43	7.16	6.94	8.8	38.39
	(D,R)	(0.18,1.6)	(0.23,2.98)	(0.26,4.32)	(0.44,4.73)	(1.75,6.48)	(2.86,20.11)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	11.8	12.2	12.6	9.7	53.7	100
	N	7.46	7.1	6.31	5.39	4.51	30.78
	(D,R)	(0.07,3.85)	(0.13,4.41)	(0.13,4.47)	(0.12,4.2)	(0.13,3.93)	(0.58,20.86)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	6.8	11.2	19.6	20.6	38.2	96.4
	N	12.07	12.11	11.74	9.89	7.13	52.94
	(D,R)	(2.16,0.6)	(2.57,2.39)	(3.12,4.15)	(2.93,4.64)	(2.33,4.11)	(13.11,15.89)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	3.3	14.4	27.1	30.9	24.1	99.8
	N	10.07	11.56	12.22	10.09	5.98	49.91
	(D,R)	(0.82,1.52)	(2.07,4.39)	(3.09,6.39)	(2.95,5.9)	(2.04,3.71)	(10.98,21.92)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	3.6	21.6	18.5	16.1	40.2	100
	N	7.51	7.15	7.05	5.42	4.18	31.3
	(D,R)	(0.08,0.38)	(0.13,1.79)	(0.2,1.75)	(0.21,1.38)	(0.22,1.06)	(0.85,6.37)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	3.4	9.2	21.4	17.3	48.7	100
	N	7.5	7.13	7.09	6.27	4.86	32.84
	(D,R)	(0.07,0.36)	(0.14,1.81)	(0.2,2.84)	(0.23,2.48)	(0.21,1.92)	(0.86,9.41)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	4.2	8.2	5	17.6	65	100
	N	7.48	7.16	7.03	6.38	6.14	34.19
	(D,R)	(0.08,0.41)	(0.15,1.76)	(0.19,2.69)	(0.26,3.82)	(0.32,3.67)	(1,12.35)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	2.4	67.9	11.9	6.4	11.4	100
	N	7.57	7.2	6.96	2.19	1.34	25.27
	(D,R)	(0.09,0.37)	(0.15,1.82)	(0.22,0.32)	(0.08,0.12)	(0.08,0.07)	(0.62,2.69)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	6.1	4.8	54.5	12.2	22.4	100
	N	7.48	7.13	6.86	6.37	2.57	30.41
	(D,R)	(0.08,0.36)	(0.14,1.41)	(0.19,2.77)	(0.24,0.99)	(0.13,0.42)	(0.78,5.95)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	6.7	10.9	2.5	52.7	27.2	100
	N	7.5	7.09	6.9	6.03	5.71	33.23
	(D,R)	(0.08,0.38)	(0.12,1.4)	(0.22,2.06)	(0.23,3.62)	(0.26,1.75)	(0.92,9.21)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	18.2	1.9	60.2	10.7	9	100
	N	7.38	7.14	5.96	5.68	1.44	27.59
	(D,R)	(0.06,0.37)	(0.15,0.36)	(0.16,1.75)	(0.24,0.27)	(0.07,0.06)	(0.69,2.82)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	19.1	20.2	1.4	43.3	16	100
	N	7.5	7.15	5.94	4.54	4.29	29.42
	(D,R)	(0.08,0.37)	(0.15,0.37)	(0.16,0.3)	(0.19,1.37)	(0.22,0.25)	(0.81,2.67)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	21.2	19.3	14.2	1.5	43.8	100
	N	7.57	7.18	5.8	4.46	3.49	28.5
	(D,R)	(0.09,0.39)	(0.17,0.36)	(0.17,0.3)	(0.18,0.21)	(0.17,1.05)	(0.78,2.3)

Table 27: Simulation results using the parametric model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\mathscr{C}_3 = 0.55$

Scenario		Dose Level 1	2	3	4	5	Total
1	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.35)	(0.04,0.6)	**(0.05,0.8)**	
	%	6.1	8.3	3.2	4.1	78.3	100
	N	7.49	7.13	6.92	6.26	6.18	33.98
	(D,R)	(0.08,0.37)	(0.15,1.45)	(0.21,2.46)	(0.25,3.77)	(0.29,4.96)	(0.98,13.01)
2	(p,q)	(0.01,0.57)	(0.03,0.58)	(0.06,0.6)	**(0.2,0.62)**	(0.32,0.64)	
	%	20.6	20.1	17.8	21.8	19.7	100
	N	7.39	7.18	6.19	6.47	4.51	31.75
	(D,R)	(0.06,4.22)	(0.2,4.17)	(0.39,3.73)	(1.29,4.03)	(1.44,2.86)	(3.39,19.01)
3	(p,q)	(0.02,0.2)	(0.03,0.4)	(0.04,0.6)	(0.06,0.68)	**(0.2,0.74)**	
	%	5.6	6.3	13.6	15.5	59	100
	N	7.76	7.34	7.22	6.96	8.78	38.06
	(D,R)	(0.14,1.58)	(0.2,2.9)	(0.27,4.39)	(0.45,4.78)	(1.78,6.47)	(2.85,20.11)
4	(p,q)	(0.01,0.52)	(0.01,0.62)	(0.02,0.71)	(0.03,0.79)	**(0.03,0.86)**	
	%	10.5	14	12.4	13	50.1	100
	N	7.53	7.14	6.43	5.32	4.49	30.91
	(D,R)	(0.08,3.87)	(0.12,4.45)	(0.13,4.56)	(0.12,4.2)	(0.11,3.85)	(0.57,20.93)
5	(p,q)	(0.18,0.05)	(0.22,0.2)	**(0.26,0.35)**	(0.3,0.47)	(0.33,0.58)	
	%	7	12.7	22.2	18.2	35.8	95.9
	N	11.83	11.98	11.49	9.39	6.54	51.23
	(D,R)	(2.05,0.55)	(2.71,2.39)	(3.04,4.13)	(2.8,4.36)	(2.13,3.8)	(12.73,15.22)
6	(p,q)	(0.08,0.15)	(0.18,0.38)	**(0.25,0.52)**	(0.3,0.59)	(0.35,0.62)	
	%	4.6	16.9	25.4	29.4	23.7	100
	N	10.14	11.4	11.75	9.55	5.7	48.54
	(D,R)	(0.8,1.49)	(2.18,4.47)	(2.9,6.08)	(2.93,5.71)	(1.96,3.54)	(10.76,21.29)
7	(p,q)	(0.01,0.05)	**(0.02,0.25)**	**(0.03,0.25)**	**(0.04,0.25)**	**(0.05,0.25)**	
	%	3.6	22.2	20.7	14.4	39.1	100
	N	7.43	7.17	6.99	5.36	4	30.96
	(D,R)	(0.07,0.4)	(0.15,1.84)	(0.21,1.75)	(0.22,1.35)	(0.2,1.04)	(0.85,6.37)
8	(p,q)	(0.01,0.05)	(0.02,0.25)	**(0.03,0.4)**	**(0.04,0.4)**	**(0.05,0.4)**	
	%	4.2	8.3	21.9	17.1	48.5	100
	N	7.46	7.13	7.07	6.26	4.84	32.76
	(D,R)	(0.07,0.38)	(0.14,1.79)	(0.23,2.81)	(0.24,2.44)	(0.23,1.93)	(0.91,9.35)
9	(p,q)	(0.01,0.05)	(0.02,0.25)	(0.03,0.4)	**(0.04,0.6)**	**(0.05,0.6)**	
	%	2.9	8.2	4.9	19	65	100
	N	7.52	7.13	7.12	6.45	6.24	34.47
	(D,R)	(0.08,0.4)	(0.15,1.82)	(0.2,2.86)	(0.25,3.88)	(0.31,3.79)	(0.99,12.75)
10	(p,q)	(0.01,0.05)	**(0.02,0.25)**	(0.03,0.05)	(0.04,0.05)	(0.05,0.05)	
	%	4.4	65	15.4	7.1	8.1	100
	N	7.5	7.17	6.91	2.16	1.11	24.84
	(D,R)	(0.08,0.37)	(0.16,1.8)	(0.24,0.38)	(0.08,0.12)	(0.05,0.05)	(0.61,2.72)
11	(p,q)	(0.01,0.05)	(0.02,0.2)	**(0.03,0.4)**	(0.04,0.15)	(0.05,0.15)	
	%	5.8	4.4	57.6	12.9	19.3	100
	N	7.45	7.17	6.93	6.42	2.37	30.34
	(D,R)	(0.08,0.36)	(0.15,1.43)	(0.2,2.81)	(0.25,0.93)	(0.11,0.35)	(0.79,5.88)
12	(p,q)	(0.01,0.05)	(0.02,0.2)	(0.03,0.3)	**(0.04,0.6)**	(0.05,0.3)	
	%	5.5	13.9	1.8	52.9	25.9	100
	N	7.5	7.2	6.94	5.9	5.69	33.23
	(D,R)	(0.08,0.36)	(0.14,1.42)	(0.21,2.01)	(0.24,3.55)	(0.28,1.73)	(0.94,9.07)
13	(p,q)	(0.01,0.05)	(0.02,0.05)	**(0.03,0.3)**	(0.04,0.05)	(0.05,0.05)	
	%	17	3	59.8	11.7	8.5	100
	N	7.45	7.11	6.06	5.63	1.46	27.71
	(D,R)	(0.07,0.33)	(0.13,0.36)	(0.16,1.76)	(0.21,0.27)	(0.06,0.08)	(0.64,2.8)
14	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	**(0.04,0.3)**	(0.05,0.05)	
	%	19.9	18.6	2.5	46.4	12.6	100
	N	7.62	7.17	5.89	4.49	4.27	29.46
	(D,R)	(0.1,0.36)	(0.15,0.33)	(0.17,0.3)	(0.16,1.34)	(0.25,0.2)	(0.83,2.54)
15	(p,q)	(0.01,0.05)	(0.02,0.05)	(0.03,0.05)	(0.04,0.05)	**(0.05,0.3)**	
	%	19.2	22.5	16.7	2	39.6	100
	N	7.46	7.1	5.83	4.2	3.13	27.71
	(D,R)	(0.07,0.39)	(0.14,0.38)	(0.17,0.3)	(0.18,0.19)	(0.13,0.88)	(0.68,2.15)

Table 28: Simulation results using the parametric+ model. Cohort size 7 and maximum size 14 for each dose level. $r^{(l)} = 10$. $\mathscr{C}_3 = 0.55$

4 Simulation settings for comparing to existing methods

In this section, we describe the simulation settings for the *EffTox* design and the *UAROET* design. For the statistical details of the two designs, we refer users to Thall and Cook [2004] for the *EffTox* design and Thall and Nguyen [2012] for the *UAROET* design. The software `EffTox V5.0.0` and `UAROET V1.8` were downloaded from Biostat MDAnderson.

4.1 EffTox Design

1. The dose values are set to be $1, 2, 4, 8, 16$
2. Max sample size is set to be 42
3. Number of Sim Repetitions is set to be 1000
4. Random seed is set to 10502
5. The probability of toxicity and efficacy limits for dose acceptability rules are as follows:
   - Probability of toxicity upper limit 0.3 with lower probability threshold 0.7
   - Probability of efficacy lower limit 0.1 with lower probability threshold 0.1
6. The trade-off function elicited points, (π_E, π_T), are chosen to be $(0.5, 0)$, $(1, 0.65)$, and $(0.7, 0.25)$
7. The standard deviations of the hyper-parameters are set to be 20

4.2 UAROET Design

1. Number of toxicity and efficacy levels are set to be 2
2. Max sample size is set to be 42
3. Number of Sim Repetitions is set to be 1000
4. The utility function is set to be
   - not toxic and not efficacious: 50
   - not toxic and efficacious: 100
   - toxic and efficacious: 5
   - toxic and not efficacious: 0
5. Correlation of toxicity and efficacy is set to 0
6. The standard deviations of the hyper-parameters are set to be 100

References

Peter F Thall and John D Cook. Dose-finding based on efficacy–toxicity trade-offs. *Biometrics*, 60(3): 684–693, 2004.

Peter F Thall and Hoang Q Nguyen. Adaptive randomization to improve utility-based dose-finding with bivariate ordinal outcomes. *Journal of biopharmaceutical statistics*, 22(4):785–801, 2012.