Supplementary Information
for
Jumping dynamics of aquatic animals

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Supplemental Text

Geometric relations of axisymmetric body. Two distinct regimes exist when a body exits water. The first regime is when the body is still partially submerged, or $z(t) < L$ where $z(t)$ is the top most part of the body and L is the length (or major diameter in the case of spheroid). The submerged portion of the body experiences a buoyancy force of the form $F_{sub} = (\rho_b - \rho_w)\mathcal{V}_{sub}\, g$. For a body exiting water, we approximate the submerged volume as $\mathcal{V}_{sub} \approx \pi\, \mathrm{R}^2\, (\mathrm{L} - \mathrm{z(t)})$. The portion that is not submerged will experience growing body weight as $F_{out} = m_b\, g\frac{z(t)}{L}$. Entrained fluid will create a downward force on the body as $F_f = \mathrm{m_f(z)\ g} + \frac{\mathrm{d}}{\mathrm{dt}}(\mathrm{m_f}\dot{z})$. The volume of entrained water is modeled as a cylinder, or more exactly $\mathcal{V}_w = \mathcal{V}_{cyl} - \left(\frac{1}{2}\mathcal{V}_{sph} - \mathcal{V}_{cap}\right)$ where $\mathcal{V}_{cyl}$ is the volume of a liquid cylinder, $\mathcal{V}_{sph}$ is the volume of the spheroid, and $\mathcal{V}_{cap}$ is the volume of the spheroidal cap submerged under water (see Fig. S2). Below are the volume relations used when $z(t) < L$.

$$\mathcal{V}_{cyl} = \pi\, \mathrm{R}^2\, (\mathrm{z(t)} - \mathrm{c}) \qquad [1]$$

$$\mathcal{V}_{sph} = \frac{4}{3}\pi\, \mathrm{R}^2\, \mathrm{c} \qquad [2]$$

$$\mathcal{V}_{cap} = \pi\, \mathrm{R}^2 \frac{h_{sub}^2}{3c^2}(3c - h_{sub}) \qquad [3]$$

Additional geometric relations are the half-length of the spheroid's major diameter, $c = L/2$, and the height of the spheroid still submerged underwater, $h_{sub} = L - z(t)$. Therefore, the mass of the entrained water is simply $m_f(z) = \rho_w\, \mathcal{V}_w$. The second regime is when the body has completely exited water, or $z(t) > L$. Here, $\mathcal{V}_w = \mathcal{V}_{cyl} - \mathcal{V}_{cap}$, where $\mathcal{V}_{cyl}$ is still the volume of a water cylinder, and $\mathcal{V}_{cap}$ is the volume of the spheroidal cap that is wetted. Below are the volume relations used when $z(t) > L$.

$$\mathcal{V}_{cyl} = \pi\, \mathrm{r_w^2}\, (\mathrm{z(t)} - \mathrm{L} + \mathrm{h_w}) \qquad [4]$$

$$\mathcal{V}_{cap} = \pi\, \mathrm{R}^2 \frac{h_w^2}{3c^2}(3c - h_w) \qquad [5]$$

Additional geometric relations are the wetted height $h_w = c\left(1 - \sqrt{1 - (r_w/R)^2}\right)$ and the wetted radius, $r_w = \beta R$, where R is the original radius of the body and β is a parameter to test the influence of the water column.

The streamlined body is geometrically a half spheroid on the top half and a cone on the bottom half, both with heights of c. When considering the volume of entrained fluid for $z(t) < L$, we have $\mathcal{V}_w = \mathcal{V}_{cyl} - \mathcal{V}_{fru}$, where $\mathcal{V}_{cyl}$ is the same as Eq. 1 and $\mathcal{V}_{fru}$ is the volume of the cone frustrum that has escaped water.

$$\mathcal{V}_{fru} = \frac{1}{3}\pi(z(t) - c)(R^2 + R\, a + a^2) \qquad [6]$$

where $a = \frac{R}{c}\left(c - (z(t) - c)\right)$ is the frustrum height. When $z(t) > L$, then the volume of entrained water simply becomes $\cancel{V}_w = \cancel{V}_{cyl} - \cancel{V}_{cone}$. These volumes are also dependent on the wetted radius, r_w, which contains the testing parameter β.

$$\cancel{V}_{cyl} = \pi\, \mathrm{r_w^2} \left(\mathrm{z(t)} - \mathrm{c}\left(1 - \frac{\mathrm{r_w}}{\mathrm{R}}\right)\right) \qquad [7]$$

$$\cancel{V}_{cone} = \frac{1}{3}\pi\, r_w^3 \frac{c}{R} \qquad [8]$$

Added mass and drag on axisymmetric bodies and robot. The added mass of the axisymmetric bodies is approximated as $m_a \approx \frac{2}{3}\rho_f\, \pi\, R^3$. The radius is fixed at a value of $R = 2$ cm. The drag coefficient is 0.47 for a sphere, 0.27 for an ellipsoid of L=4 cm, 0.22 for an ellipsoid of L=6 cm, and 0.05 for a streamlined body. The added mass for the robot is based on two flat plates at an angle. The drag coefficient for the robot is 0.1, which is the same coefficient as a wedge.

Supplementary Figures

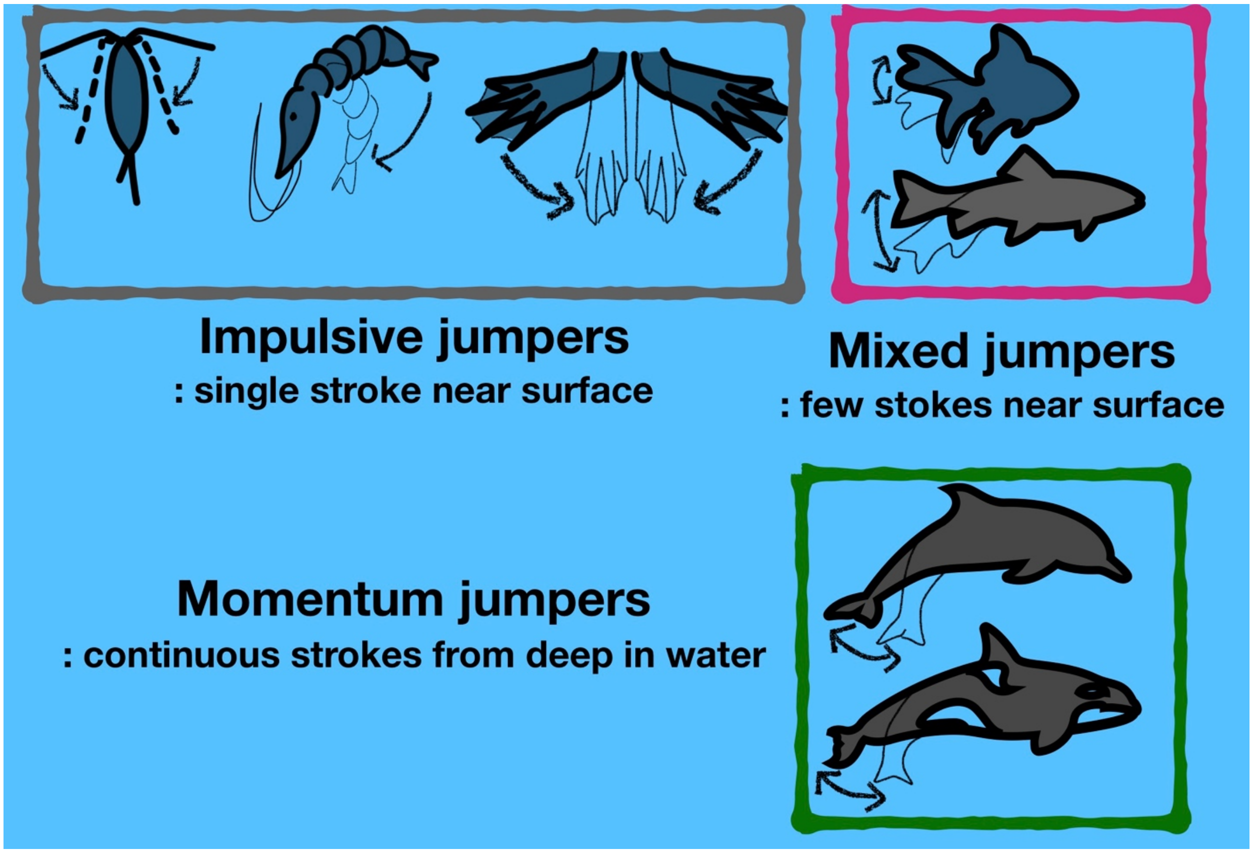

Figure S1. Schematic of the different jumping behaviors with brief descriptions. The robot described in this manuscript is inspired by the impulsive jumpers.

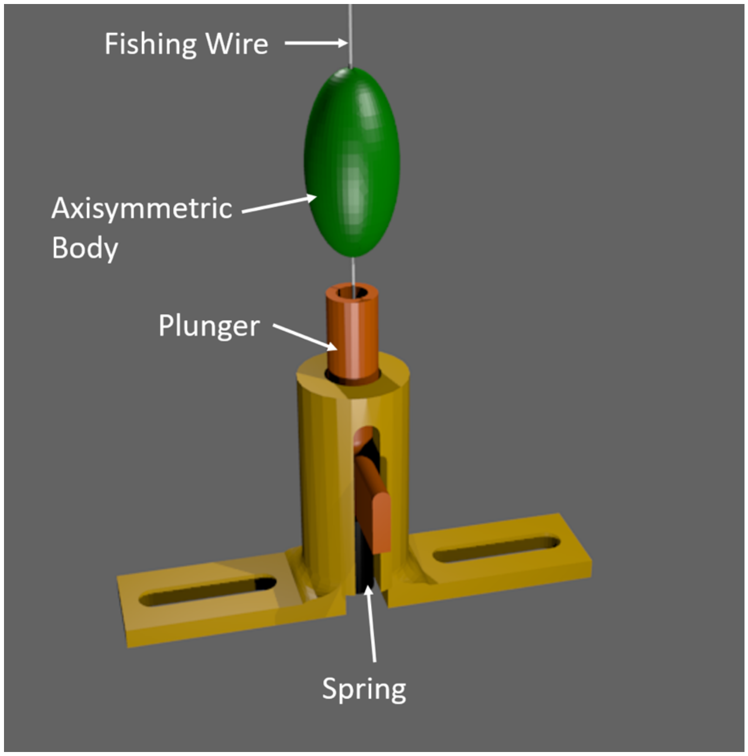

Figure S2. Experimental setup for shooting axisymmetric bodies through the water surface.

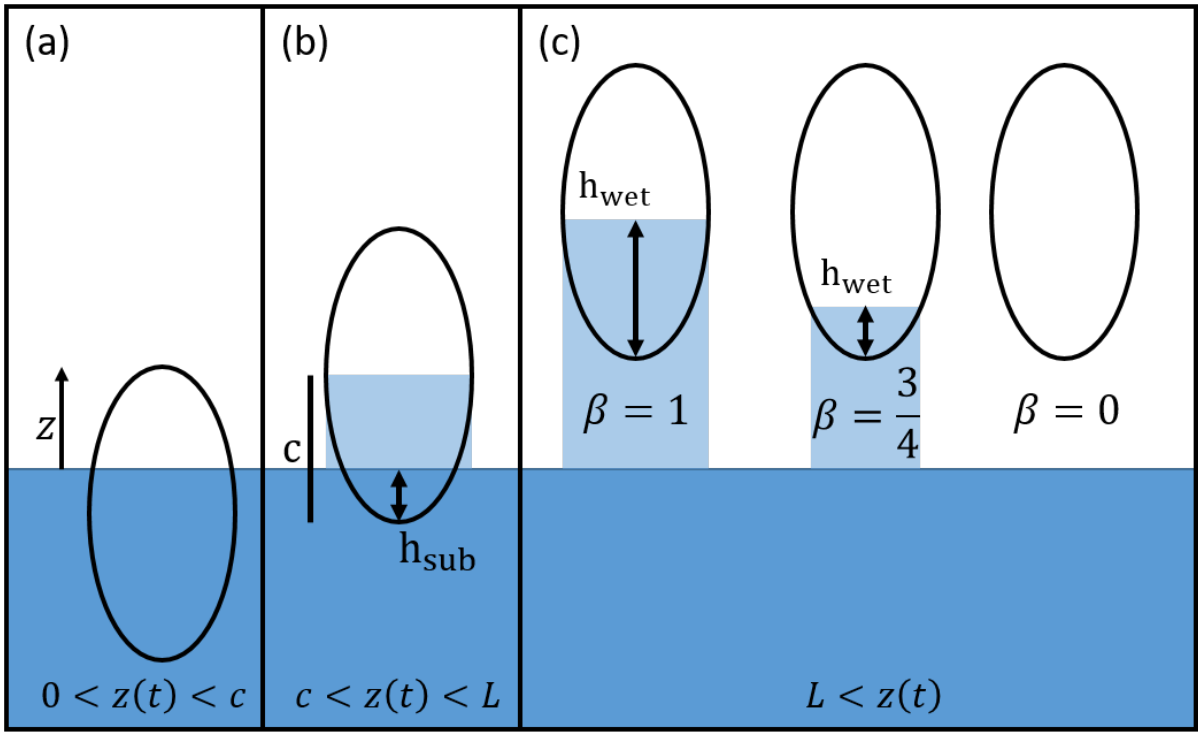

Figure S3. Schematic of spheroidal body exiting water at different times. (a) z(t)<c, the body is partially submerged, but no fluid entrainment. (b) c<z(t)<L, beginning of entrained fluid formation, but body is still partially submerged. The height of the submerged spheroidal cap is denoted as h_{sub}. (c) z(t)>L, the body completely escapes and entrains fluid. The testing parameter, β, is a constant that determines the effect of the entrained fluid. By decreasing β, the height of the wetted region, h_{wet}, decreases, which also decreases the importance of the entrained fluid mass.

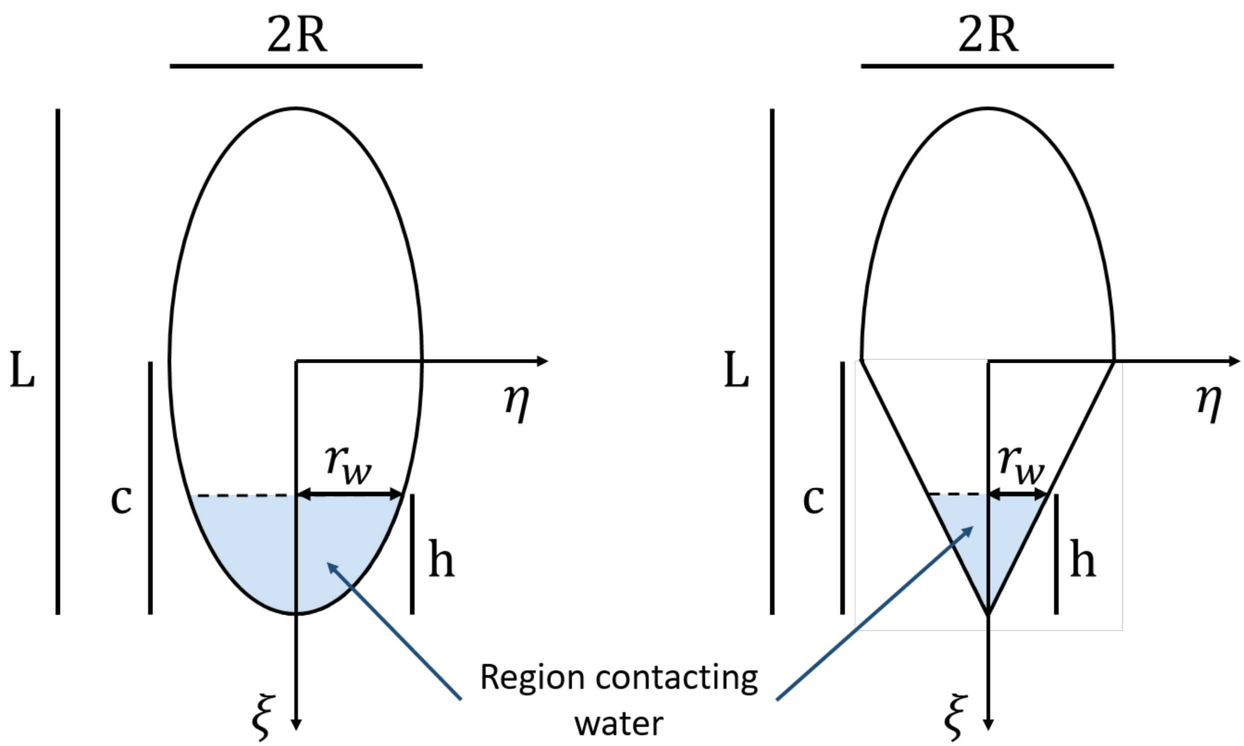

Figure S4. The local coordinates of the spheroid and streamlined body. For the spheroid, $\xi(\eta) = c\sqrt{1-(\eta/R)^2}$, so $h = c - \xi(r_w)$.

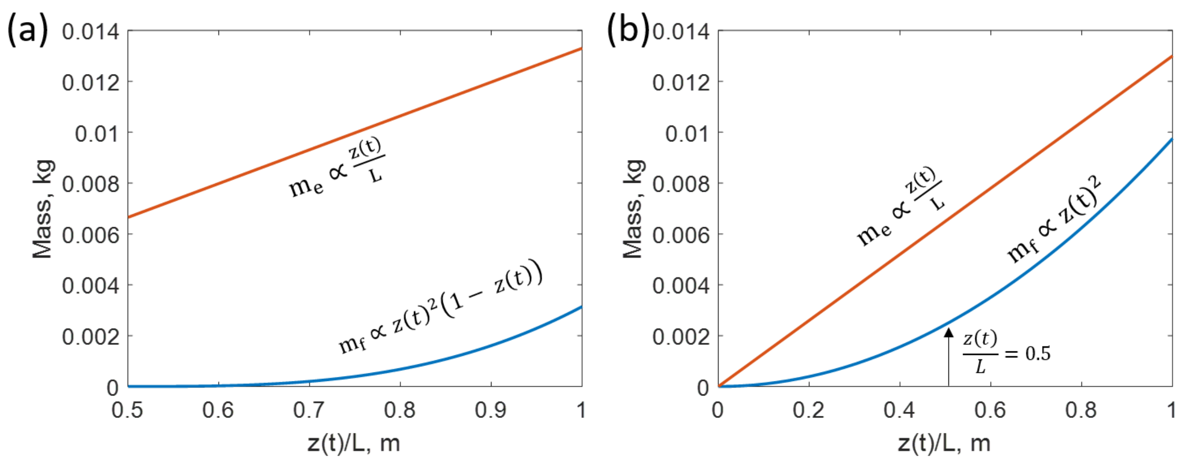

Figure S5. A comparison between body mass and growing entrained fluid mass from simulations. (a) Axisymmetric bodies with U_0 = 1.7 m/s, L = 6 cm, R = 1 cm. Here, fluid mass begins to grow when z(t)/L = 0.5. (b) Robot with U0 = 1.7 m/s, L = 6 cm. Here, fluid mass begins to grow at z(t)/L = 0.

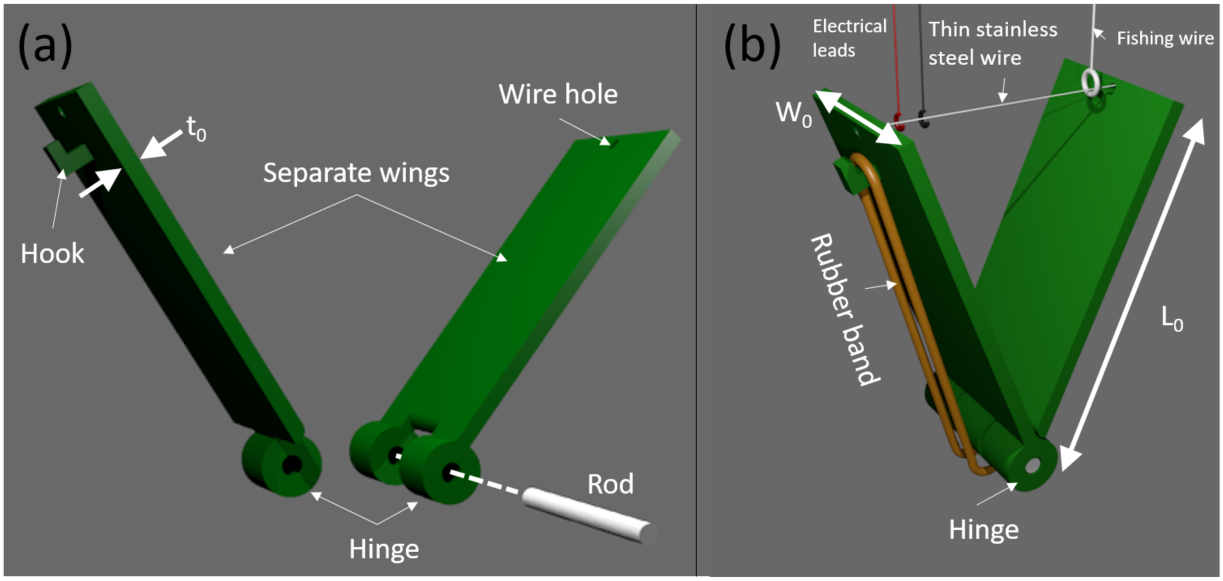

Figure S6. Robot schematic. (a) The two wings are joined together by a hinge. The thickness, t_0, of the wings is 3 mm. (b) Experimental setup. The wing ends are constrained together with a thin stainless steel wire while a rubber band is used to pull the wings apart. To make the wings flap, 24 volts is sent through the thin wire using electrical leads, enough to burn it. This, in turn, will allow the rubber band to pull the wings downward to create the flapping motion. When the stainless steel wire is threaded through the wire holes, the robot is now able to hang freely from the hook shaped electrical leads the fishing wire.

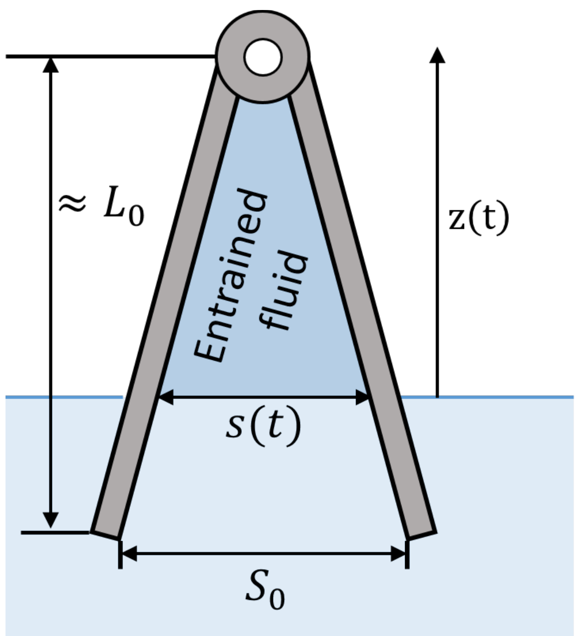

Figure S7. Schematic of robot leaping out of water. Volume of entrained fluid is calculated based on a fixed geometry.

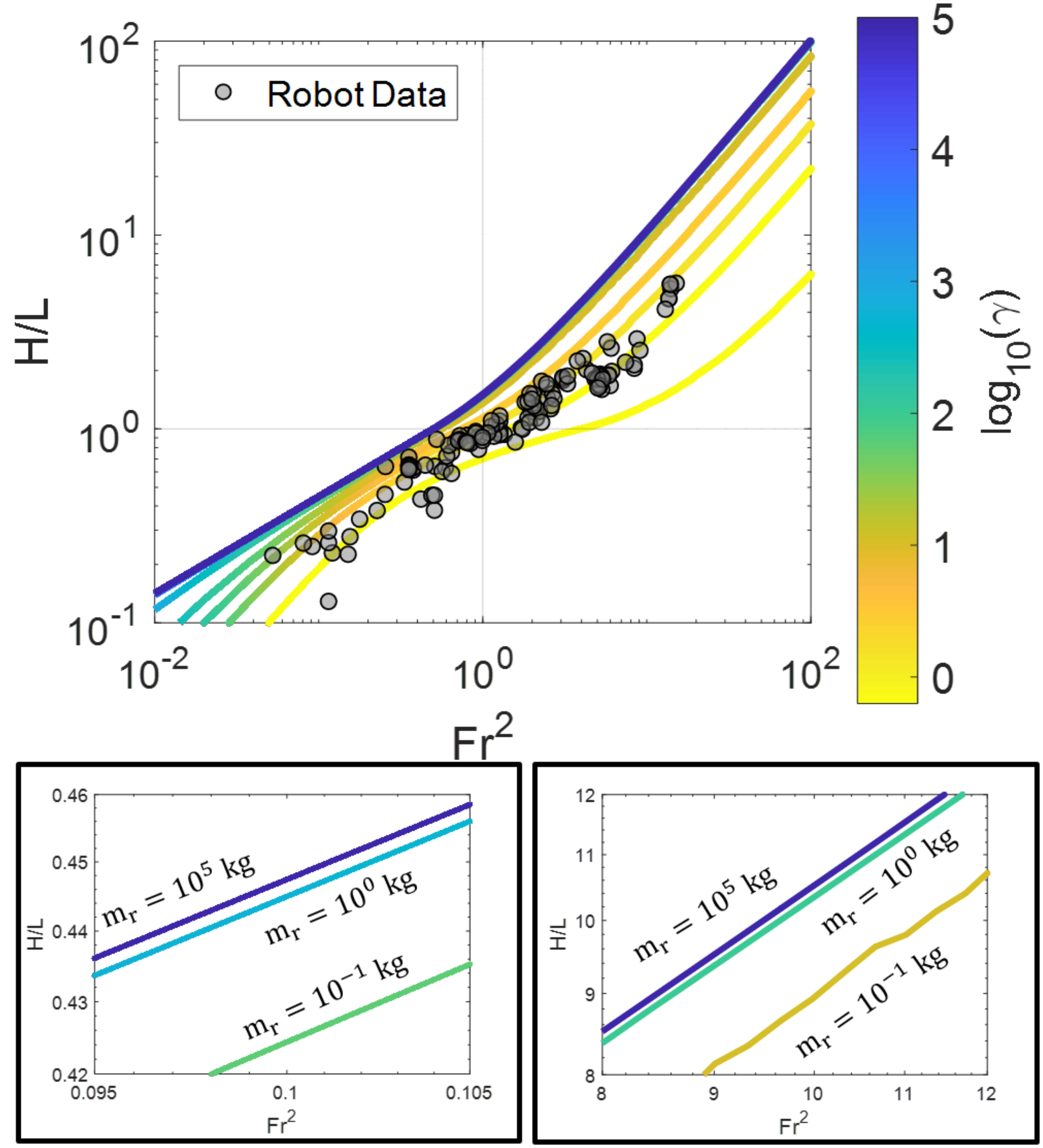

Figure S8. Sensitivity of robot mass on the jumping height. Experimental robot mass ranges from 0.006 - 0.026 kg. Hypothetical robot masses are input into the model. When the mass of the robot increases past 10^0 kg, the numerical solutions quickly converge.

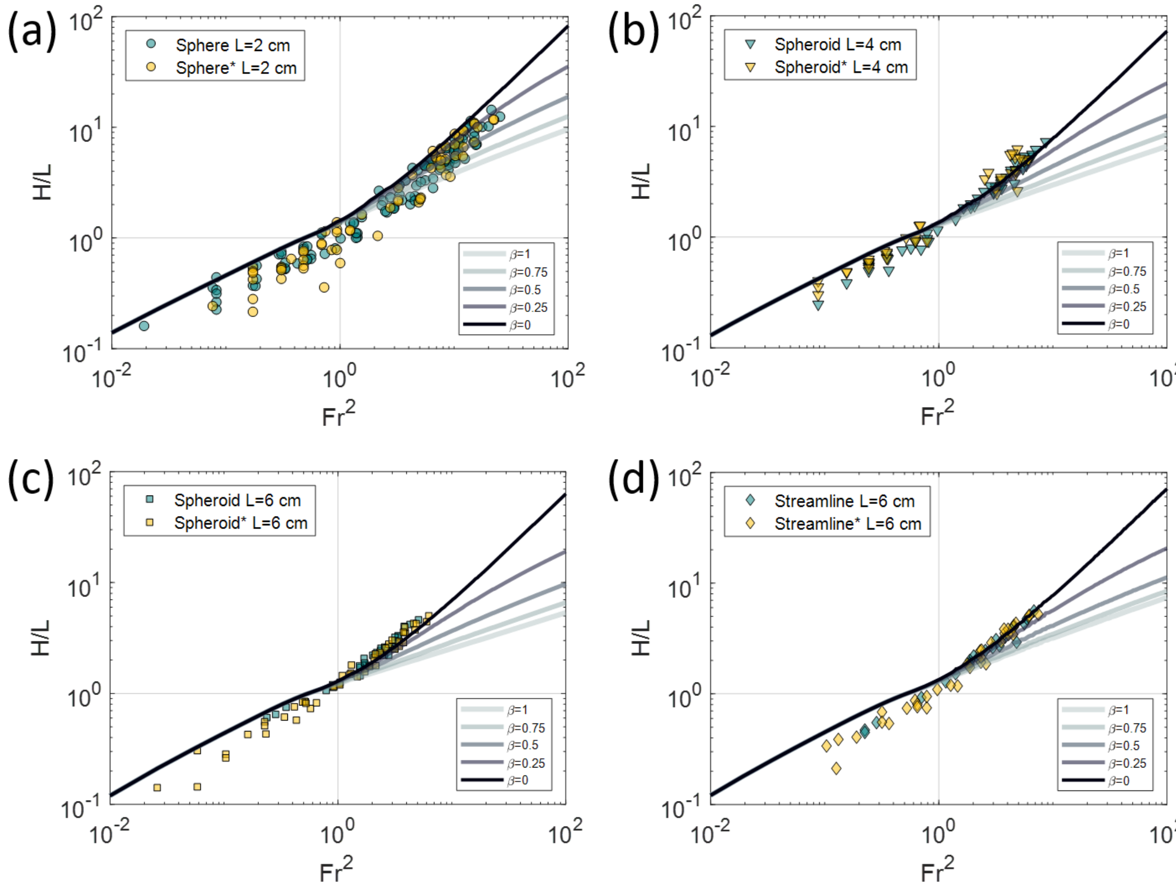

Figure S9. A detailed look at data from the jumping axisymmetric bodies. (a) Spheres with a diameter of L=2 cm. (b) Spheroids with a major diameter of L=4 cm and a minor diameter of 2R=2 cm. (c) Spheroids with a major diameter of L=6 cm and a minor diameter of 2R=2 cm.

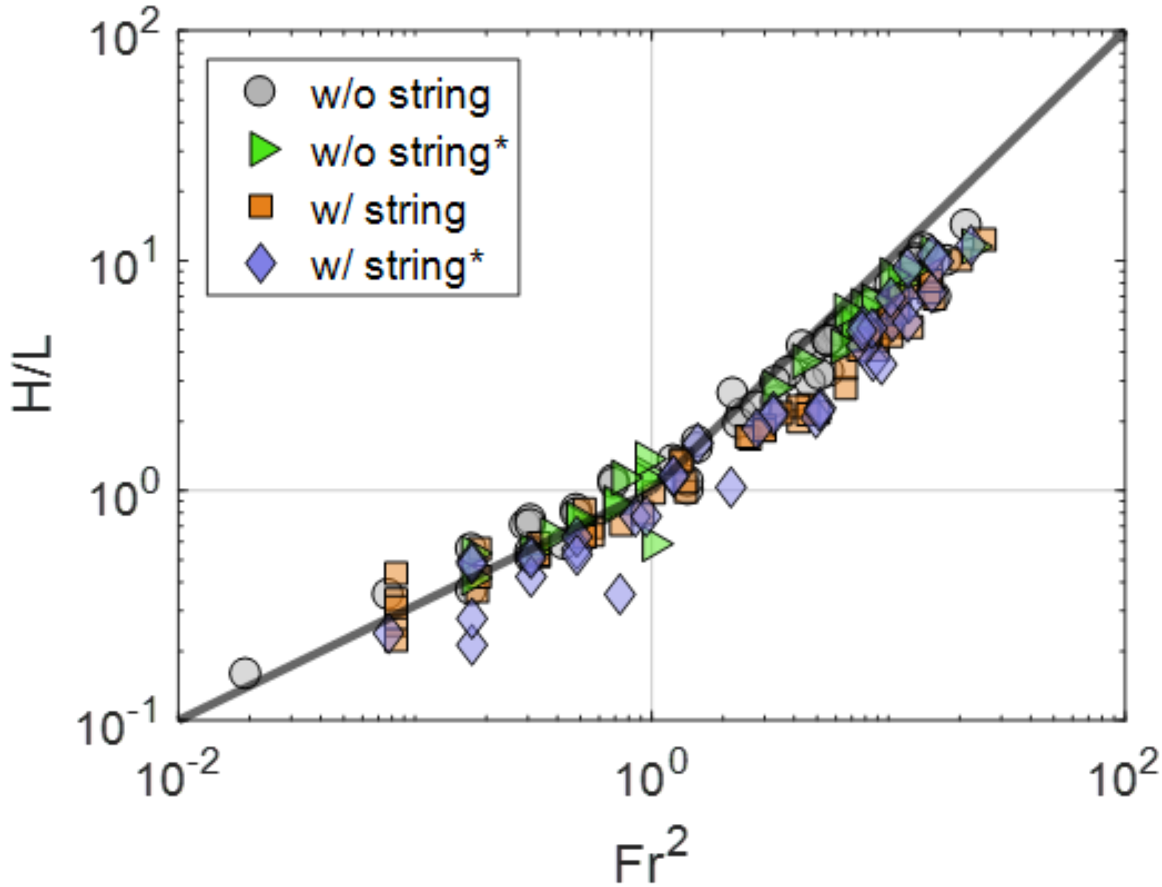

Figure S10. Distinguishing the different conditions that spheres are launched out of water. Here, there appears to be very little difference between spheres launched out of water with and without a guiding string.

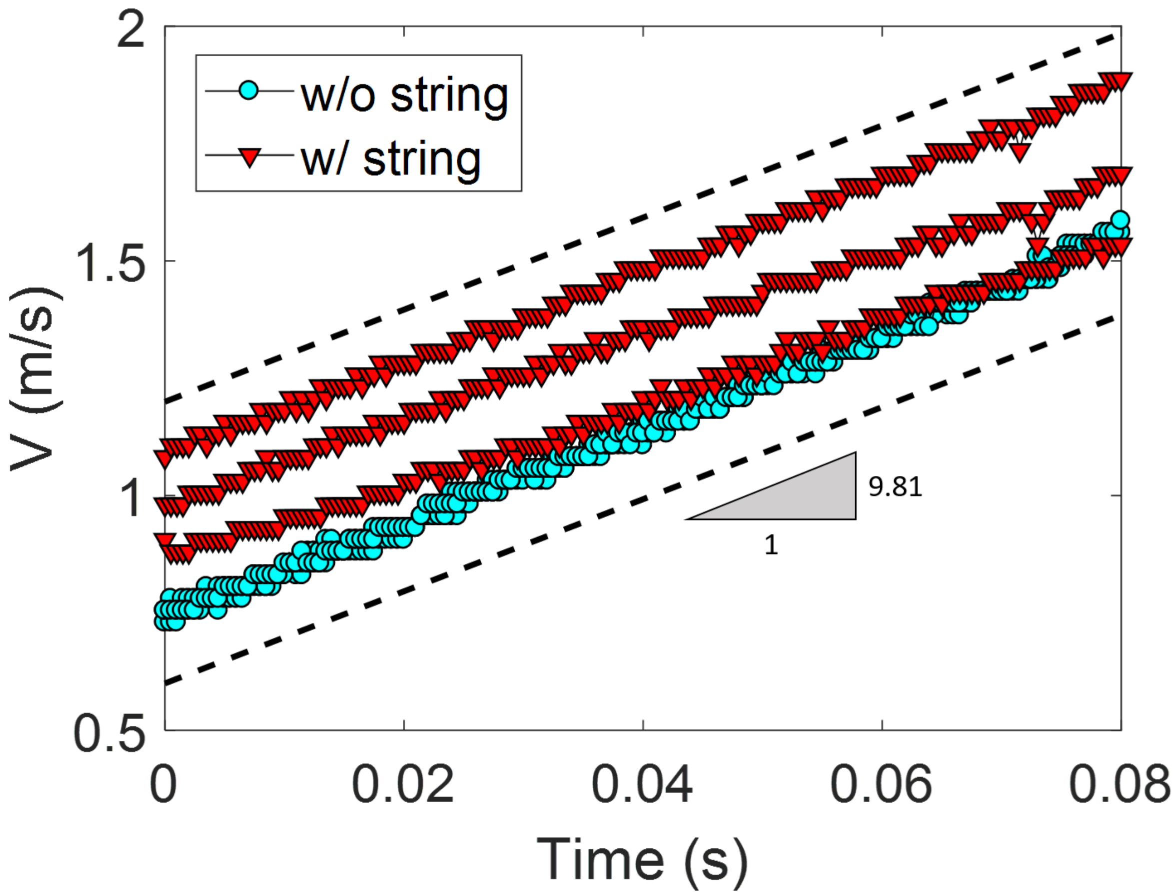

Figure S2. Data from dropping spheres with and without the thin string. Dotted lines represent the slope for gravity. Three drop trials were conducted for each case, all falling nearly at a rate of 9.81 m/s^2. This suggests that the string has little to no effect of the jumping dynamics.

Supplementary Movies and Data

Movie S1. Water exit of a prolate spheroid (major diameter is 6 cm, minor diameter is 2 cm, U_0=1.72 m/s, Fr^2=2.51). Same as image sequence from main text Fig. 3(a).

Movie S2. Water exit of a sphere (diameter is 2 cm, U_0=0.27 m/s, Fr^2=0.19).

Movie S3. Water exit of a sphere (diameter is 2 cm, U_0=2.1 m/s, Fr^2=11).

Movie S4. Robot jumping out of water. Same as image sequence from main text Fig. 3(c) (L=6.5 cm, U_0=1.52 m/s, Fr^2=1.96).

Additional data table S1 (AnimalData.xlsx). Dataset used to produce Fig. 2(a,b) in main text. Includes data for mass (kg), power production (W), and corresponding references for Fig. 2a. Includes data for length, jumping height, Froude number calculation, and corresponding references.

Additional data table S2 (SpheroidData.xlsx). Dataset used to produce Fig. 3b in main text and SI Fig. S8. Includes data for jumping height, velocity, Froude number, and Weber number.

Additional data table S3 (RobotData.xlsx). Dataset used to produce Fig. 3d in main text. Includes data for jumping height, velocity, and Froude number.