

Supplemental Data

Facilitated Transport of a Dpp/Scw

Heterodimer by Sog/Tsg Leads to Robust

Patterning of the *Drosophila* Blastoderm Embryo

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Sog/Tsg Formation Compensates for Reductions in Sog or Tsg

Experimentally, we find that Sog and Tsg are both required for proper amnioserosa specification and that they form a multimeric complex Sog/Tsg that has enhanced inhibitory activity and a high affinity for binding to Dpp/Scw. While the complex Sog/Tsg forms in a spatially distributed system, we analyzed the local dynamics of this type of binding reaction. Here, degradation must be considered for there to be a steady state. The local dynamics for Sog/Tsg formation are described by Equations 1–3 (main text) by setting the homodimer rates ($k_{2,-2}, k_{3,-3}$) equal to zero. Now, defining $\lambda \equiv \phi_{Tsg}^{perturbed} / \phi_{Tsg}^{wt}$ and $\beta \equiv \phi_{Sog}^{wt} / \phi_{Tsg}^{wt}$, we can measure the Sog/Tsg output to changes in input. Defining two new dimensionless variables

$u = \frac{\delta_X}{\phi_X^{wt}} X$, $v = \frac{\delta_Y}{\phi_Y^{wt}} Y$ and the dimensionless parameter $\Lambda = \frac{K_1 \phi_X^{wt}}{2\delta_X \delta_Y}$ leads to Equations S1 and S2 given

below. These are analogous to Equations 6 and 7 (Figure 6, main text), and the primary difference is that the quadratic term is replaced by linear terms. Solving for u and v leads to Equations S3 and S4 where $a = 2\Lambda$ and $b = 1 + 2\Lambda(\beta - \lambda)$.

$$(S1) \quad u + 2\Lambda uv = \lambda \qquad (S3) \quad u = \frac{-b(\lambda, \beta, \Lambda) + \sqrt{b(\lambda, \beta, \Lambda)^2 + 4\lambda a(\Lambda)}}{2a(\Lambda)}$$

$$(S2) \quad v + 2\Lambda uv = \beta \qquad (S4) \quad v = \frac{\beta}{1 + 2\Lambda u}$$

Now the output ratio is $\frac{XY^{mut}}{XY^{wt}} = \frac{\lambda - u(\lambda, \beta, \Lambda)}{1 - u(1, \beta, \Lambda)}$ where XY denotes Sog/Tsg. Solutions for the case $\Lambda = 1/2$

are shown as green lines in Figure 7F for different β s. Solutions for $\Lambda = 1/2$ can be computed as before, and a histogram summarizing these results is shown in Figure 7D along with a plot of the *mut* to *wt* ratio versus β for the case $\lambda = 1/2$. For small β , the Sog/Tsg ratio is bounded again by a maximum of 1; however, unlike before, the minimum is now equal to the amount perturbed, suggesting some choices of parameters lead to no compensation. For large β , all solutions collapse to a Sog/Tsg ratio equal to the perturbation, which suggests that there is no compensation for perturbations of the limiting species. For reductions of Sog and Tsg, this result suggests that the system could compensate for only one of Sog or Tsg and not both. However, these results are based on the local dynamics, when in the fact the spatially distributed system where Sog and Tsg are produced in separate adjacent domains (Sog in the lateral neurogenic ectoderm and Tsg in the dorsal region). It turns out the spatial segregation of Sog and Tsg production increases the robustness at the level of morphogen gradient formation as a result of this heterodimer formation and as a result of the heterodimer formation occurring at the interface between opposing gradients of Sog and Tsg (D.U., O.S., M.B.O., and H.O., unpublished data). Additionally, there are a number of other reactions downstream of Sog/Tsg formation that may provide additional compensation leading to robust output by the BMP signal.