

## Supplementary Materials for

### **Holographic imaging of electromagnetic fields via electron-light quantum interference**

I. Madan, G. M. Vanacore, E. Pomarico, G. Berruto, R. J. Lamb, D. McGrouther, T. T. A. Lummen, T. Latychevskaia, F. J. García de Abajo, F. Carbone\*

\*Corresponding author. Email: [fabrizio.carbone@epfl.ch](mailto:fabrizio.carbone@epfl.ch)

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#### **The PDF file includes:**

Note S1. Calculation of group and phase velocities.

Fig. S1. Wavefronts of the plasmons forming the interference pattern for arbitrary light wavefront tilt (incidence direction in the sample plane).

#### **Other Supplementary Material for this manuscript includes the following:**

(available at [advances.sciencemag.org/cgi/content/full/5/5/eaav8358/DC1](https://advances.sciencemag.org/cgi/content/full/5/5/eaav8358/DC1))

Movie S1 (.avi format). Plasmon hologram evolution with 0.33-fs time step.

Movie S2 (.avi format). Plasmon hologram evolution over 145-fs delay span.

## Note S1. Calculation of group and phase velocities.

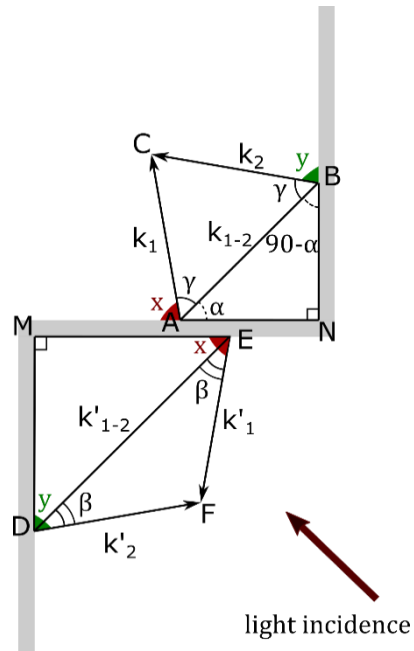
As noted in the main text, the experiments on local interference of two plasmons are conducted under critical-angle conditions. Specifically, the sample-surface normal  $\mathbf{n}$ , the incident light wave vector  $\mathbf{k}$ , and the electron wave vector  $\mathbf{k}_{el}$  all lie in the same plane, while the angles between these vectors are  $\mathbf{n} \wedge \mathbf{k} \approx 7^\circ$  and  $\mathbf{n} \wedge \mathbf{k}_{el} \approx 12^\circ$ . We introduce this tilt of the sample in order to reduce the electron interaction with inverse transition radiation. As one can see from Fig. 2a of the main text, no observable contrast from nonlocal interference is observed under these conditions. Nevertheless, two corrections need to be introduced in the calculations of the phase velocity:

- 1) Because the recorded holograms are projections of the sample plane onto the plane orthogonal to the electron propagation direction, we need to change the scale in the direction of the tilt by a factor of  $1/\cos(12^\circ) = 1.022$ .
- 2) The tilt of the emitting slit with respect to the light wavefront results in retardation of the plasmon emission, as different parts of the slit are exposed to the light wavefront at different times. Effectively, this results in a tilt of the plasmon wavefront with respect to the direction orthogonal to the emitter. Consequently, the periodicity of the standing wave pattern is observed at a wave vector  $|\mathbf{k}_1 - \mathbf{k}_2| = 2k_{SPP} \sin((\mathbf{k}_1 \wedge \mathbf{k}_2)/2)$ , which depends on the tilt of both  $\mathbf{k}_1$  and  $\mathbf{k}_2$ .

In general, the tilt direction is not aligned with any of the emitters, and the angle  $\mathbf{k}_1 \wedge \mathbf{k}_2$  is not known precisely. We have addressed this problem by devising a simple extension of our experiment involving a stair-like emitter, which allows us to discriminate between plasmons emitted with different tilts of the wavefront.

In Supplementary Fig. 1 we consider incident light having an arbitrary angle of incidence (the direction of in-plane component of the light momentum is indicated by a red arrow). The plasmon wavefronts are tilted along directions  $\mathbf{k}_1$ ,  $\mathbf{k}_2$ ,  $\mathbf{k}'_1$ , and  $\mathbf{k}'_2$ , and the interference patterns are formed along the line defined by  $k_{1-2} = |\mathbf{k}_1 - \mathbf{k}_2|$  and  $k'_{1-2} = |\mathbf{k}'_1 - \mathbf{k}'_2|$  with experimentally measurable periodicities. We note that wave vectors  $\mathbf{k}_1, \mathbf{k}'_1$  form the same angle  $x$  with the emitter. The same is true for wave vectors  $\mathbf{k}_2, \mathbf{k}'_2$  and the angle  $y$ . Furthermore, we can deduce the angles  $\angle DFE = 270^\circ - x - y$  and  $\angle ACB = x + y - 90^\circ$ , which allow us to write  $\angle DFE = 180^\circ - \angle ACB$ .

Alternatively, we can write  $\gamma = 90^\circ - \beta$ , and finally from the relations  $k_{1-2} = 2k_{SPP} \cos \gamma$  and  $k'_{1-2} = 2k_{SPP} \cos \beta$ , we can write  $k_{1-2}^2 + k'^2_{1-2} = 4k_{SPP}^2$ . Thus, knowing both  $k_{1-2}$  and  $k'_{1-2}$ , we can extract the actual value of  $k_{SPP}$  used in the calculation of the phase velocity.



**Fig. S1. Wavefronts of the plasmons forming the interference pattern for arbitrary light wavefront tilt (incidence direction in the sample plane).** Retardation effects lead to a plasmon wavefront tilt with respect to the direction orthogonal to the slits. Angles relevant to the derivation presented in this note are marked in the figure.