APPENDIX

A ESTIMATION UNDER GENERALIZED LINEAR MODELS

Here we consider extending the CoFu method to the generalized linear models (GLMs). Consider K independent datasets. In dataset $k(= 1, \dots, K)$, denote $\boldsymbol{y}^k$ as the response variable, and $\boldsymbol{X}^k$ as the length-p vector of covariates. There are n_k i.i.d observations in dataset k. The response variable $\boldsymbol{y}^k$ is generated from a distribution in the exponential family with mean $\boldsymbol{\mu}^k \equiv E(\boldsymbol{y}^k)$ dependent on $\boldsymbol{X}^k$ via

$$g(\boldsymbol{\mu}^k) = \boldsymbol{X}^k \boldsymbol{\beta}^k,$$

where $g(\cdot)$ is the link function. In the Logit model, a representative of GLM, $g(\mu) = \log \frac{\mu}{1-\mu}$.

The CoFu estimator $\{\hat{\boldsymbol{\beta}}^k : k = 1, \dots, K\}$ is defined as the minimizer of

$$-\sum_{k=1}^{K} \frac{1}{n_k} L_k(\boldsymbol{\beta}^k) + \lambda_1 \sum_{k=1}^{K} \parallel \boldsymbol{\beta}^k \parallel_1 + \lambda_2 \sum_{k=1}^{K-1} \sum_{l=1}^{L} \parallel \boldsymbol{\beta}^k_{(l)} - \boldsymbol{\beta}^{k+1}_{(l)} \parallel_2, \tag{A1}$$

where $L_k(\cdot)$ is the log-likelihood function of dataset k. In the Logit model, $L_k(\boldsymbol{\beta}^k) = \sum_{i=1}^{n_k} y_i^k \boldsymbol{X}_i^k \boldsymbol{\beta}^k - \log[1 + \exp(\boldsymbol{X}_i^k \boldsymbol{\beta}^k)]$.

We compute the minimizer of (A1) by using the ADMM algorithm:

$$\begin{aligned}
\boldsymbol{\beta}(t+1) &= argmin_{\beta} \left[-\sum_{k=1}^{K} \frac{1}{n_k} L_k(\boldsymbol{\beta}^k) + \frac{\sigma}{2} \Big(\parallel \boldsymbol{A}\boldsymbol{\beta} - \boldsymbol{\eta}(t) + \boldsymbol{u}(t)/\sigma \parallel_2^2 + \parallel \boldsymbol{\beta} - \boldsymbol{\delta}(t) + \boldsymbol{v}(t)/\sigma \parallel_2^2 \Big) \right], \\
\boldsymbol{\delta}(t+1) &= \mathrm{ST}_{\lambda_1/\sigma}[\boldsymbol{\beta}(t+1) + \boldsymbol{v}(t)/\sigma], \\
\boldsymbol{\eta}^k_{(l)}(t+1) &= \left(1 - \frac{\lambda_2}{\sigma \parallel \boldsymbol{\beta}^k_{(l)}(t+1) - \boldsymbol{\beta}^{k+1}_{(l)}(t+1) + \boldsymbol{u}^k_{(l)}(t)/\sigma \parallel_2}\right)_+ \times [\boldsymbol{\beta}^k_{(l)}(t+1) - \boldsymbol{\beta}^{k+1}_{(l)}(t+1) + \boldsymbol{u}^k_{(l)}(t)/\sigma], \\
\boldsymbol{u}(t+1) &= \boldsymbol{u}(t) + \sigma[\boldsymbol{A}\boldsymbol{\beta}(t+1) - \boldsymbol{\eta}(t+1)], \\
\boldsymbol{v}(t+1) &= \boldsymbol{v}(t) + \sigma[\boldsymbol{\beta}(t+1) - \boldsymbol{\delta}(t+1)].
\end{aligned}$$

These update equations are highly similar to that in the linear regression model. The $\boldsymbol{\beta}$ update involves solving a non-linear equation and can be efficiently realized with the L-BFGS algorithm[35], a limited memory version of the BFGS algorithm.

B ADDITIONAL TABLES AND FIGURES

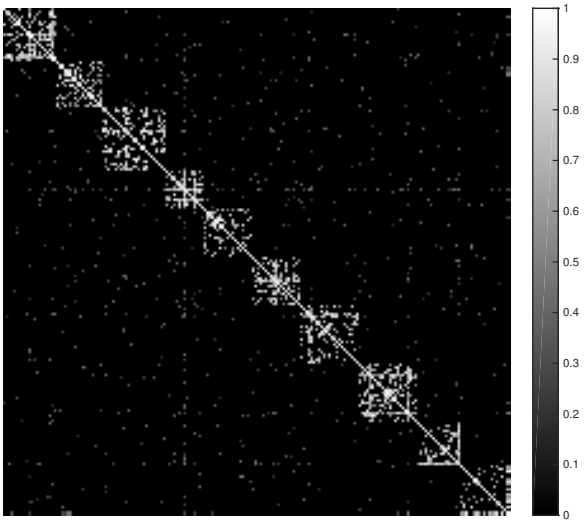

FIGURE B1 Greyscale image of a structured correlation matrix. The lighter the color is, the stronger the correlation is.

TABLE B1 Simulation under the LR model: mean(sd) of AUC for effect identification. Nonzero coefficients are randomly drawn from $\mathcal{U}[0.2, 1]$.

	$r = 100$			$r = 150$		
	structured	unstructured	independence	structured	unstructured	independence
$(\rho_a, \rho_h, \rho_n) = (0.1, 0, 0.9)$						
P.Lasso	0.605(0.018)	0.57(0.01)	0.612(0.017)	0.578(0.016)	0.576(0.02)	0.595(0.019)
S.Lasso	0.746(0.012)	0.795(0.008)	0.745(0.01)	0.677(0.015)	0.75(0.011)	0.675(0.017)
CoFu	0.761(0.011)	0.848(0.01)	0.757(0.012)	0.695(0.013)	0.746(0.015)	0.688(0.012)
$(\rho_a, \rho_h, \rho_n) = (0.1, 0.9, 0)$						
P.Lasso	0.744(0.021)	0.698(0.014)	0.745(0.015)	0.659(0.017)	0.646(0.013)	0.661(0.014)
S.Lasso	0.757(0.014)	0.807(0.013)	0.752(0.014)	0.683(0.017)	0.756(0.011)	0.676(0.015)
CoFu	0.839(0.013)	0.848(0.007)	0.844(0.012)	0.763(0.013)	0.795(0.009)	0.744(0.01)
$(\rho_a, \rho_h, \rho_n) = (0.2, 0.6, 0.2)$						
P.Lasso	0.721(0.023)	0.827(0.012)	0.713(0.019)	0.667(0.015)	0.647(0.015)	0.658(0.016)
S.Lasso	0.757(0.019)	0.806(0.009)	0.756(0.011)	0.689(0.01)	0.763(0.017)	0.677(0.013)
CoFu	0.824(0.015)	0.882(0.008)	0.82(0.013)	0.751(0.012)	0.786(0.014)	0.738(0.014)
$(\rho_a, \rho_h, \rho_n) = (0.4, 0.1, 0.5)$						
P.Lasso	0.723(0.01)	0.713(0.013)	0.706(0.014)	0.675(0.014)	0.666(0.014)	0.673(0.018)
S.Lasso	0.752(0.009)	0.801(0.012)	0.748(0.008)	0.681(0.011)	0.757(0.012)	0.668(0.012)
CoFu	0.82(0.009)	0.843(0.011)	0.804(0.014)	0.74(0.014)	0.779(0.01)	0.733(0.013)
$(\rho_a, \rho_h, \rho_n) = (0.5, 0.5, 0)$						
P.Lasso	0.845(0.02)	0.827(0.012)	0.838(0.018)	0.787(0.019)	0.777(0.015)	0.781(0.021)
S.Lasso	0.76(0.015)	0.806 (0.009)	0.753(0.013)	0.691(0.011)	0.759(0.016)	0.682(0.013)
CoFu	0.88(0.014)	0.882(0.008)	0.876(0.01)	0.805(0.01)	0.825(0.012)	0.806(0.008)
$(\rho_a, \rho_h, \rho_n) = (0.6, 0.2, 0.2)$						
P.Lasso	0.852(0.012)	0.819(0.015)	0.838(0.016)	0.79(0.015)	0.765(0.021)	0.788(0.017)
S.Lasso	0.755(0.013)	0.803(0.011)	0.753(0.011)	0.684(0.022)	0.755(0.014)	0.677(0.018)
CoFu	0.883(0.012)	0.877(0.009)	0.866(0.01)	0.803(0.016)	0.811(0.013)	0.802(0.016)
$(\rho_a, \rho_h, \rho_n) = (0.9, 0, 0.1)$						
P.Lasso	0.955(0.015)	0.889(0.014)	0.954(0.014)	0.886(0.016)	0.853(0.017)	0.895(0.014)
S.Lasso	0.748(0.016)	0.795(0.012)	0.741(0.009)	0.696(0.016)	0.747(0.013)	0.687(0.016)
CoFu	0.967(0.013)	0.93(0.01)	0.964(0.012)	0.92(0.014)	0.887(0.011)	0.92(0.012)

TABLE B2 Simulation under the LR model: mean(sd) of AUC for community differentiation. Nonzero coefficients are randomly drawn from $\mathcal{U}[0.2, 1]$.

	$r = 100$			$r = 150$		
	structured	unstructured	independence	structured	unstructured	independence
$(\rho_a, \rho_h, \rho_n) = (0.1, 0, 0.9)$						
S.Lasso	0.524(0.066)	0.551(0.065)	0.554(0.082)	0.563(0.052)	0.591(0.072)	0.537(0.045)
CoFu	0.774(0.042)	0.723(0.039)	0.795(0.038)	0.714(0.045)	0.711(0.048)	0.724(0.037)
$(\rho_a, \rho_h, \rho_n) = (0.1, 0.9, 0.1)$						
S.Lasso	0.626(0.062)	0.53(0.075)	0.595(0.088)	0.627(0.067)	0.652(0.082)	0.675(0.042)
CoFu	0.801(0.045)	0.722(0.032)	0.735(0.034)	0.76(0.041)	0.781(0.035)	0.724(0.052)
$(\rho_a, \rho_h, \rho_n) = (0.2, 0.6, 0.2)$						
S.Lasso	0.573(0.082)	0.54(0.105)	0.549(0.074)	0.477(0.06)	0.515(0.088)	0.488(0.037)
CoFu	0.742(0.028)	0.698(0.042)	0.73(0.04)	0.711(0.035)	0.734(0.031)	0.711(0.033)
$(\rho_a, \rho_h, \rho_n) = (0.4, 0.1, 0.5)$						
S.Lasso	0.534(0.072)	0.552(0.086)	0.567(0.041)	0.537(0.058)	0.542(0.055)	0.544(0.066)
CoFu	0.785(0.021)	0.738(0.046)	0.777(0.036)	0.727(0.033)	0.731(0.041)	0.715(0.033)
$(\rho_a, \rho_h, \rho_n) = (0.5, 0.5, 0)$						
S.Lasso	0.542(0.057)	0.487(0.059)	0.546(0.061)	0.515(0.088)	0.492(0.124)	0.513(0.084)
CoFu	0.812(0.037)	0.754(0.023)	0.81(0.036)	0.711(0.043)	0.705(0.053)	0.703(0.034)
$(\rho_a, \rho_h, \rho_n) = (0.6, 0.2, 0.2)$						
S.Lasso	0.5(0.074)	0.49(0.072)	0.539(0.103)	0.47(0.075)	0.529(0.08)	0.504(0.068)
CoFu	0.791(0.046)	0.742(0.029)	0.802(0.046)	0.723(0.049)	0.721(0.036)	0.713(0.055)
$(\rho_a, \rho_h, \rho_n) = (0.9, 0, 0.1)$						
S.Lasso	0.481(0.062)	0.45(0.088)	0.453(0.092)	0.52(0.072)	0.478(0.082)	0.444(0.062)
CoFu	0.975(0.031)	0.968(0.033)	0.975(.034)	0.927(0.035)	0.91(0.045)	0.911(0.031)

TABLE B3 Simulation under the LR model: mean(sd) of AUC for effect identification. Nonzero coefficients in dataset 2 are drawn from $\mathcal{U}[0.4, 0.7]$, and those specific to datasets 1 and 3 are drawn from $\mathcal{U}[0.1, 0.3]$ and $\mathcal{U}[0.8, 1]$, respectively.

	$r = 100$			$r = 150$		
	structured	unstructured	independence	structured	unstructured	independence
$(\rho_a, \rho_h, \rho_n) = (0.1, 0, 0.9)$						
P.Lasso	0.598(0.02)	0.549(0.022)	0.591(0.012)	0.559(0.02)	0.546(0.014)	0.556(0.021)
S.Lasso	0.757(0.011)	0.812(0.012)	0.738(0.015)	0.675(0.021)	0.757(0.011)	0.666(0.015)
CoFu	0.759(0.012)	0.805(0.009)	0.744(0.012)	0.677(0.018)	0.734(0.012)	0.661(0.018)
$(\rho_a, \rho_h, \rho_n) = (0.1, 0.9, 0)$						
P.Lasso	0.721(0.02)	0.684(0.022)	0.744(0.012)	0.675(0.016)	0.621(0.012)	0.667(0.015)
S.Lasso	0.768(0.013)	0.832(0.012)	0.761(0.015)	0.689(0.018)	0.755(0.012)	0.68(0.014)
CoFu	0.811(0.012)	0.846(0.01)	0.813(0.013)	0.761(0.016)	0.786(0.01)	0.754(0.017)
$(\rho_a, \rho_h, \rho_n) = (0.2, 0.6, 0.2)$						
P.Lasso	0.724(0.023)	0.678(0.017)	0.726(0.009)	0.66(0.022)	0.624(0.01)	0.662(0.022)
S.Lasso	0.761(0.01)	0.819(0.014)	0.756(0.007)	0.69(0.014)	0.764(0.013)	0.681(0.012)
CoFu	0.823(0.015)	0.853(0.011)	0.82(0.012)	0.754(0.011)	0.785(0.014)	0.741(0.01)
$(\rho_a, \rho_h, \rho_n) = (0.4, 0.1, 0.5)$						
P.Lasso	0.712(0.018)	0.685(0.027)	0.708(0.014)	0.672(0.012)	0.626(0.018)	0.67(0.012)
S.Lasso	0.751(0.012)	0.806(0.009)	0.731(0.022)	0.681(0.013)	0.758(0.009)	0.673(0.013)
CoFu	0.809(0.013)	0.847(0.007)	0.804(0.014)	0.815(0.02)	0.786(0.008)	0.734(0.022)
$(\rho_a, \rho_h, \rho_n) = (0.5, 0.5, 0)$						
P.Lasso	0.868(0.016)	0.845(0.012)	0.868(0.013)	0.799(0.021)	0.766(0.021)	0.796(0.019)
S.Lasso	0.769(0.014)	0.82(0.006)	0.757(0.019)	0.693(0.011)	0.768(0.013)	0.684(0.009)
CoFu	0.889(0.014)	0.884(0.006)	0.885(0.013)	0.816(0.014)	0.829(0.01)	0.809(0.016)
$(\rho_a, \rho_h, \rho_n) = (0.6, 0.2, 0.2)$						
P.Lasso	0.861(0.016)	0.827(0.019)	0.858(0.014)	0.807(0.018)	0.751(0.015)	0.805(0.024)
S.Lasso	0.764(0.012)	0.814(0.009)	0.751(0.013)	0.686(0.023)	0.761(0.01)	0.677(0.018)
CoFu	0.878(0.008)	0.877(0.007)	0.872(0.01)	0.815(0.02)	0.817(0.011)	0.813(0.02)
$(\rho_a, \rho_h, \rho_n) = (0.9, 0, 0.1)$						
P.Lasso	0.878(0.02)	0.889(0.022)	0.861(0.012)	0.909(0.019)	0.855(0.019)	0.913(0.016)
S.Lasso	0.761(0.013)	0.813(0.008)	0.749(0.018)	0.68(0.018)	0.75(0.012)	0.672(0.015)
CoFu	0.857(0.012)	0.883(0.009)	0.848(0.012)	0.932(0.017)	0.899(0.013)	0.93(0.021)

TABLE B4 Simulation under the LR model: mean(sd) of AUC for community differentiation. Nonzero coefficients in dataset 2 are drawn from $\mathcal{U}[0.4, 0.7]$, and those specific to datasets 1 and 3 are drawn from $\mathcal{U}[0.1, 0.3]$ and $\mathcal{U}[0.8, 1]$, respectively.

	$r = 100$			$r = 150$		
	structured	unstructured	independence	structured	unstructured	independence
$(\rho_a, \rho_h, \rho_n) = (0.1, 0, 0.9)$						
S.Lasso	0.598(0.066)	0.609(0.086)	0.553(0.062)	0.578(0.068)	0.515(0.082)	0.462(0.088)
CoFu	0.793(0.036)	0.782(0.052)	0.787(0.042)	0.824(0.038)	0.84(0.038)	0.853(0.035)
$(\rho_a, \rho_h, \rho_n) = (0.1, 0.9, 0)$						
S.Lasso	0.449(0.055)	0.441(0.082)	0.489(0.056)	0.658(0.061)	0.629(0.082)	0.6(0.058)
CoFu	0.765(0.035)	0.754(0.031)	0.789(0.040)	0.79(0.037)	0.737(0.042)	0.733(0.035)
$(\rho_a, \rho_h, \rho_n) = (0.2, 0.6, 0.2)$						
S.Lasso	0.578(0.083)	0.516(0.093)	0.574(0.078)	0.499(0.077)	0.544(0.1)	0.506(0.094)
CoFu	0.733(0.042)	0.713(0.057)	0.734(0.037)	0.711(0.035)	0.734(0.031)	0.711(0.033)
$(\rho_a, \rho_h, \rho_n) = (0.4, 0.1, 0.5)$						
S.Lasso	0.553(0.038)	0.546(0.071)	0.558(0.046)	0.516(0.063)	0.516(0.06)	0.541(0.061)
CoFu	0.755(0.041)	0.754(0.044)	0.753(0.027)	0.724(0.034)	0.719(0.047)	0.695(0.029)
$(\rho_a, \rho_h, \rho_n) = (0.5, 0.5, 0)$						
S.Lasso	0.53(0.072)	0.48(0.063)	0.549(0.073)	0.474(0.063)	0.515(0.094)	0.502(0.052)
CoFu	0.811(0.039)	0.769(0.026)	0.818(0.047)	0.698(0.04)	0.692(0.059)	0.701(0.028)
$(\rho_a, \rho_h, \rho_n) = (0.6, 0.2, 0.2)$						
S.Lasso	0.495(0.071)	0.482(0.082)	0.476(0.073)	0.476(0.065)	0.51(0.073)	0.504(0.075)
CoFu	0.787(0.037)	0.762(0.031)	0.747(0.031)	0.724(0.029)	0.702(0.054)	0.721(0.038)
$(\rho_a, \rho_h, \rho_n) = (0.9, 0, 0.1)$						
S.Lasso	0.529(0.062)	0.539(0.067)	0.543(0.071)	0.449(0.068)	0.519(0.082)	0.584(0.082)
CoFu	0.975(0.036)	0.968(0.036)	0.975(0.035)	0.899(0.03)	0.908(0.042)	0.908(0.035))

TABLE B5 Simulation under the LR model: summary statistics of results obtained by using V-fold (V=5) CV. Nonzero coefficients are equal to 0.5. In each cell, mean(sd).

	Individual effect		Community			
	TPR	FPR	TPR	FPR	ERMSE	PRMSE
$(\rho_a, \rho_h, \rho_n) = (0.1, 0, 0.9)$						
P.Lasso	0.176 (0.054)	0.05 (0.025)	1 (0)	1 (0)	0.154 (0.003)	4.181 (0.414)
S.Lasso	0.238 (0.07)	0.041 (0.015)	0.179 (0.118)	0.022 (0.032)	0.151 (0.002)	3.335 (0.533)
CoFu	0.625 (0.033)	0.21 (0.026)	0.579 (0.158)	0.072 (0.059)	0.144 (0.001)	0.838 (0.109)
$(\rho_a, \rho_h, \rho_n) = (0.1, 0.9, 0)$						
P.Lasso	0.449(0.06)	0.081(0.018)	1 (0)	1 (0)	0.132 (0.002)	2.937(0.167)
S.Lasso	0.291(0.074)	0.036(0.013)	0.23(0.157)	0.065(0.09)	0.135(0.003)	2.88 (0.463)
CoFu	0.732(0.024)	0.211(0.017)	0.67(0.106)	0.205(0.037)	0.122(0.003)	0.685(0.047)
$(\rho_a, \rho_h, \rho_n) = (0.2, 0.6, 0.2)$						
P.Lasso	0.496 (0.071)	0.104 (0.035)	1 (0)	1 (0)	0.133 (0.002)	2.868 (0.441)
S.Lasso	0.319 (0.069)	0.045 (0.016)	0.177 (0.162)	0.016 (0.019)	0.138 (0.002)	2.601 (0.63)
CoFu	0.77 (0.029)	0.214 (0.017)	0.685 (0.099)	0.16 (0.076)	0.122 (0.003)	0.69 (0.057)
$(\rho_a, \rho_h, \rho_n) = (0.4, 0.1, 0.5)$						
P.Lasso	0.523 (0.043)	0.111 (0.025)	1 (0)	1 (0)	0.135 (0.002)	2.750 (0.251)
S.Lasso	0.279 (0.06)	0.039 (0.014)	0.142 (0.071)	0.012 (0.026)	0.146 (0.002)	2.926 (0.554)
CoFu	0.76 (0.032)	0.206 (0.011)	0.579 (0.069)	0.104 (0.051)	0.126 (0.003)	0.703 (0.039)
$(\rho_a, \rho_h, \rho_n) = (0.5, 0.5, 0)$						
P.Lasso	0.772 (0.03)	0.12 (0.019)	1 (0)	1 (0)	0.111 (0.004)	1.945 (0.203)
S.Lasso	0.316 (0.069)	0.047 (0.012)	0.107 (0.082)	0.029 (0.051)	0.142 (0.003)	2.697 (0.557)
CoFu	0.909 (0.022)	0.168 (0.013)	0.769 (0.08)	0.138 (0.057)	0.1 (0.004)	0.669 (0.027)
$(\rho_a, \rho_h, \rho_n) = (0.6, 0.2, 0.2)$						
P.Lasso	0.729 (0.032)	0.141 (0.028)	1 (0)	1 (0)	0.118 (0.004)	1.9 (0.294)
S.Lasso	0.28 (0.11)	0.041 (0.027)	0.158 (0.154)	0.029 (0.057)	0.146 (0.003)	2.879 (1.19)
CoFu	0.87 (0.02)	0.182 (0.009)	0.797 (0.088)	0.135 (0.068)	0.107 (0.004)	0.655 (0.024)
$(\rho_a, \rho_h, \rho_n) = (0.9, 0, 0.1)$						
P.Lasso	0.915 (0.037)	0.132 (0.034)	1 (0)	1 (0)	0.105 (0.004)	1.685 (0.308)
S.Lasso	0.275 (0.077)	0.044 (0.02)	0.12 (0.098)	0.03 (0.048)	0.149 (0.002)	2.996 (0.721)
CoFu	0.925 (0.025)	0.161 (0.02)	0.925 (0.09)	0.05 (0.071)	0.095 (0.006)	0.65 (0.036)

TABLE B6 Simulation under the LR model: summary statistics of results obtained by using V-fold (V=5) CV. Nonzero coefficients are randomly drawn from $\mathcal{U}[0.2, 1]$. In each cell, mean(sd).

	Individual effect		Community			
	TPR	FPR	TPR	FPR	ERMSE	RMSPE
$(\rho_a, \rho_h, \rho_n) = (0.1, 0, 0.9)$						
P.Lasso	0.232 (0.047)	0.08 (0.029)	1 (0)	1 (0)	0.197 (0.001)	4.875 (0.599)
S.Lasso	0.296 (0.065)	0.044 (0.015)	0.121 (0.089)	0.011 (0.019)	0.185 (0.007)	3.657 (0.55)
CoFu	0.617 (0.044)	0.228 (0.02)	0.583 (0.138)	0.058 (0.021)	0.174 (0.004)	0.913 (0.112)
$(\rho_a, \rho_h, \rho_n) = (0.1, 0.9, 0)$						
P.Lasso	0.497 (0.054)	0.117 (0.027)	1 (0)	1 (0)	0.162 (0.003)	3.388 (0.4)
S.Lasso	0.413 (0.061)	0.059 (0.015)	0.057 (0.056)	0.017 (0.03)	0.159 (0.006)	2.459 (0.673)
CoFu	0.748 (0.033)	0.202 (0.011)	0.642 (0.084)	0.114 (0.031)	0.138 (0.003)	0.766 (0.028)
$(\rho_a, \rho_h, \rho_n) = (0.2, 0.6, 0.2)$						
P.Lasso	0.535 (0.052)	0.121 (0.038)	1 (0)	1 (0)	0.162 (0.004)	3.249 (0.436)
S.Lasso	0.353 (0.071)	0.051 (0.015)	0.107 (0.077)	0.025 (0.021)	0.173 (0.006)	2.97 (0.652)
CoFu	0.759 (0.023)	0.205 (0.01)	0.629 (0.121)	0.125 (0.038)	0.142 (0.006)	0.771 (0.036)
$(\rho_a, \rho_h, \rho_n) = (0.4, 0.1, 0.5)$						
P.Lasso	0.457 (0.047)	0.097 (0.023)	1 (0)	1 (0)	0.162 (0.003)	3.615 (0.401)
S.Lasso	0.374 (0.058)	0.062 (0.013)	0.013 (0.033)	0.038 (0)	0.172 (0.004)	2.69 (0.679)
CoFu	0.725 (0.037)	0.204 (0.018)	0.67 (0.096)	0.096 (0.042)	0.143 (0.003)	0.821 (0.056)
$(\rho_a, \rho_h, \rho_n) = (0.5, 0.5, 0)$						
P.Lasso	0.752 (0.05)	0.114 (0.023)	1 (0)	1 (0)	0.115 (0.004)	2.114 (0.264)
S.Lasso	0.427 (0.05)	0.064 (0.008)	0.032 (0.03)	0.016 (0.025)	0.152 (0.007)	2.234 (0.583)
CoFu	0.848 (0.019)	0.139 (0.011)	0.668 (0.087)	0.116 (0.054)	0.097 (0.003)	0.732 (0.037)
$(\rho_a, \rho_h, \rho_n) = (0.6, 0.2, 0.2)$						
P.Lasso	0.701 (0.038)	0.149 (0.028)	1 (0)	1 (0)	0.142 (0.005)	2.748 (0.307)
S.Lasso	0.376 (0.054)	0.062 (0.02)	0.049 (0.029)	0.012 (0.025)	0.178 (0.007)	2.707 (1.072)
CoFu	0.849 (0.035)	0.205 (0.016)	0.8 (0.087)	0.147 (0.064)	0.126 (0.003)	0.986 (0.036)
$(\rho_a, \rho_h, \rho_n) = (0.9, 0, 0.1)$						
P.Lasso	0.784 (0.034)	0.126 (0.027)	1 (0)	1 (0)	0.116 (0.003)	2.01 (0.267)
S.Lasso	0.336 (0.092)	0.056 (0.02)	0.066 (0.085)	0.02 (0.047)	0.181 (0.007)	3.099 (1.032)
CoFu	0.886 (0.027)	0.16 (0.019)	0.912 (0.048)	0.133 (0.126)	0.098 (0.006)	0.971 (0.027)

TABLE B7 Simulation under the LR model: summary statistics of results obtained by using V-fold (V=5) CV. Nonzero coefficients in dataset 2 are drawn from $\mathcal{U}[0.4, 0.7]$, and those specific to datasets 1 and 3 are drawn from $\mathcal{U}[0.1, 0.3]$ and $\mathcal{U}[0.8, 1]$, respectively. In each cell, mean(sd).

	Individual effect		Community			
	TPR	FPR	TPR	FPR	ERMSE	PRMSE
$(\rho_a, \rho_h, \rho_n) = (0.1, 0, 0.9)$						
P.Lasso	0.213 (0.083)	0.057 (0.04)	1 (0)	1 (0)	0.192 (0.002)	4.807 (1.013)
S.Lasso	0.253 (0.067)	0.025 (0.01)	0.207 (0.154)	0.002 (0.01)	0.181 (0.005)	3.694 (0.723)
CoFu	0.664 (0.044)	0.186 (0.027)	0.597 (0.124)	0.094 (0.004)	0.168 (0.005)	0.997 (0.187)
$(\rho_a, \rho_h, \rho_n) = (0.1, 0.9, 0)$						
P.Lasso	0.462 (0.088)	0.101 (0.032)	1 (0)	1 (0)	0.155 (0.004)	3.421 (0.466)
S.Lasso	0.38 (0.075)	0.051 (0.014)	0.114 (0.096)	0.008 (0.013)	0.149 (0.003)	2.471 (0.738)
CoFu	0.759 (0.02)	0.191 (0.013)	0.612 (0.096)	0.117 (0.041)	0.132 (0.003)	0.772 (0.049)
$(\rho_a, \rho_h, \rho_n) = (0.2, 0.6, 0.2)$						
P.Lasso	0.513 (0.05)	0.124 (0.025)	1 (0)	1 (0)	0.158 (0.003)	3.22 (0.269)
S.Lasso	0.363 (0.073)	0.055 (0.017)	0.044 (0.057)	0.009 (0.015)	0.158 (0.004)	2.43 (0.801)
CoFu	0.743 (0.026)	0.2 (0.013)	0.63 (0.064)	0.113 (0.052)	0.138 (0.004)	0.765 (0.026)
$(\rho_a, \rho_h, \rho_n) = (0.4, 0.1, 0.5)$						
P.Lasso	0.547 (0.067)	0.11 (0.028)	1 (0)	1 (0)	0.166 (0.004)	3.604 (0.339)
S.Lasso	0.378 (0.061)	0.056 (0.023)	0.056 (0.044)	0.004 (0.014)	0.172 (0.002)	2.75 (1)
CoFu	0.836 (0.026)	0.189 (0.02)	0.632 (0.075)	0.108 (0.048)	0.144 (0.003)	0.844 (0.059)
$(\rho_a, \rho_h, \rho_n) = (0.5, 0.5, 0)$						
P.Lasso	0.793 (0.038)	0.132 (0.021)	1 (0)	1 (0)	0.124 (0.004)	2.175 (0.291)
S.Lasso	0.316 (0.05)	0.05 (0.015)	0.069 (0.066)	0.017 (0.038)	0.16 (0.004)	2.996 (0.755)
CoFu	0.905 (0.019)	0.16 (0.011)	0.689 (0.117)	0.056 (0.026)	0.106 (0.004)	0.715 (0.028)
$(\rho_a, \rho_h, \rho_n) = (0.6, 0.2, 0.2)$						
P.Lasso	0.811 (0.04)	0.148 (0.042)	1 (0)	1 (0)	0.126 (0.005)	1.971 (0.468)
S.Lasso	0.306 (0.073)	0.045 (0.013)	0.078 (0.061)	0.021 (0.048)	0.167 (0.005)	3.35 (0.793)
CoFu	0.884 (0.021)	0.193 (0.018)	0.867 (0.05)	0.164 (0.117)	0.112 (0.007)	0.902 (0.032)
$(\rho_a, \rho_h, \rho_n) = (0.9, 0, 0.1)$						
P.Lasso	0.969 (0.007)	0.104 (0.016)	1 (0)	1 (0)	0.073 (0.003)	1.17 (0.107)
S.Lasso	0.33 (0.048)	0.053 (0.013)	0.067 (0.055)	0 (0)	0.166 (0.004)	1.985 (0.59)
CoFu	0.982 (0.005)	0.083 (0.025)	0.998 (0.007)	0 (0)	0.054 (0.005)	0.801 (0.041)

TABLE B8 Simulation under the Logit model: mean(sd) of AUC for effect identification. (a) Nonzero coefficients are equal to 0.5. (b) Nonzero coefficients are drawn from $\mathcal{U}[0.2, 1]$. (c) Nonzero coefficients in dataset 2 are drawn from $\mathcal{U}[0.4, 0.7]$, and those specific to datasets 1 and 3 are drawn from $\mathcal{U}[0.1, 0.3]$ and $\mathcal{U}[0.8, 1]$, respectively.

	$r = 100$			$r = 150$		
	a	b	c	a	b	c
$(\rho_a, \rho_h, \rho_n) = (0.1, 0, 0.9)$						
P.Lasso	0.561(0.02)	0.613(0.022)	0.552(0.018)	0.533(0.016)	0.582(0.017)	0.547(0.014)
S.Lasso	0.616(0.08)	0.618(0.079)	0.609(0.051)	0.541(0.084)	0.557(0.071)	0.538(0.074)
CoFu	0.632(0.027)	0.646(0.026)	0.623(0.015)	0.579(0.011)	0.623(0.028)	0.594(0.021)
$(\rho_a, \rho_h, \rho_n) = (0.1, 0.9, 0)$						
P.Lasso	0.624(0.012)	0.623(0.023)	0.619(0.038)	0.538(0.016)	0.569(0.021)	0.579(0.016)
S.Lasso	0.617(0.071)	0.587(0.08)	0.576(0.083)	0.521(0.076)	0.584(0.066)	0.581(0.067)
CoFu	0.658(0.027)	0.656(0.03)	0.652(0.026)	0.606(0.015)	0.626(0.029)	0.636(0.067)
$(\rho_a, \rho_h, \rho_n) = (0.2, 0.6, 0.2)$						
P.Lasso	0.621(0.029)	0.627(0.02)	0.622(0.024)	0.544(0.016)	0.592(0.023)	0.587(0.011)
S.Lasso	0.621(0.089)	0.584(0.089)	0.579(0.075)	0.56(0.095)	0.569(0.085)	0.553(0.098)
CoFu	0.648(0.022)	0.65(0.024)	0.646(0.035)	0.61(0.028)	0.616(0.022)	0.627(0.023)
$(\rho_a, \rho_h, \rho_n) = (0.4, 0.1, 0.5)$						
P.Lasso	0.645(0.03)	0.648(0.019)	0.631(0.031)	0.567(0.023)	0.595(0.024)	0.589(0.019)
S.Lasso	0.595(0.086)	0.591(0.082)	0.614(0.067)	0.55(0.09)	0.544(0.088)	0.548(0.066)
CoFu	0.672(0.028)	0.668(0.014)	0.652(0.034)	0.619(0.025)	0.64(0.021)	0.639(0.019)
$(\rho_a, \rho_h, \rho_n) = (0.5, 0.5, 0)$						
P.Lasso	0.679(0.042)	0.687(0.022)	0.668(0.036)	0.614(0.028)	0.611(0.026)	0.623(0.017)
S.Lasso	0.625(0.081)	0.635(0.046)	0.62(0.072)	0.57(0.08)	0.586(0.084)	0.568(0.076)
CoFu	0.72(0.025)	0.699(0.028)	0.717(0.031)	0.668(0.028)	0.656(0.018)	0.654(0.032)
$(\rho_a, \rho_h, \rho_n) = (0.6, 0.2, 0.2)$						
P.Lasso	0.672(0.022)	0.689(0.022)	0.69(0.015)	0.62(0.033)	0.647(0.027)	0.642(0.017)
S.Lasso	0.607(0.079)	0.611(0.083)	0.592(0.072)	0.533(0.07)	0.544(0.086)	0.543(0.08)
CoFu	0.678(0.021)	0.707(0.022)	0.707(0.017)	0.65(0.017)	0.676(0.026)	0.685(0.022)
$(\rho_a, \rho_h, \rho_n) = (0.9, 0, 0.1)$						
P.Lasso	0.77(0.016)	0.746(0.034)	0.774(0.022)	0.69(0.034)	0.695(0.03)	0.691(0.02)
S.Lasso	0.603(0.071)	0.613(0.071)	0.612(0.087)	0.563(0.087)	0.575(0.066)	0.558(0.072)
CoFu	0.764(0.028)	0.735(0.034)	0.778(0.033)	0.707(0.021)	0.709(0.015)	0.703(0.021)

TABLE B9 Simulation under the Logit model: mean(sd) of AUC for community differentiation. (a) Nonzero coefficients are equal to 0.5. (b) Nonzero coefficients are drawn from $\mathcal{U}[0.2, 1]$. (c) Nonzero coefficients in dataset 2 are drawn from $\mathcal{U}[0.4, 0.7]$, and those specific to datasets 1 and 3 are drawn from $\mathcal{U}[0.1, 0.3]$ and $\mathcal{U}[0.8, 1]$, respectively.

	$r = 100$			$r = 150$		
	a	b	c	a	b	c
$(\rho_a, \rho_h, \rho_n) = (0.1, 0, 0.9)$						
S.Lasso	0.591(0.077)	0.613(0.083)	0.537(0.081)	0.585(0.071)	0.606(0.108)	0.545(0.071)
CoFu	0.663(0.067)	0.659(0.075)	0.623(0.065)	0.631(0.067)	0.687(0.063)	0.639(0.058)
$(\rho_a, \rho_h, \rho_n) = (0.1, 0.9, 0.1)$						
S.Lasso	0.569(0.064)	0.605(0.09)	0.604(0.073)	0.536(0.097)	0.576(0.081)	0.539(0.138)
CoFu	0.639(0.052)	0.688(0.053)	0.661(0.056)	0.59(0.071)	0.623(0.07)	0.596(0.083)
$(\rho_a, \rho_h, \rho_n) = (0.2, 0.6, 0.2)$						
S.Lasso	0.547(0.075)	0.577(0.08)	0.586(0.084)	0.52(0.078)	0.573(0.108)	0.557(0.06)
CoFu	0.627(0.073)	0.633(0.078)	0.645(0.079)	0.608(0.08)	0.644(0.062)	0.615(0.068)
$(\rho_a, \rho_h, \rho_n) = (0.4, 0.1, 0.5)$						
S.Lasso	0.536(0.08)	0.594(0.052)	0.523(0.134)	0.529(0.177)	0.544(0.056)	0.551(0.085)
CoFu	0.627(0.073)	0.648(0.049)	0.619(0.102)	0.612(0.112)	0.626(0.064)	0.628(0.076)
$(\rho_a, \rho_h, \rho_n) = (0.5, 0.5, 0)$						
S.Lasso	0.516(0.076)	0.513(0.079)	0.512(0.063)	0.568(0.108)	0.556(0.054)	0.541(0.051)
CoFu	0.595(0.048)	0.614(0.048)	0.633(0.069)	0.602(0.086)	0.6(0.064)	0.619(0.062)
$(\rho_a, \rho_h, \rho_n) = (0.6, 0.2, 0.2)$						
S.Lasso	0.522(0.076)	0.558(0.096)	0.498(0.06)	0.537(0.081)	0.564(0.085)	0.452(0.081)
CoFu	0.652(0.043)	0.647(0.057)	0.685(0.06)	0.635(0.062)	0.644(0.069)	0.636(0.051)
$(\rho_a, \rho_h, \rho_n) = (0.9, 0, 0.1)$						
S.Lasso	0.584(0.114)	0.596(0.088)	0.538(0.146)	0.598(0.168)	0.577(0.107)	0.565(0.107)
CoFu	0.71(0.087)	0.707(0.042)	0.699(0.065)	0.704(0.121)	0.688(0.056)	0.676(0.064)

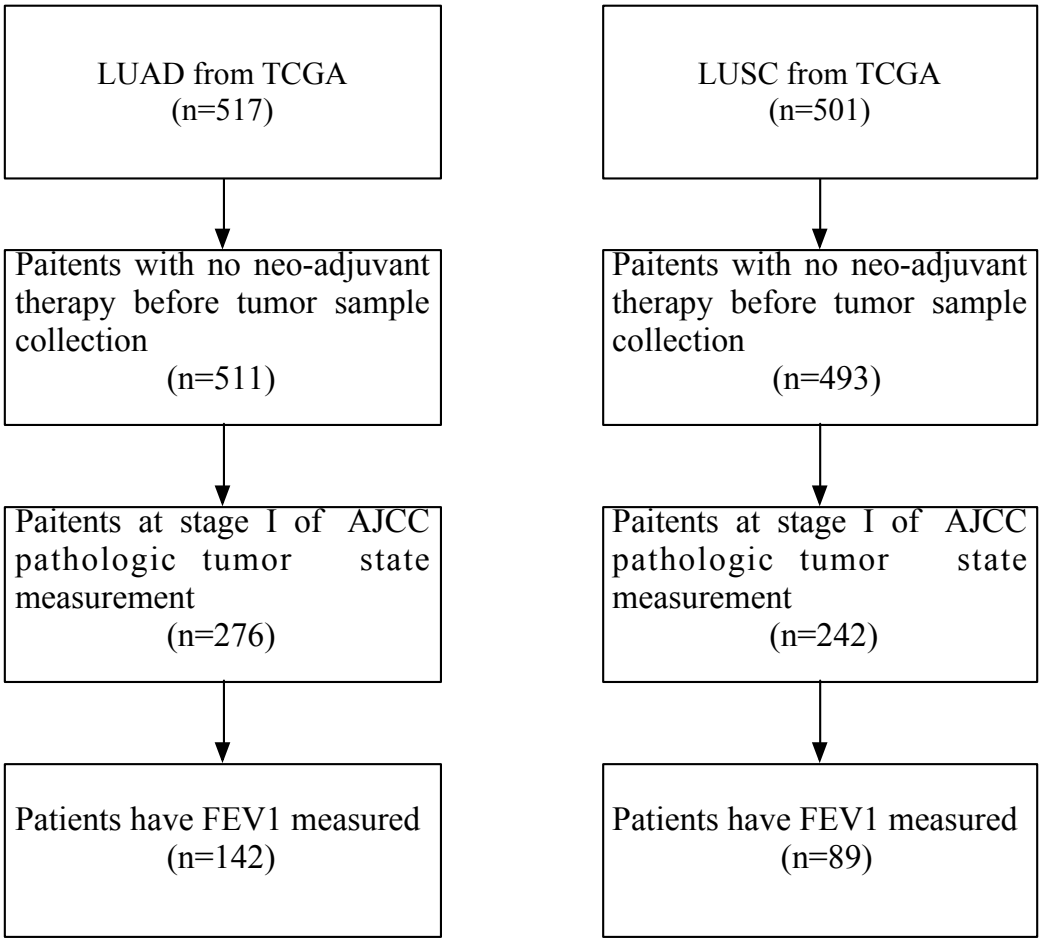

FIGURE B2 Flowcharts of sample selection for the analysis of lung cancer data (left: LUAD, right: LUSC).

TABLE B10 Simulation based on the SKCM data: mean(sd) of AUC.

community		individual effect		
S.Lasso	CoFu	P.Lasso	S.Lasso	CoFu
$(\rho_a, \rho_h, \rho_n) = (0.1, 0, 0.9)$				
0.772(0.051)	0.882(0.032)	0.576(0.011)	0.659(0.012)	0.673(0.01)
$(\rho_a, \rho_h, \rho_n) = (0.1, 0.9, 0)$				
0.758(0.081)	0.819(0.035)	0.625(0.009)	0.651(0.016)	0.706(0.012)
$(\rho_a, \rho_h, \rho_n) = (0.2, 0.6, 0.2)$				
0.514(0.062)	0.631(0.032)	0.656(0.012)	0.655(0.009)	0.679(0.008)
$(\rho_a, \rho_h, \rho_n) = (0.4, 0.1, 0.5)$				
0.6(0.048)	0.71(0.042)	0.686(0.018)	0.661(0.014)	0.723(0.011)
$(\rho_a, \rho_h, \rho_n) = (0.5, 0.5, 0)$				
0.563(0.055)	0.676(0.038)	0.665(0.009)	0.658(0.011)	0.724(0.006)
$(\rho_a, \rho_h, \rho_n) = (0.6, 0.2, 0.2)$				
0.633(0.046)	0.712(0.043)	0.706(0.013)	0.659(0.008)	0.704(0.014)
$(\rho_a, \rho_h, \rho_n) = (0.9, 0, 0.1)$				
0.566(0.066)	0.829(0.046)	0.752(0.015)	0.66(0.012)	0.749(0.01)

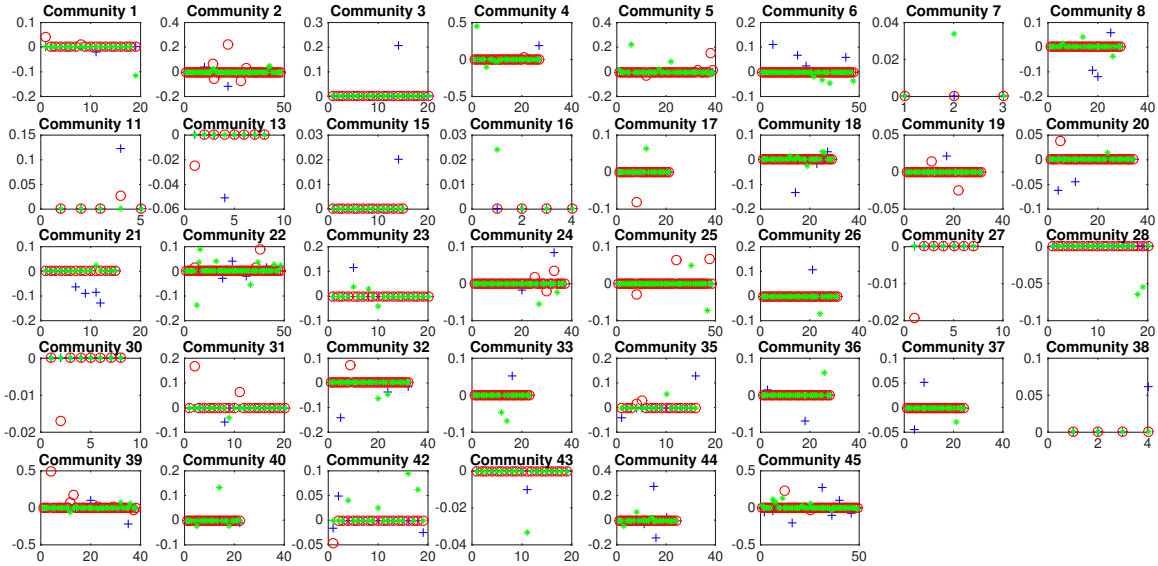

FIGURE B3 Analysis of the TCGA SKCM data using S.Lasso. Blue crosses correspond to stage I, red circles to stage II, and green filled circles to stages III and IV.

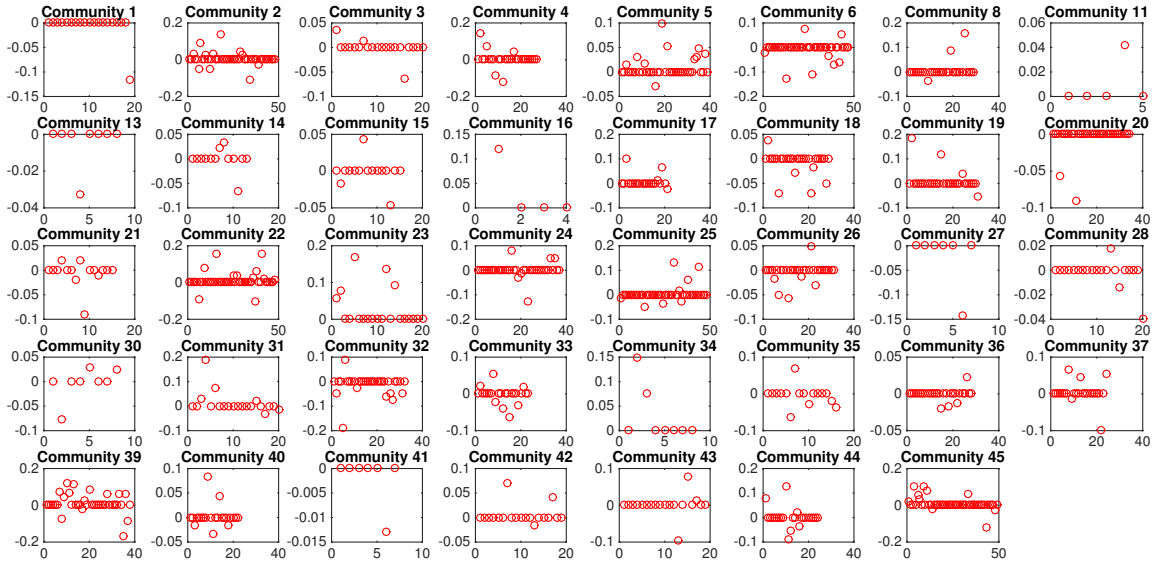

FIGURE B4 Analysis of the TCGA SKCM data using P.Lasso.

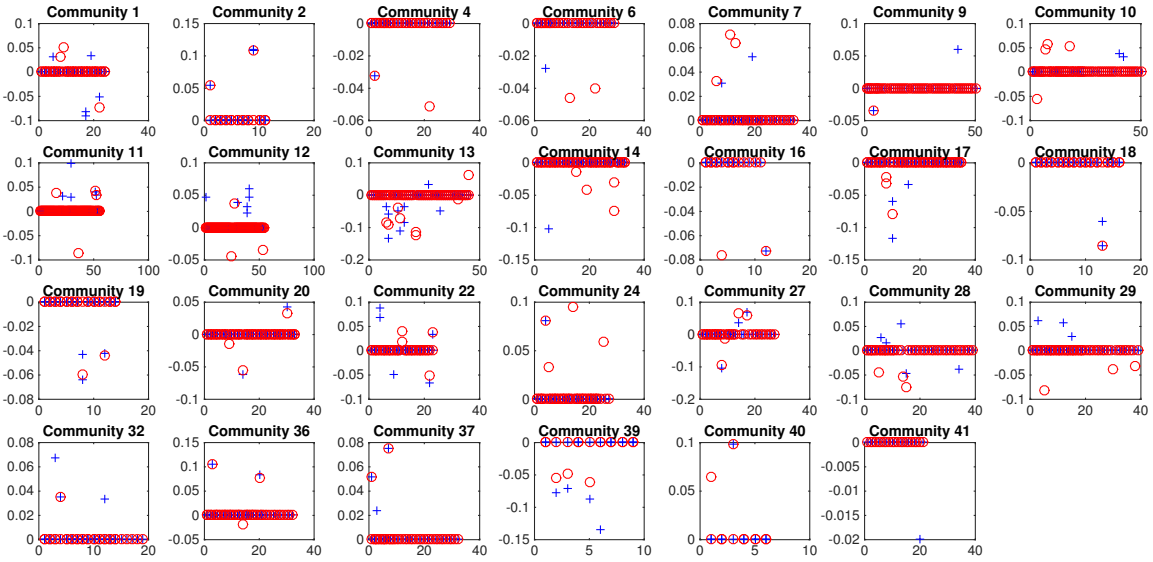

FIGURE B5 Analysis of the TCGA lung cancer data using S.Lasso. Blue crosses correspond to LUAD and red circles to LUSC.

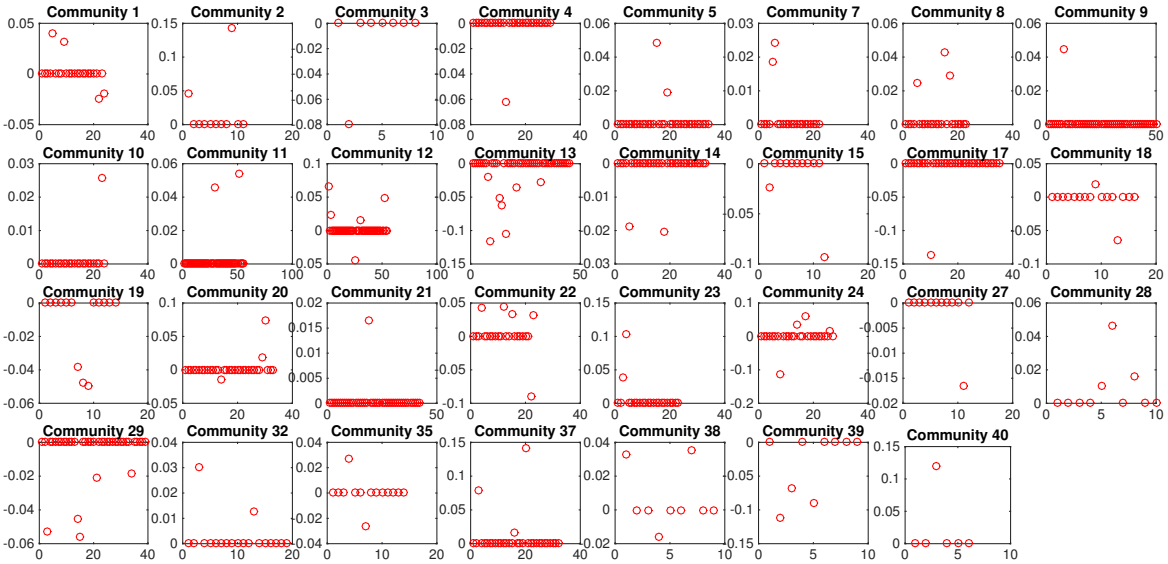

FIGURE B6 Analysis of the TCGA lung cancer data using P.Lasso.

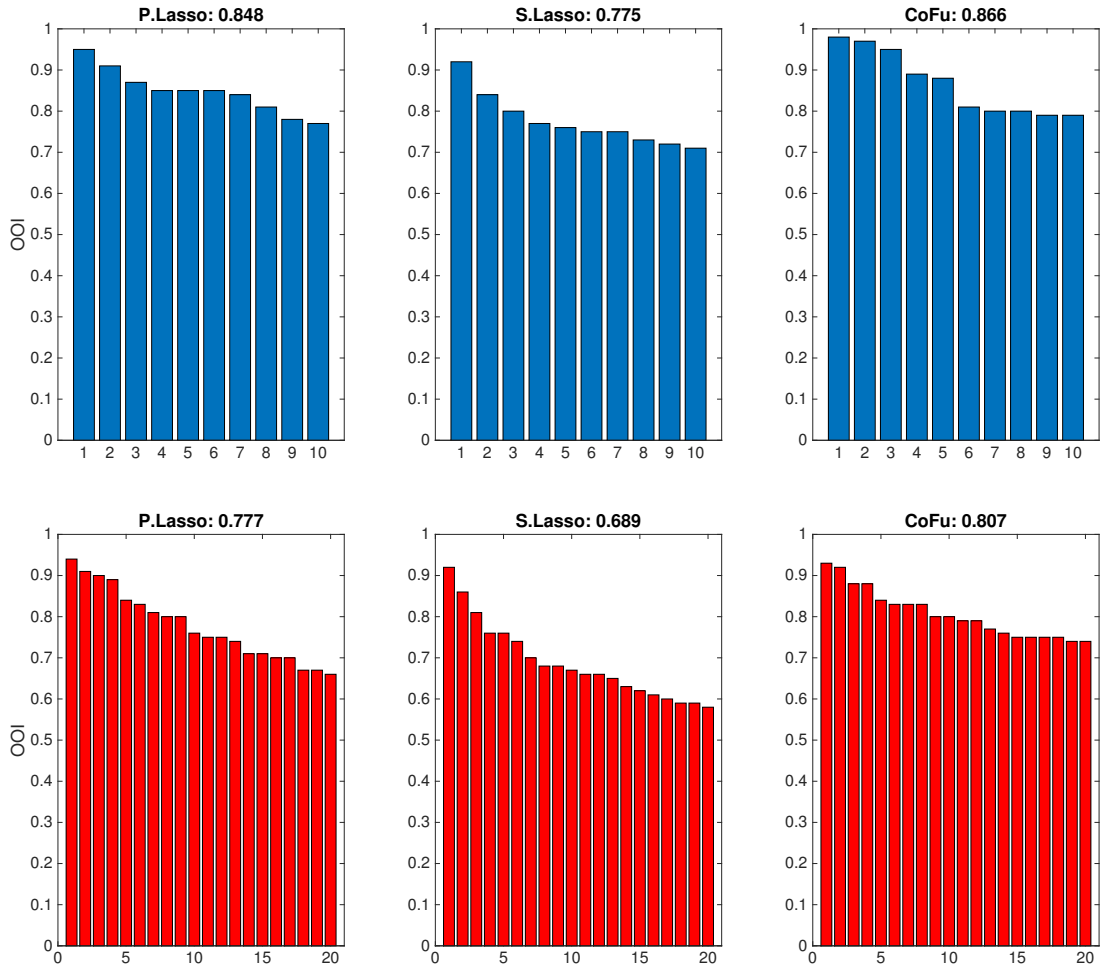

FIGURE B7 OOI in data analysis. Top: SKCM. Bottom: lung cancer data.

C COMPUTATION OF AUC IN SIMULATION

For the proposed CoFu method, we fit the model under 20×6 $[\lambda_1, \lambda_2]$ values, where $\lambda_2 \in (0, 0.001, 0.01, 0.1, 1, 10)$. Let $\lambda_1^{\max}$ be the minimal λ_1 such that all regression coefficients shrink to zero. We generate a sequence of λ_1 with length 20 that are equally-spaced in logarithm from $0.001\lambda_1^{\max}$ to $\lambda_1^{\max}$. We calculate the TPR and FPR for each $[\lambda_1, \lambda_2]$, and extract the envelope, which is the ROC curve. From this, we obtain the AUC. For the two alternatives, the calculation of the ROC and AUC can be conducted in a similar manner.

D EXAMINATION OF CENSORING IN THE ANALYSIS OF LUNG CANCER DATA

We take a closer look at the vital status in the analysis of the lung cancer data. The contingency table is shown below. For the LUAD data, stage I has a higher percentage of censoring, whereas for the LUSC data, there is no significant association between vital status and stage. In our analysis, we focus on the continuous FEV1 variable, which is not subject to censoring. All subjects with FEV1 measurements are used in analysis, regardless of their vital status. Censoring is expected to play a more direct role in the analysis of prognosis.

TABLE D11 Contingency table for LUAD and LUSC

	LUAD			LUSC		
	Stage I	Stage II	Stage III	Stage I	Stage II	Stage III
Deceased	68	53	61	99	63	48
Alive	208	68	45	143	94	42