SUPPLEMENTARY INFORMATION

S1: Derivation of Equation 8:

The energy deposited into a medium (E) by an electron is the sum of its electron linear energy transfer ($\mu(E)=dE/dx$) integrated along its path:

$$E = \int \mu(E_x)dx$$

In this expression, the particle is emitted with initial energy E, and its energy decreases as it deposits energy in the medium: E_x represents the energy of the particle after having traveled a distance x. For a mixture of tissue and nanoparticles (NP):

$$E_{NP} = \int_{\mathrm{NP}} \mu_{\mathrm{NP}}(E_x)dx$$

$$E_{\mathrm{tissue}} = \int_{\mathrm{tissue}} \mu_{\mathrm{tissue}}(E_x)dx$$

It is important to note that the integral is taken only over the portion of the electron path that overlaps with the material of interest (tissue or NP).

Because E_x decreases along the integration path, it is not straightforward to calculate this expression. Instead, we consider an upper bound for E_{NP} by noting the following inequality:

$$\frac{\mu_{\mathrm{NP}}(E)}{\mu_{\mathrm{tissue}}(E)} < \frac{\mu_{\mathrm{NP}}(E_{\mathrm{max}})}{\mu_{\mathrm{tissue}}(E_{\mathrm{max}})},$$

where $E<E_{max}$. This relationship holds true for virtually all inorganic scintillator materials. An example is provided in **Figure S1** for the LaF_3 scintillator. It can be seen that the ratio of the stopping power of the two materials (tissue and scintillator) increase monotonically. Furthermore, the ratio vsries little over 4 orders of magnitude (especially for E>10 keV). Therefore, the electron stopping power of the nanoscintillator can approximated by the stopping power of tissue times a constant term:

$$\mu_{\mathrm{NP}}(E) < \frac{\mu_{\mathrm{NP}}(E_{\mathrm{max}})}{\mu_{\mathrm{tissue}}(E_{\mathrm{max}})} \times \mu_{\mathrm{tissue}}(E)$$

This approximation provides an upper bound on the stopping power of the nanoparticle; therefore, the energy transfer computed in this manner is in fact a rather optimistic estimate. It follows from this relationship that:

$$E_{\mathrm{NP}} \leq \frac{\mu_{\mathrm{NP}}(E_{\mathrm{max}})}{\mu_{\mathrm{tissue}}(E_{\mathrm{max}})} \int_{\mathrm{NP}} \mu_{\mathrm{tissue}}(E_x)dx.$$

Furthermore, we can establish a relationship between the integration of μ_{tissue} over the portion of the path that corresponds to the volume occupied by NP, and the same integration for the volume occupied by tissue:

$$\int_{\mathrm{NP}} \mu_{\mathrm{tissue}}(E_x)dx = \frac{v_{\mathrm{NP}}}{v_{\mathrm{tissue}}} \int_{\mathrm{tissue}} \mu_{\mathrm{tissue}}(E_x)dx$$

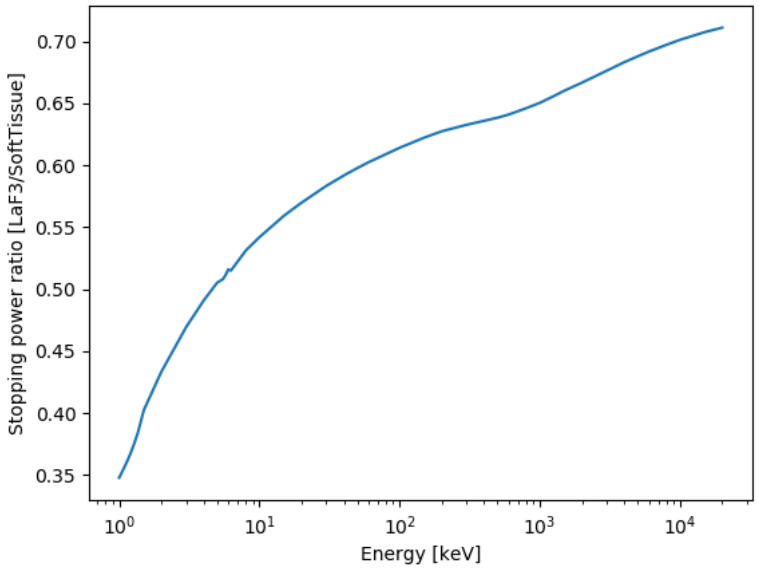

Figure S1. Plot showing the ratio of the stopping power of LaF_3 to the stopping power of tissue, as a function of energy. The curve suggests a relationship between the two quantities.

where v_{NP} and v_{tissue} correspond to the fractional volume of NP and tissue. (By definition, $v_{\mathrm{NP}} + v_{\mathrm{tissue}} = 1$.) This relationship holds because the path length of the electron in NP and tissue is directly proportional to the fractional volume of NP and tissue, respectively. From the last two equations, we conclude that

$$E_{\mathrm{NP}} \leq \frac{v_{\mathrm{NP}}}{v_{\mathrm{tissue}}} \frac{\mu_{\mathrm{NP}}(E_{\mathrm{max}})}{\mu_{\mathrm{tissue}}(E_{\mathrm{max}})} \int_{\mathrm{tissue}} \mu_{\mathrm{tissue}}(E_x)dx$$

and, therefore,

$$E_{\mathrm{NP}} \leq \frac{v_{\mathrm{NP}}}{v_{\mathrm{tissue}}} \frac{\mu_{\mathrm{NP}}(E_{\mathrm{max}})}{\mu_{\mathrm{tissue}}(E_{\mathrm{max}})} E_{\mathrm{tissue}}.$$

Since cross-sections are typically provided as mass-attenuation coefficient (μ/ρ – unit: MeV·cm^2/g), we rewrite the previous equation as

$$E_{\mathrm{NP}} \leq \frac{v_{\mathrm{NP}}}{v_{\mathrm{tissue}}} \frac{\rho_{\mathrm{NP}}}{\rho_{\mathrm{tissue}}} \frac{\left(\mu(E_{\mathrm{max}})/\rho\right)_{\mathrm{NP}}}{\left(\mu(E_{\mathrm{max}})/\rho\right)_{\mathrm{tissue}}} E_{\mathrm{tissue}}.$$

We can further simplify this equation by noting that

$$v_{\mathrm{NP}} \times \rho_{\mathrm{NP}} = \frac{V_{\mathrm{NP}} \times \rho_{\mathrm{NP}}}{V} = \frac{m_{\mathrm{NP}}}{V} = C_{\mathrm{NP}},$$

the mass concentration of nanoparticle in the tissue (unit: g/cm^3). The relationship between the dose deposited in the NP and tissue can finally be expressed as:

$$E_{\mathrm{NP}} \leq \frac{C_{\mathrm{NP}}}{\rho_{\mathrm{tissue}}} \frac{(\mu(E_{\mathrm{max}})/\rho)_{\mathrm{NP}}}{(\mu(E_{\mathrm{max}})/\rho)_{\mathrm{tissue}}} E_{\mathrm{tissue}}$$

which was used to derive Equation 9.

To verify this formula, we consider the Monte-Carlo simulations of Bulin et al (2015), who computed the energy deposited in a nanoparticle following the interaction of various energy X-rays. We consider the following case, taken from Table 3:

- Gd_2O_3, which has a density of 7.41 g/cm^3
- Volume fraction of 0.002, equivalent to a mass-concentration C_{NP} = 14.8 mg/cm^3
- Excitation using E_{max} = 500 keV X-ray
- $(\mu/\rho)_{\mathrm{Gd2O3}} = 1.25$ MeV·cm^2/g at E_{max} = 500 keV
- $(\mu/\rho)_{\mathrm{H2O}} = 2.04$ MeV·cm^2/g at E_{max} = 500 keV
- Deposition in tissue of E_{tissue} = 173.85 keV
- Deposition in the nanoparticle (10 nm diameter) of E_{NP} = 1.10 keV

Using the analytical formula provided above,

$$\frac{C_{\mathrm{NP}}}{\rho_{\mathrm{tissue}}} \frac{(\mu(E_{\mathrm{max}})/\rho)_{\mathrm{NP}}}{(\mu(E_{\mathrm{max}})/\rho)_{\mathrm{tissue}}} E_{\mathrm{tissue}} = 1.57 \text{ keV} > E_{\mathrm{NP}} = 1.10 \text{ keV},$$

we verify that the energy deposition in the nanoparticle, as computed by Monte-Carlo simulation, is consistent with the formula derived from electron cross-sections. We note that the formula is also valid for the other conditions considered by Bulin et al (2015).